New concepts and new devices for microelectronics

Maud Vinet, Marc Sanquer

2017

Quantum transport and Coulomb blockade in silicon nanodevices

marc.sanquer@cea.fr

CEA-DSM-INAC-PHELIQS

LaTEQS (Laboratoire de Transport Electronique Quantique et Supraconductivité)

http://inac.cea.fr/Pisp/58/marc.sanquer.html
http://inac.cea.fr/Cours/marc.sanquer/

Lundi 11 Avril 2017 PHELMA Maud Vinet

8h30-12h30

Mercredi 12 Avril 2017 Z403(V) PHELMA Marc Sanquer

14h00-15h00	**Scaling CMOS; energy-time consideration; Shrinking the MOSFET: single dopant variability ; Single dopant –single electron transport** ***(slides 3-28)***
15h00-15h10	**Pause**
15h10-16h00	**SET-FET convergence, MOS-SET @300K** **What is the physics involved in single dopant transport?** **Resonant Tunnelling;** ***(slides 29-68)***
16h00-16h10	**Pause**
16h10-17h00	**Coulomb blockade (Orthodox theory)** ***(slides 69-92)***

Vendredi 14 Avril 2017 Z403(V) PHELMA M. Sanquer

14h00-15h00	**Coulomb blockade (non-orthodox), quantum confinement effects** ***(slides 93-109)***
15h00-15h10	**Pause**
15h10-16h00	**Coupled dopants, Coupled atom transistor, Double dots, Pumps** ***(slides 110-132)***
16h00-16h10	**Pause**
16h10-17h00	**CMOS qubits** ***(slides 133-150)***

Down-Scaling CMOS:
energy-time considerations

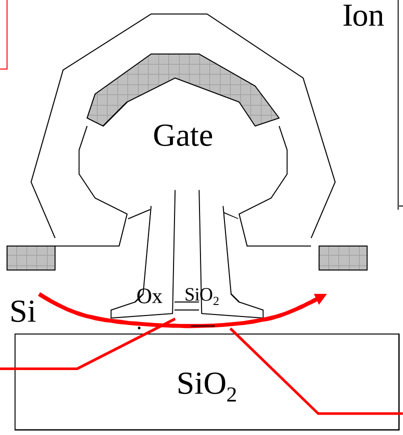
ultimate MOSFET

Ion density* α,
delay time/ α

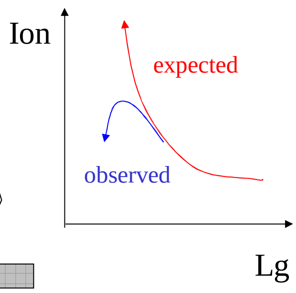
⇩

Lgate/α
Wgate/α
Tox/α
Taccess/α..

Gate length
6nm, Tsi=7nm

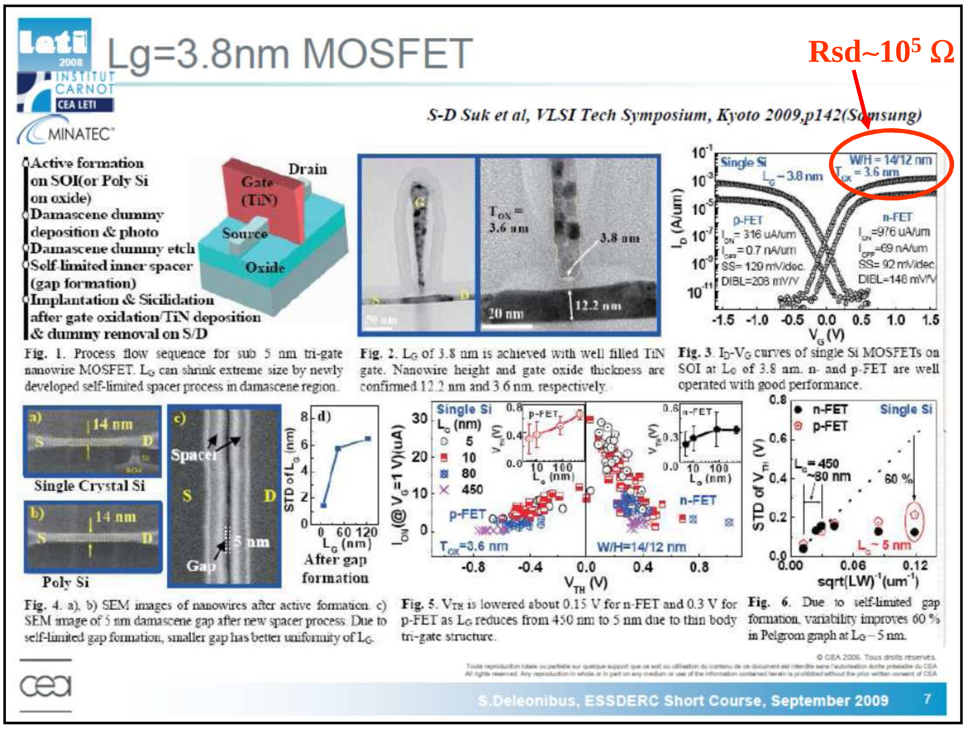


thin access



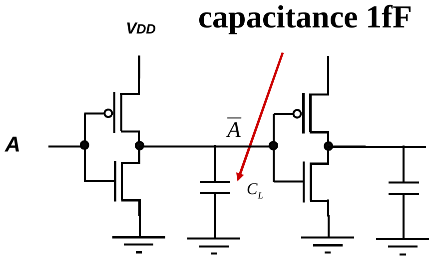
B. Doris et al. IBM IEDM 2002

Ion saturation: ballisticity ? Ion decrease = parasitic access resistance ?



(CMOS inverter)

Typical interconnect capacitance 1fF



Switching energy (intrinsic device)	80 aJ
Dynamic Power (intrinsic device) (Vdd*Ion)	0.7(mW/µm)
Static Power (intrinsic device)	5.3pW
Drive voltage Vdd	0.85V
Ion	800 (µ A/ µ m)
Ioff	6.3 pA/switch

Charging C_L at constant Voltage V_{DD} dissipated energy:

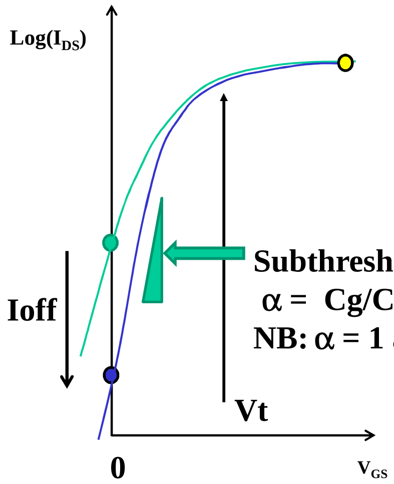
$E = ½ CV^2 = 0.5 * 10^{-15}$ (F)* $0.85(V)^2 = 0.36\ 10^{-15}$ (J)

