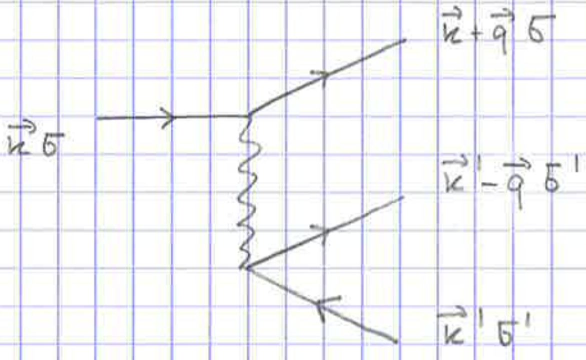
## interaction :

$$\hat{V} = \frac{1}{2} \sum_{\vec{k}, \vec{k}', \vec{q}; \sigma, \sigma'} V(q)\, c^{+}_{\vec{k}+\vec{q}\,\sigma}\, c^{+}_{\vec{k}'-\vec{q}\,\sigma'}\, c_{\vec{k}'\sigma'}\, c_{\vec{k}\sigma}$$

avec $V(q) = \dfrac{e^2}{\varepsilon_0} \dfrac{1}{q^2 + \lambda^2}$ (Yukawa : $V(r) \sim \frac{1}{r} e^{-\lambda r}$)

[ Coulomb : $\lambda \to 0$ ]

$1/\tau_{\vec{k}\sigma}$ ?




règle d'or de Fermi :

$$1/\tau_{\vec{k}\sigma} = \frac{2\pi}{\hbar} \frac{1}{V^2} \sum_{\vec{k}', \vec{q}; \sigma'} |V(q)|^2 f(\varepsilon_{\vec{k}'})$$
$$\times \left(1 - f(\varepsilon_{\vec{k}'-\vec{q}})\right)\left(1 - f(\varepsilon_{\vec{k}+\vec{q}})\right) \delta\left(\varepsilon_{\vec{k}} + \varepsilon_{\vec{k}'} - \varepsilon_{\vec{k}'-\vec{q}} - \varepsilon_{\vec{k}+\vec{q}}\right)$$

avec $\varepsilon_{\vec{k}} = \dfrac{\hbar^2 k^2}{2m} - \mu$ et $f(\varepsilon_{\vec{k}}) = \dfrac{1}{1 + e^{\beta \varepsilon_{\vec{k}}}}$

on va évaluer $1/\tau_{\vec{k}\sigma}$ à $T = 0$
et dans la limite continue ...

$$1/\tau_{\vec{k}\sigma} = \frac{2\pi}{\hbar} \underset{\substack{\uparrow \\ \text{spin}}}{2} \int \frac{d^3 q}{(2\pi)^3} |V(q)|^2\, \theta(\varepsilon_{\vec{k}+\vec{q}})$$
$$\times \int \frac{d^3 k'}{(2\pi)^3}\, \theta(-\varepsilon_{\vec{k}'})\, \theta(\varepsilon_{\vec{k}'-\vec{q}})\, \delta\left(\varepsilon_{\vec{k}} + \varepsilon_{\vec{k}'} - \varepsilon_{\vec{k}'-\vec{q}} - \varepsilon_{\vec{k}+\vec{q}}\right)$$

pour simplifier, on pose $\vec{k}' = k_{\parallel} \hat{q} + \vec{k}_{\perp}$

projection de $\vec{k}'$ sur $\vec{q}$ — composante de $\vec{k}'$ perpendiculaire à $\vec{q}$

$\theta(-\varepsilon_{\vec{k}'}) \curvearrowright k_{\parallel}^2 + k_{\perp}^2 < k_F^2$

$\theta(\varepsilon_{\vec{k}'-\vec{q}}) \curvearrowright (k_{\parallel} - q)^2 + k_{\perp}^2 > k_F^2$

$$\delta\left(\varepsilon_{\vec{k}} + \varepsilon_{\vec{k}'} - \varepsilon_{\vec{k}'-\vec{q}} - \varepsilon_{\vec{k}+\vec{q}}\right)$$

$$= \frac{2m}{\hbar^2}\,\delta\left(k_{\parallel}^2 + k_{\perp}^2 - \left((k_{\parallel}-q)^2 + k_{\perp}^2\right) + \frac{2m}{\hbar^2}\left(\varepsilon_{\vec{k}} - \varepsilon_{\vec{k}+\vec{q}}\right)\right)$$

$$= \frac{2m}{\hbar^2}\,\delta\left(2k_{\parallel}q - q^2 + \frac{2m}{\hbar^2}\left(\varepsilon_{\vec{k}} - \varepsilon_{\vec{k}+\vec{q}}\right)\right)$$

$$\int \frac{d^3k'}{(2\pi)^3} \longrightarrow \frac{1}{(2\pi)^3}\int dk_{\parallel}\; 2\pi \int k_{\perp}\, dk_{\perp}$$

donc

$$\int \frac{d^3k'}{(2\pi)^3}\,\theta(-\varepsilon_{\vec{k}'})\,\theta(\varepsilon_{\vec{k}'-\vec{q}})\,\delta\left(\varepsilon_{\vec{k}} + \varepsilon_{\vec{k}'} - \varepsilon_{\vec{k}'-\vec{q}} - \varepsilon_{\vec{k}+\vec{q}}\right)$$

