Fonctions de Green à T=0

évaluation de $\langle T\hat{\psi}_{\alpha_1}(t_1)\psi^\dagger_{\alpha_2}(t_2)\ldots\hat{\psi}_{\alpha_{2k-1}}(t_{2k-1})\hat{\psi}^\dagger_{\alpha_{2k}}(t_{2k})\rangle$?

- k opérateurs d'annihilation et k opérateurs de création
- sans interactions !!

théorème de Wick : $\langle T\hat{\psi}_{\alpha_1}(t_1)\psi^\dagger_{\alpha_2}(t_2)\ldots\hat{\psi}_{\alpha_{2k-1}}(t_{2k-1})\hat{\psi}^\dagger_{\alpha_{2k}}(t_{2k})\rangle$

$$= \sum_P (\pm 1)^P \prod_{l=1}^{k} \langle T\hat{\psi}_{\alpha_{2l-1}}(t_{2l-1})\psi^\dagger_{\alpha_{P_{2l}}}(t_{P_{2l}})\rangle$$

$$= i^k \sum_P (\pm 1)^P \prod_{l=1}^{k} G^0_{\alpha_{2l-1}\alpha_{P_{2l}}}(t_{2l-1}, t_{P_{2l}})$$

P : toutes les permutations des k opérateurs de création

Fonctions de Green à T=0

exemple : $k = 2$

$$
\begin{aligned}
& \langle T\hat{\psi}_{\alpha_1}(t_1)\psi^\dagger_{\alpha_2}(t_2)\hat{\psi}_{\alpha_3}(t_3)\hat{\psi}^\dagger_{\alpha_4}(t_4)\rangle \\
= \;& \langle T\hat{\psi}_{\alpha_1}(t_1)\psi^\dagger_{\alpha_2}(t_2)\rangle\langle T\hat{\psi}_{\alpha_3}(t_3)\hat{\psi}^\dagger_{\alpha_4}(t_4)\rangle \\
& -\langle T\hat{\psi}_{\alpha_1}(t_1)\psi^\dagger_{\alpha_4}(t_4)\rangle\langle T\hat{\psi}_{\alpha_3}(t_3)\hat{\psi}^\dagger_{\alpha_2}(t_2)\rangle \\
= \;& -G^0_{\alpha_1\alpha_2}(t_1,t_2)G^0_{\alpha_3\alpha_4}(t_3,t_4)+G^0_{\alpha_1\alpha_4}(t_1,t_4)G^0_{\alpha_3\alpha_2}(t_3,t_2)
\end{aligned}
$$

$\alpha \to \vec{k}$:

$$
\begin{aligned}
& \langle T\hat{\psi}_{\vec{k}_1}(t_1)\psi^\dagger_{\vec{k}_2}(t_2)\hat{\psi}_{\vec{k}_3}(t_3)\hat{\psi}^\dagger_{\vec{k}_4}(t_4)\rangle \\
= \;& -\delta_{\vec{k}_1,\vec{k}_2}\delta_{\vec{k}_3,\vec{k}_4}G^0(\vec{k}_1;t_1,t_2)G^0(\vec{k}_3;t_3,t_4) \\
& +\delta_{\vec{k}_1,\vec{k}_4}\delta_{\vec{k}_3,\vec{k}_2}G^0(\vec{k}_1;t_1,t_4)G^0(\vec{k}_3;t_3,t_2)
\end{aligned}
$$