Ne=CV ➔ N≈ 5000 carriers

N ➔ N/a²

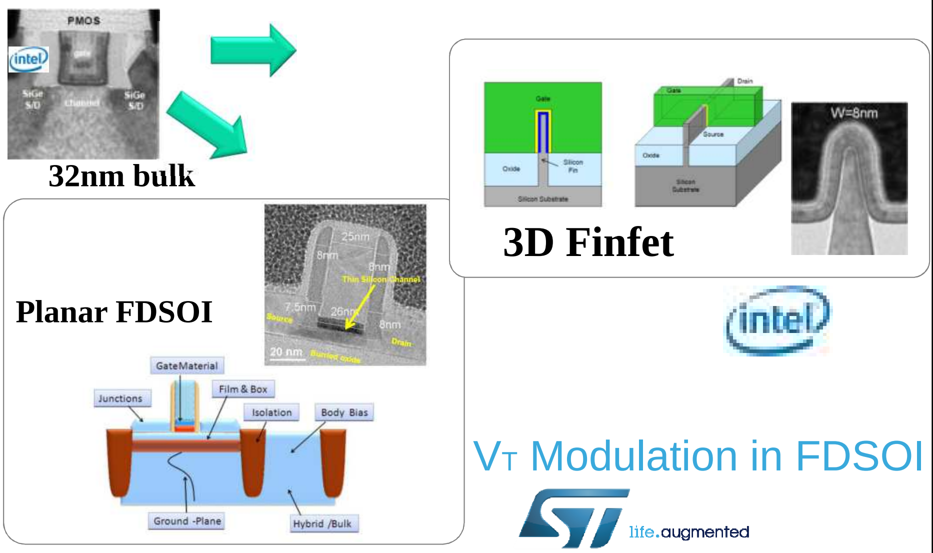
Reduction of V_D

Low sub-V_T slope (i.e. electostatic control) has become the key parameter for performance, with the need for V_d lowering.

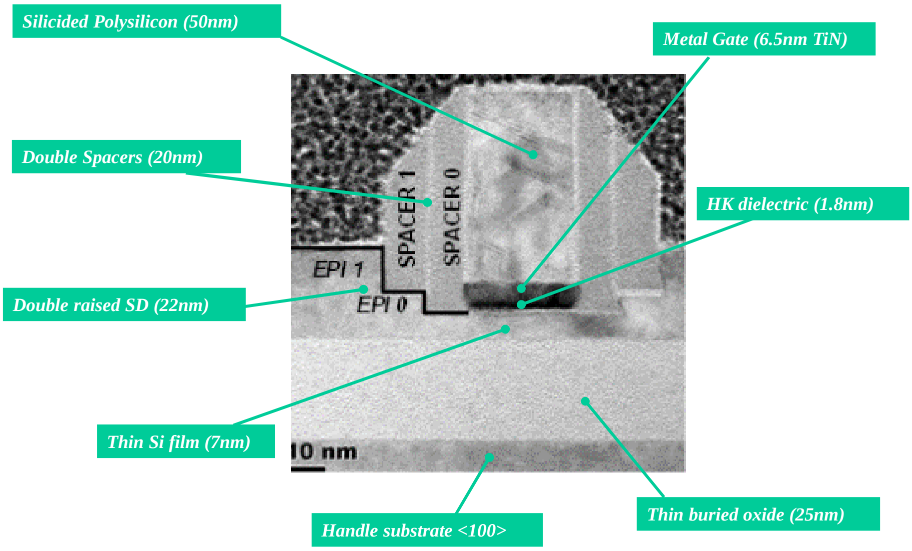


Subthreshold slope= $\ln(10)k_BT/e \times 1/\alpha$
$\alpha = Cg/C_\Sigma$
NB: $\alpha = 1$ and T=300K SSS≈ 60mV/dec

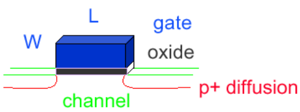
Recent technology progresses in CMOS: transition to thin body devices



28FDSOI Transistor at a Glance (ST)



Shrinking down the MOSFET:



L	$\Rightarrow$	L/α
W	$\Rightarrow$	W/α
d_{oxide}	$\Rightarrow$	d_{oxide} / α
V	$\Rightarrow$	V/ α

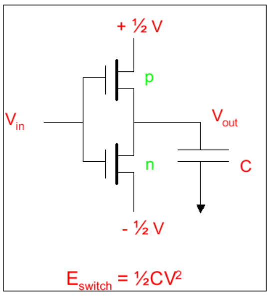
gate capacitance	$C = \varepsilon W L / d_{oxide}$	$\propto 1/\alpha$
switching energy	$E = \frac{1}{2} C V^2$	$\propto 1/\alpha^3$
switching time	$\tau \propto L^2/V$	$\propto 1/\alpha$
switching power	$P = E/\tau$	$\propto 1/\alpha^2$

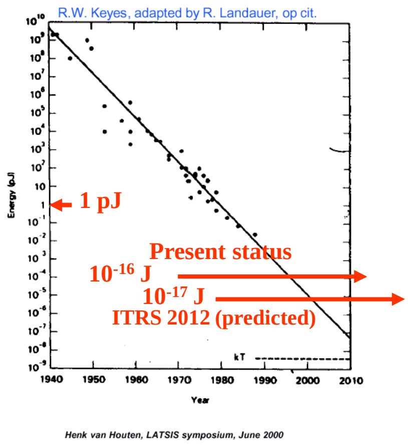
Power per unit area remains constant

Henk van Houten, LATSIS symposium, June 2000

Scaling of switching energy

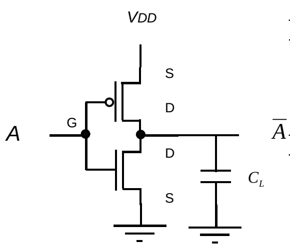
(Landauer '88)





Scaling : $E = \frac{1}{2} C V^2 \propto 1/\alpha^3$

in10 years $\alpha \approx 5$ (Moore's law)



Today (energy switching, device level)
a few 10^{-16} J (Vdd ~ 0.8V)

τ=Cg Vdd / I = 1 ps (low power)
Cg (C_L) ~ 10^{-15} F
R ~ 10^3 Ω

Energy-time product ~10^{-27} Js >> h = 10^{-34} Js

$\frac{1}{2}$ C Vdd^2 ~ $1/\alpha^3$

Reduce Vdd
But problem with the standby power
(Idd @ Vg=0V)

PHYSICAL LIMITATIONS / energy considerations

physical limitations for digital calculation:

switching energy larger than temperature and should obey Heisenberg uncertainty principle

$$E_{sw} = 1/2 \times CV^2 > kT \quad \text{et} \quad E_{sw} \times \tau > h/2\pi$$