$$= \frac{1}{(2\pi)^2}\int dk_{\parallel}\,\frac{2m}{\hbar^2}\,\delta\left(2k_{\parallel}q - q^2 + \frac{2m}{\hbar^2}\left(\varepsilon_{\vec{k}} - \varepsilon_{\vec{k}+\vec{q}}\right)\right)$$

$$\times \int_{\sqrt{k_F^2-(k_{\parallel}-q)^2}}^{\sqrt{k_F^2 - k_{\parallel}^2}} k_{\perp}\, dk_{\perp}$$

$$= \frac{1}{(2\pi)^2}\int dk_{\parallel}\,\frac{2m}{\hbar^2}\,\frac{1}{2q}\,\delta\left(k_{\parallel} - \frac{q}{2} + \frac{m}{\hbar^2 q}\left(\varepsilon_{\vec{k}} - \varepsilon_{\vec{k}+\vec{q}}\right)\right)$$

$$\times \frac{1}{2}\underbrace{\left[k_F^2 - k_{\parallel}^2 - \left(k_F^2 - (k_{\parallel}-q)^2\right)\right]}_{-2k_{\parallel}q + q^2 \,=\, -2q\left(k_{\parallel} - \frac{q}{2}\right)}$$

$$= \frac{1}{(2\pi)^2}\,\frac{m}{\hbar^2} \times \frac{m}{\hbar^2 q}\left(\varepsilon_{\vec{k}} - \varepsilon_{\vec{k}+\vec{q}}\right)$$

$$= \frac{1}{(2\pi)^2}\,\frac{m}{\hbar^2}\,\frac{1}{2q}\left(k^2 - (\vec{k}+\vec{q})^2\right) = -\frac{1}{(2\pi)^2}\,\frac{m}{2\hbar^2}\left(2k\cos\theta + q\right)$$

où $\theta$ est l'angle entre $\vec{k}$ et $\vec{q}$

donc

$$1/\tau_{\vec{k}\sigma} = \frac{4\pi}{\hbar}\,\frac{1}{(2\pi)^3}\int q^2\,dq\,|V(q)|^2$$

$$\times \int \sin\theta\, d\theta\;\theta\left(\varepsilon_{\vec{k}+\vec{q}}\right)\left[-\frac{1}{(2\pi)^2}\,\frac{m}{2\hbar^2}\left(2k\cos\theta + q\right)\right]$$

on note $(\vec{k}+\vec{q})^2 < k^2$

$(\vec{k}+\vec{q})^2 > k_F^2$

$$\curvearrowright \quad k_F^2 < k^2 + 2kq\cos\theta + q^2 < k^2$$

$$\frac{1}{2kq}\left(k_F^2 - k^2 - q^2\right) < \cos\theta < -\frac{q}{2k}$$

donc

$$\int \sin\theta \, d\theta \; \theta(\varepsilon_{\vec{k}+\vec{q}}) \left[ -\frac{1}{(2\pi)^2} \frac{m}{2\hbar^2} (2k\cos\theta + q) \right]$$

$$= -\frac{1}{(2\pi)^2} \frac{m}{2\hbar^2} \int_{\frac{1}{2kq}(k_F^2-k^2-q^2)}^{-q/(2k)} d\cos\theta \; (2k\cos\theta + q)$$

$$= \frac{1}{(2\pi)^2} \frac{m}{2\hbar^2} \Big[ k\cos^2\theta + q\cos\theta \Big]_{-q/(2k)}^{\frac{1}{2kq}(k_F^2-k^2-q^2)}$$

$$= \frac{1}{(2\pi)^2} \frac{m}{2\hbar^2} \left\{ \frac{1}{4kq^2}(k_F^2-k^2-q^2)^2 - \frac{q^2}{4k} + \frac{1}{2k}(k_F^2-k^2-q^2) + \frac{q^2}{2k} \right\}$$

$$= \frac{1}{(2\pi)^2} \frac{m}{2\hbar^2} \frac{1}{4kq^2} (k_F^2-k^2)^2$$

$$= \frac{1}{(2\pi)^2} \frac{m}{2\hbar^2} \frac{1}{4kq^2} \frac{4m^2}{\hbar^4} \varepsilon_{\vec{k}}^2$$

et

$$1/\tau_{\vec{k}\sigma} = \frac{4\pi}{\hbar} \frac{1}{(2\pi)^3} \frac{1}{(2\pi)^2} \frac{m}{2\hbar^2} \frac{1}{4k} \frac{m^2}{\hbar^4} \varepsilon_{\vec{k}}^2 \times \int dq \, |V(q)|^2$$

$$= \frac{m^3}{2(2\pi)^5 \hbar^7 k} \varepsilon_{\vec{k}}^2 \int dq \, |V(q)|^2$$

si $\lambda \neq 0$, $\int dq \, |V(q)|^2 < \infty$

$$\Rightarrow 1/\tau_{\vec{k}\sigma} = \text{cste} \times (E_{\vec{k}} - E_F)^2$$