10nm/10nm ballistic FET: Gate capacitance (Tsi=1nm): 10^{-18} F

Channel resistance 30kΩ

➔Ballistic Transit time =10nm/v_F~10^{-13}s

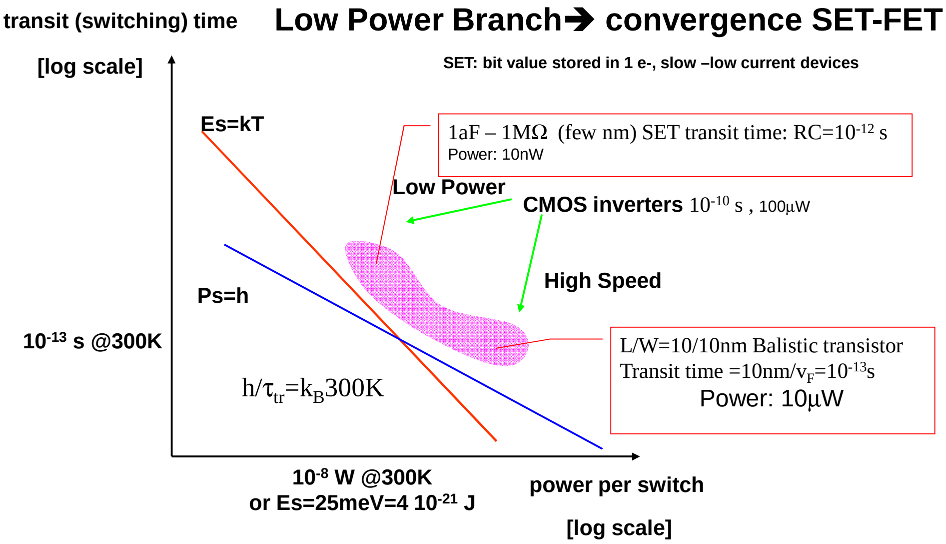
electronic circuit time constant :RC~ 10^{-13}s

➔Esw (@V=0.5V)~ 500 meV >>25meV=k_B 300K

➔Esw×transit time=10^{-32} >>h = 10^{-34} Js

still safe ....

Low Power Branch➔ convergence SET-FET



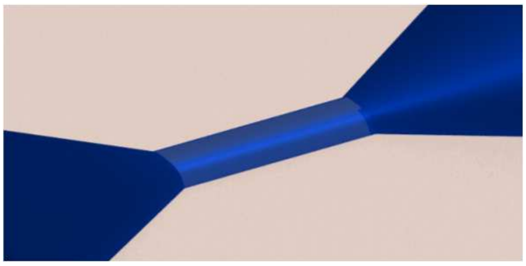
Short channel effect and dopant variability:
An exemple: the Single Atom Transistor

Threshold voltage variability due to
individual dopants in the channel

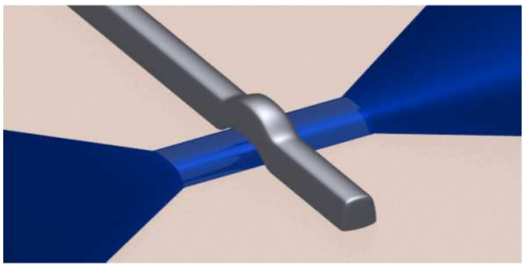
M. Pierre *et al.,* Nat. Nanotechnol. **373**, 10.1038, (2009)

M. Fuechsle et al. 2012
Nat. Nanotechnol. DOI: 10.1038/NNANO.2012.21

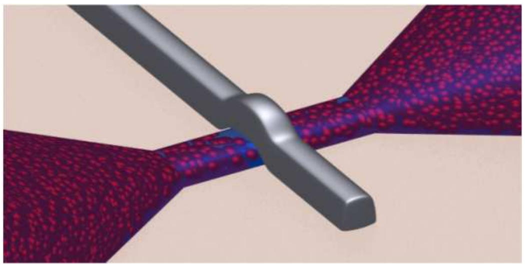
Sample fabrication: 200 mm wafers
8-20 nm Thick Silicon-on-Insulator (SOI)
e-beam litho of active layer (down to 20nm), Thermal oxidation (5nm SiO_2)



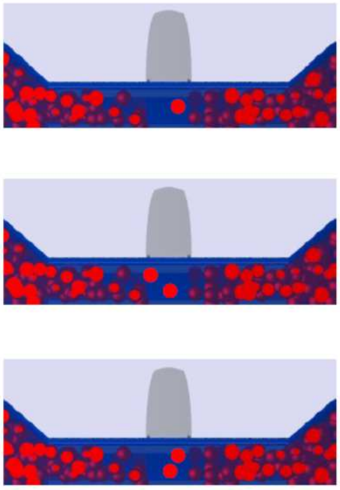
e-beam litho of poly Si gate layer gate length down to 20nm
Eventually multigate design= Pitch 70 nm

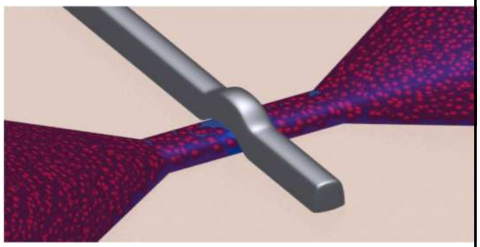


Arsenic implantation of highly doped Source and Drain
(channel masked by the gate stack)



OVERLAP GATE/SOURCE-DRAIN
No (or thin) nitride spacers

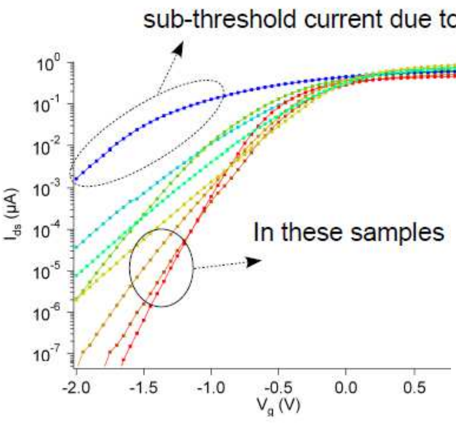




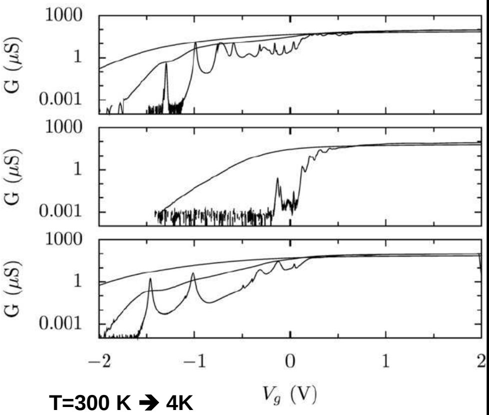
Without nitride spacers

Lateral dopant diffusion:
➔« Short channel effects »
+ Variability

Bad for transistors but…
Opportunity to study transport thru
Single shallow donors

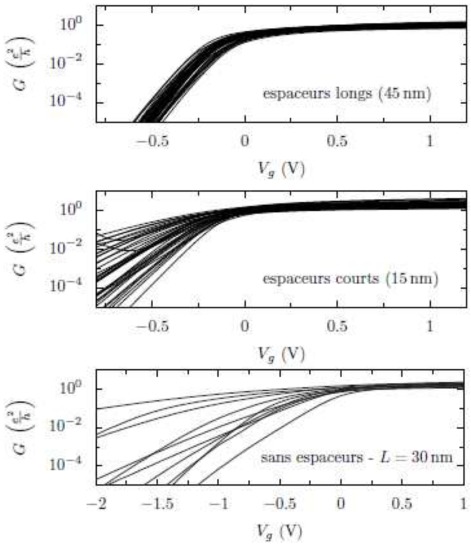


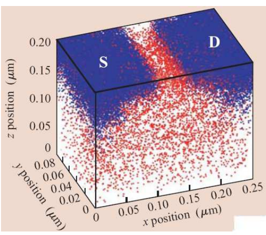
T=300 K



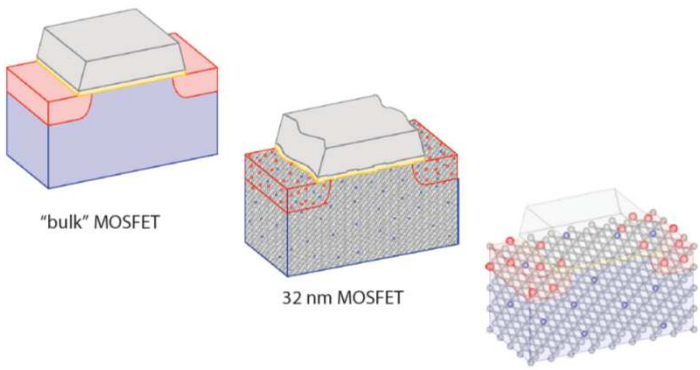
T=300 K ➔ 4K

Shrinking the mosfet: single dopant variability





Atomic-scale electronics



[Asenov IEEE Trans. Elec. Dev. 50, 1837, 2003]

Intel Technology Journal, Volume 12, Issue 2, 2008

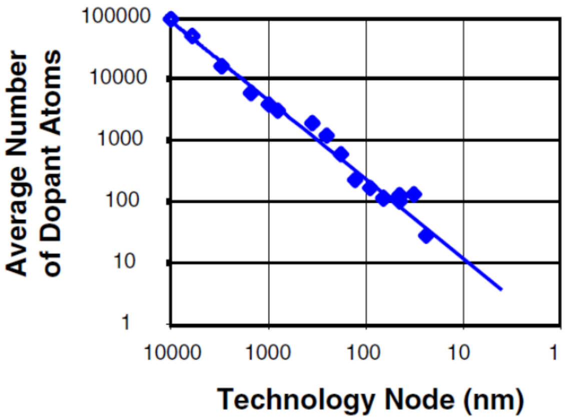
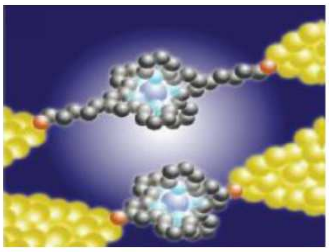
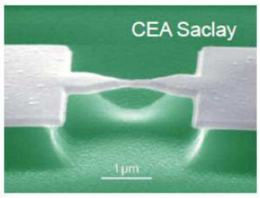


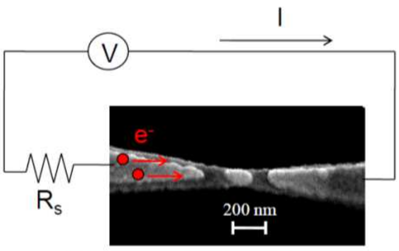
Figure 2: Average number of dopant atoms in the channel as a function of technology node

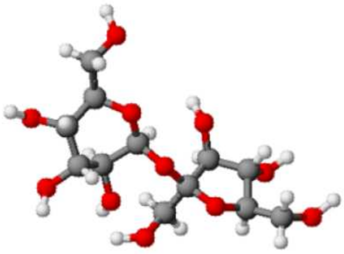




Mechanically controllable break junctions (MCBJ)

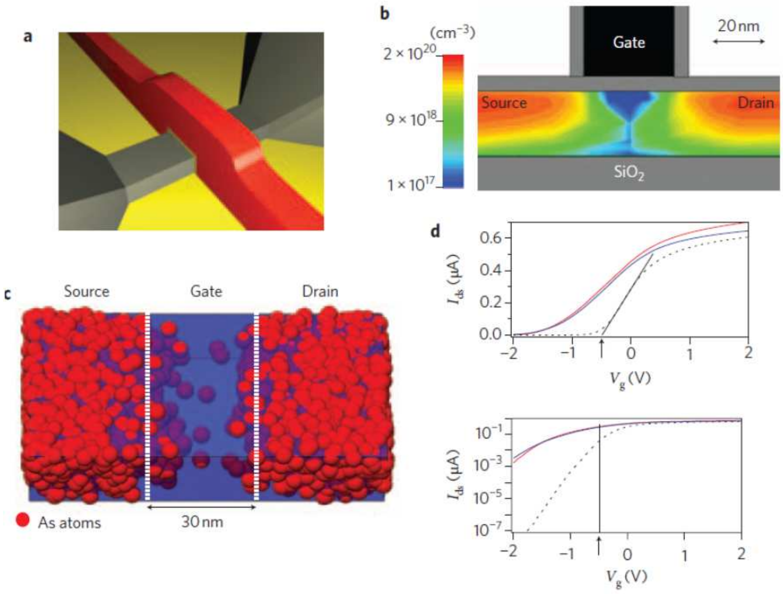
… circuit inside the molecule

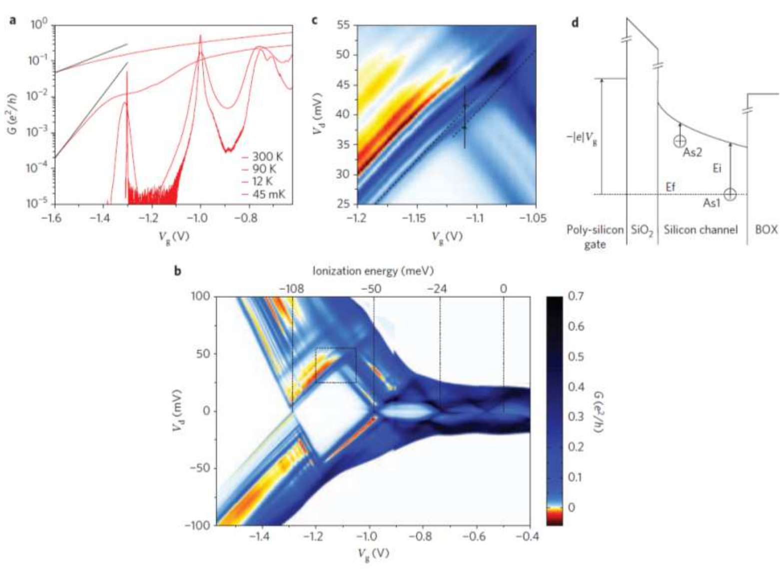




momentum transfer from the electrons

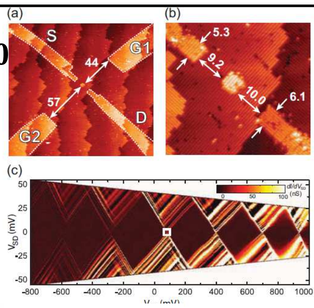
M. Pierre *et al.*, Nat. Nanotechnol. **373**, 10.1038, (2009)

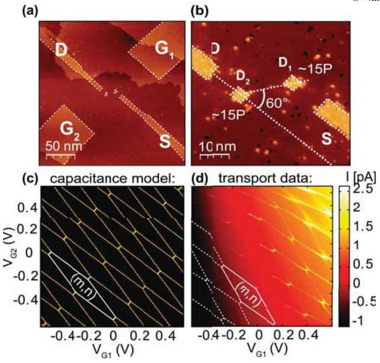


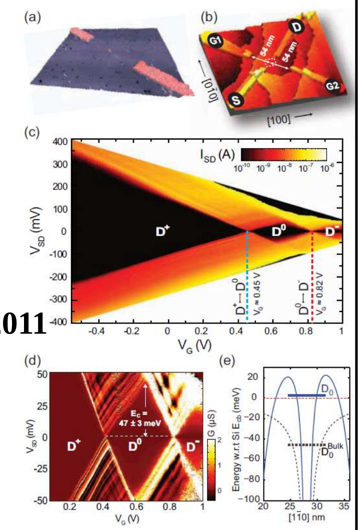


M. Pierre *et al.*, Nat. Nanotechnol. **373**, 10.1038, (2009)

Fuechsle 2010







Fuechsle 2011

Weber 2012

STM assisted lithography
Bottom-up approach of SAT

Convergence SET-FET: the MOS-SET

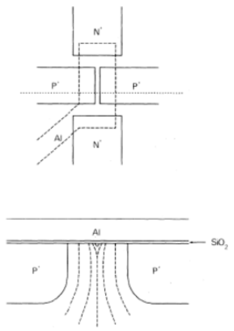
(the underlap analog of the preceding exemple)

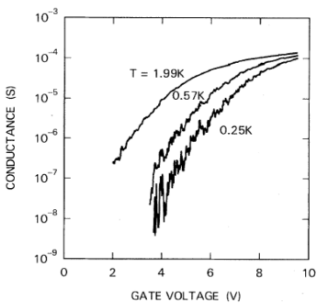
Limit variability due to
individual dopants in the channel
& push the energy stored by bit to its minimum
physical value (1 electron stored on the gate)

MOSFET scaling down at low temperature
A brief chronology (1982➔2017)

Brief chronology:

Conductance in Restricted-Dimensionality Accumulation Layers
A. B. Fowler, A. Hartstein, and R. A. Webb
Phys. Rev. Lett. 48, 196 (1982)





Mesoscopic fluctuations (« Fingerprint »)

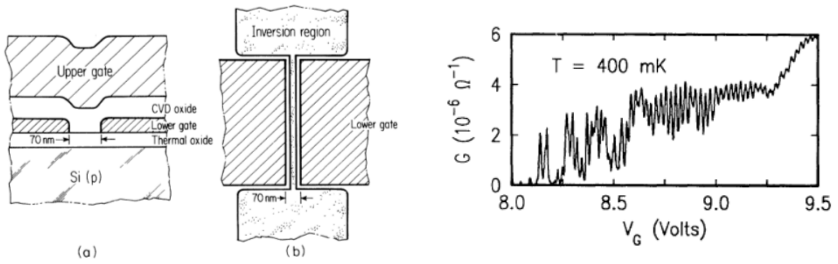
First periodic Conductance Oscillations

"Conductance Oscillations Periodic in the Density of a One-Dimensional Electron Gas"

Scott-Thomas, J.H.F., S.B. Field, M.A. Kastner, H.I. Smith, and D.A. Antonadis, 1989, Phys. Rev. Lett. 62, 583.

Abstract:

By use of x-ray lithography Si inversion layers have been fabricated with width ~25 nm and mobility ~15 000 cm2/V s. These display oscillations in their conductance that are periodic in the number of electrons per unit length, even in zero magnetic field. The oscillations reflect an oscillatory activation energy of the conductance and are accompanied by unusual nonlinearities suggestive of pinned charge-density waves.

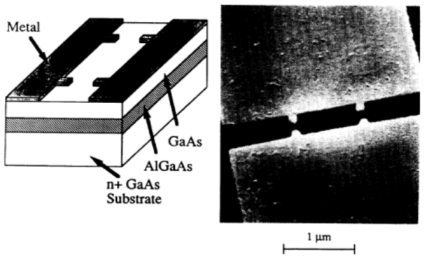


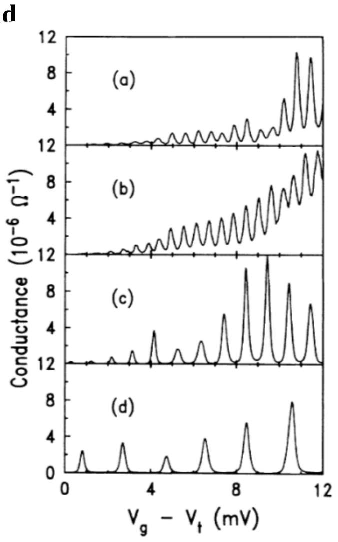
First Geometry Controlled conductance oscillations (CBO ?)

Single-electron charging and periodic conductance resonances in GaAs nanostructures

U. Meirav, M. A. Kastner, and S. J. Wind

Phys. Rev. Lett. 65, 771 (1990)





GaAs first quantum dot
2 intentional constrictions

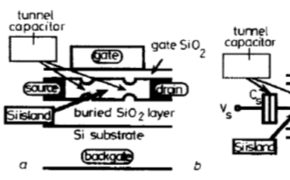
$\Delta Vg=e/Cg$

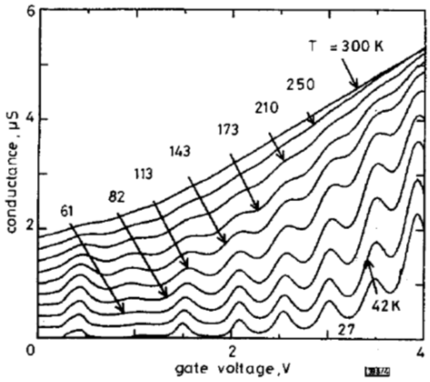
H. Matsuoka, T. Yoshimura, T. Ichiguchi and E. Takeda,"Coulomb blockade in the inversion layer of a Si metal-oxide-semiconductor field-effect transistor with a dual-gate structure," Appl. Phys. Lett., vol. 64, p. 586, 1994.

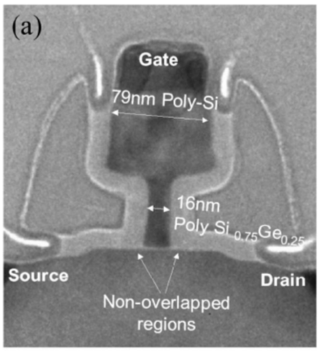
Ali, D., and H. Ahmed, 1994, Appl. Phys. Lett. 64, 2119.

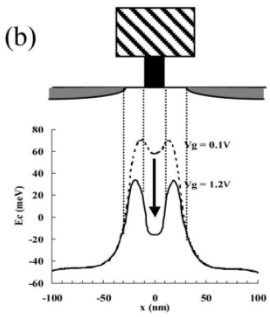
Takahashi, Y., M. Nagase, H. Namatsu, K. Kurihara, K. Iwdate, Y. Nakajima, S. Horiguchi, K. Murase, and M. Tabe, 1995, Electron. Lett. 31, 136

First silicon quantum dot
Two intentional constrictions



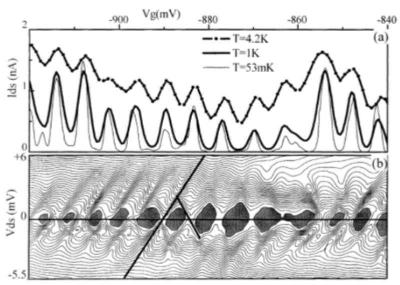






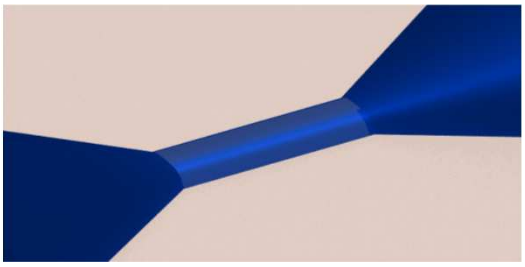
First QuDot based on CMOS (bulk)
(no constriction/ underlapped geometry)

Controlled Single-Electron Effects in Non overlapped Ultra-Short Silicon Field Effect Transistors
Frédéric Boeuf, Xavier Jehl, Marc Sanquer, and Thomas Skotnicki,



IEEE Trans. Nanotechnology 2 144 (2003)

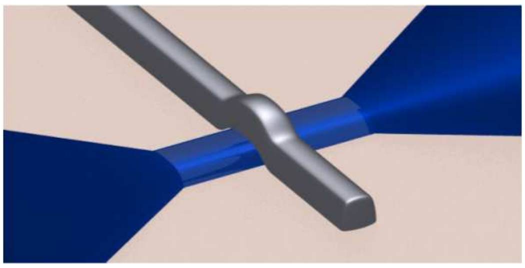
Sample fabrication: 200 mm wafers
8-20 nm Thick Silicon-on-Insulator (SOI)
e-beam litho of active layer (down to 20nm), Thermal oxidation (5nm SiO_2)



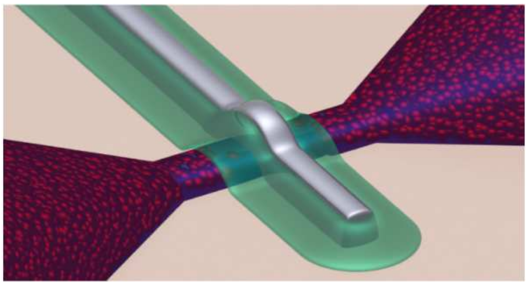
Convergence SET-FET: the MOS-SET

(underlapped geometry)

e-beam litho of poly Si gate layer gate length down to 20nm
Eventually multigate design= Pitch 70 nm

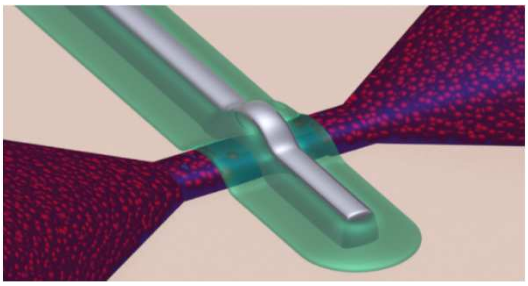


formation of self aligned Si_3N_4 spacers before Arsenic implantation of highly doped Source and Drain



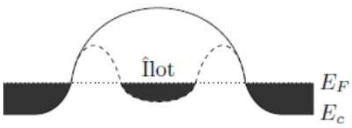
leti

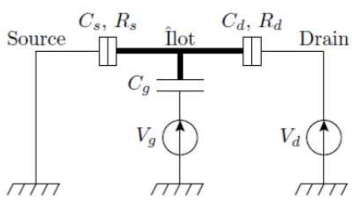
UNDERLAP GATE/SOURCE-DRAIN

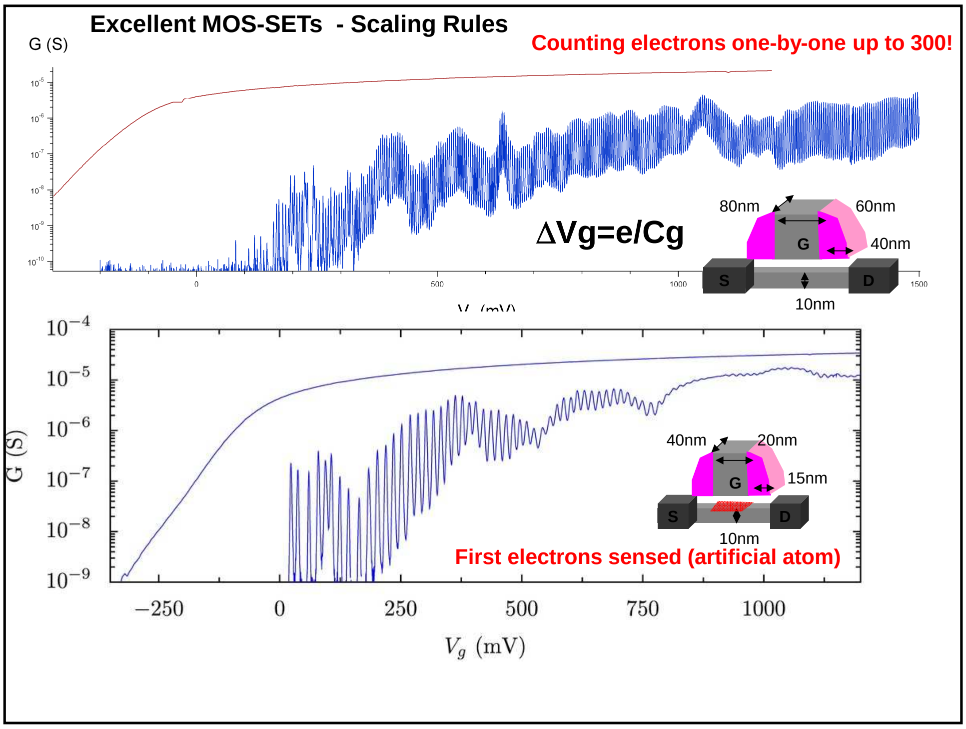


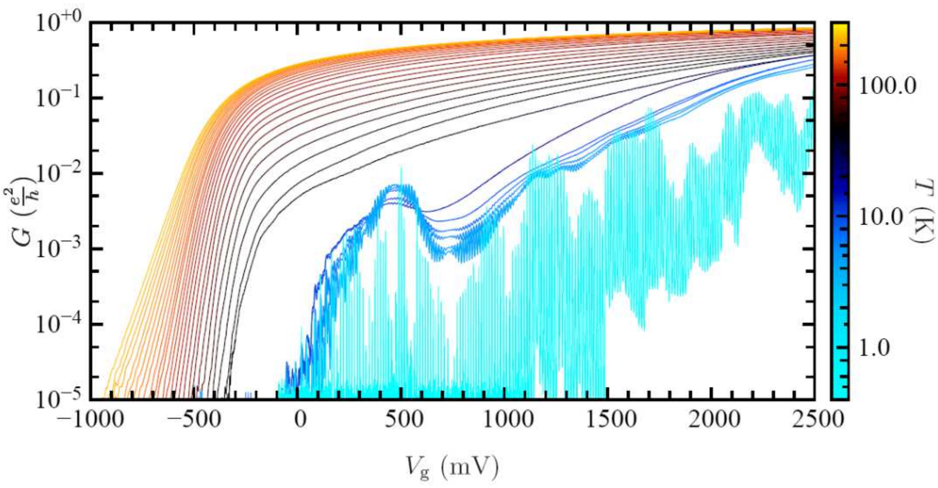
UNDERLAP GATE/SOURCE-DRAIN

The MOS-SET





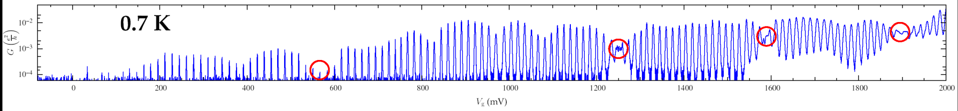




« Trigate » MOSFET

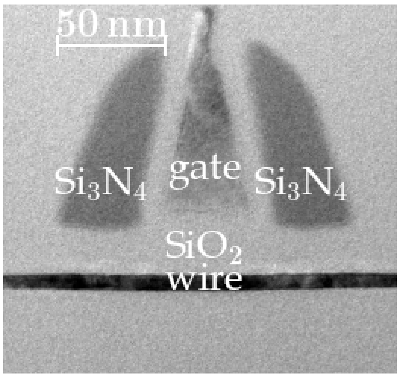
M. Hofheinz PhD, U. J. Fourier 2006

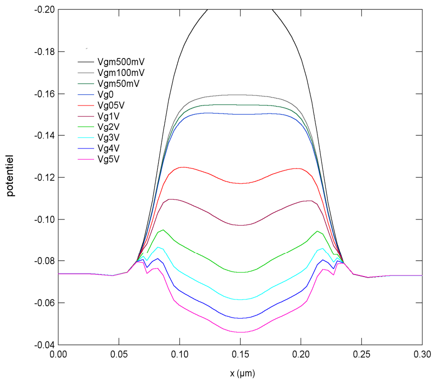
access resistances. ex: SOI Non-Overlapped nanowire FET

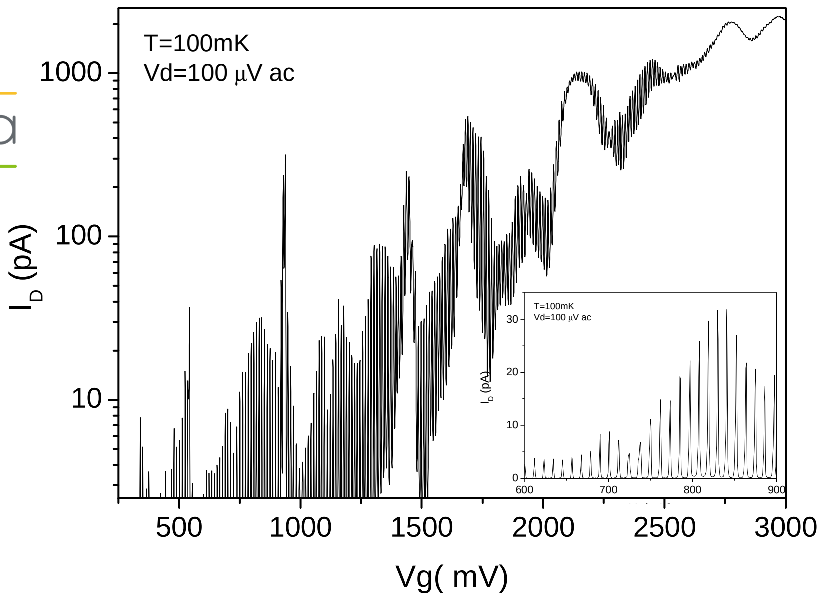


M. Hofheinz, X. Jehl , M. Sanquer , G. Molas, M. Vinet and S. Deleonibus ,
"A simple and controlled single electron transistor based on doping modulation in silicon nanowires",
Applied Physics Letters vol.89, 143504 (2006).

fabrication: **cea-leti**







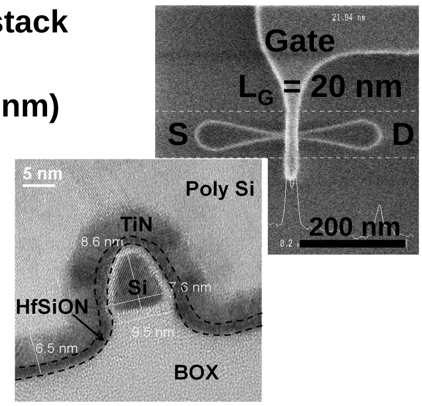
DSM/INAC + LETI

MOS-SET at room temperature

Device fabrication

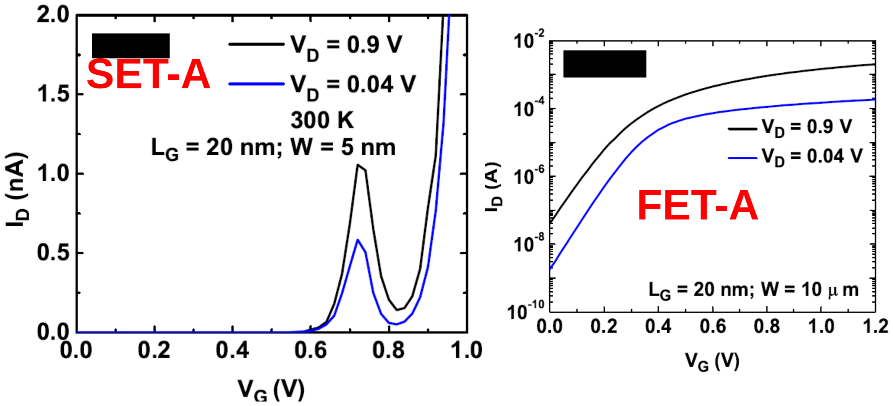
Si Nanowire Integration scheme

- **SOI Wafer (Mesa isolation)**
- **Si NW patterning (resist trimming)**
- **HfSiON/ALD TiN gate stack**
- **Gate patterning**
- **Spacer1 formation (25 nm)**
- **S/D Epitaxy (for RSD)**
- **LDD implantation**
- **Spacer2 formation**
- **HDD implantation**
- **Silicidation**
- **Back-End**



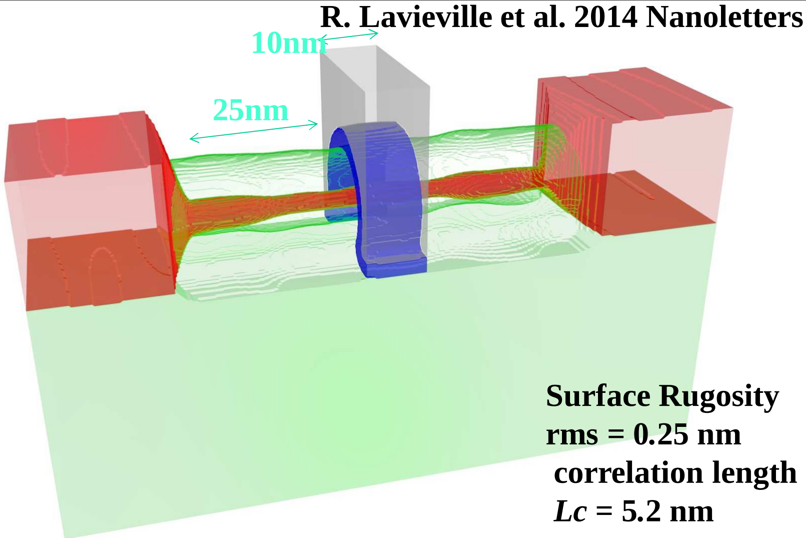
V. Deshpande et al. IEDM2012

Co-integration de FET et SET (variation sur W)



V. Deshpande et al. IEDM2012

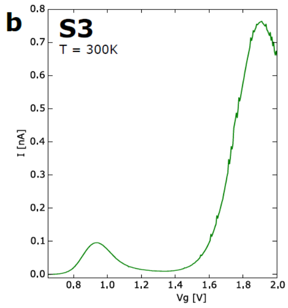
R. Lavieville et al. 2014 Nanoletters

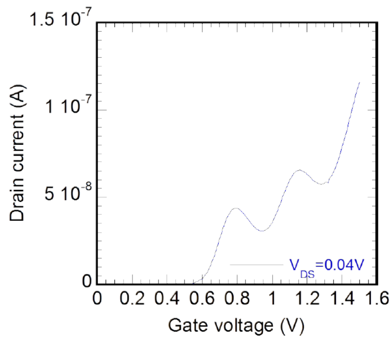


An undoped silicon nanowire Artificial atom

see also S. J. Shin *et al* APL 103101(2010).

SET @300K (also SHT)

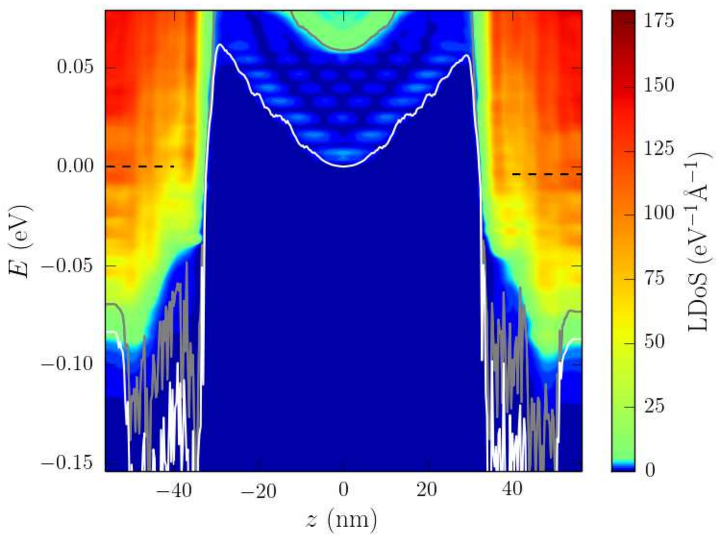




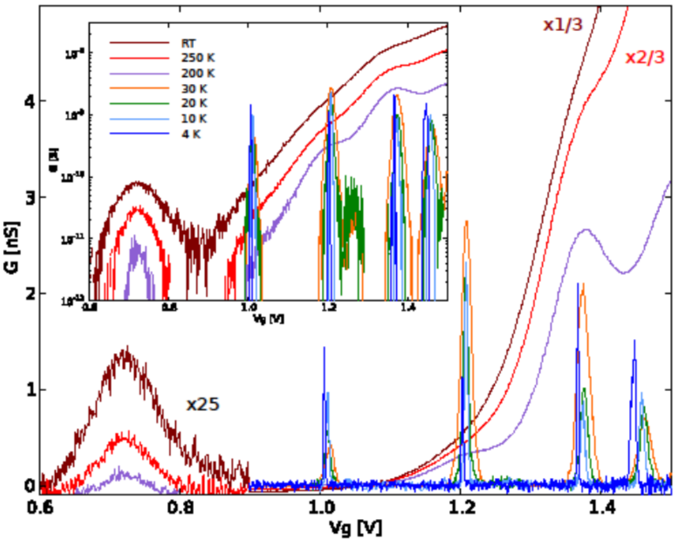
LDOS@ first e⁻

Y-M Niquet et al. INAC

(W/O SR & VOS, NEGF + effective mass approx.) Lg=10nm

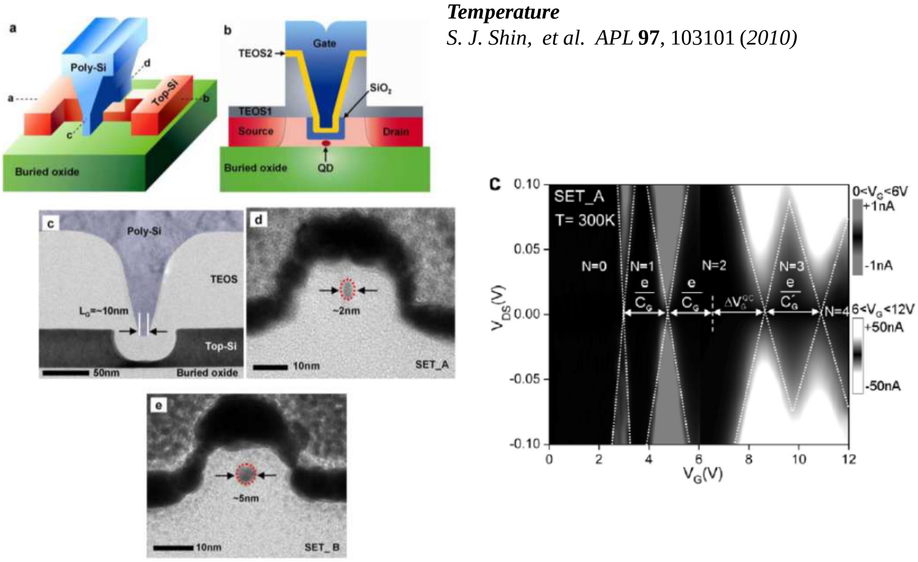


R. Lavieville et al. 2014 Nanoletters



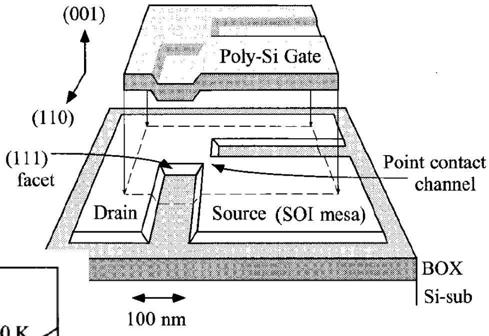
A room temperature MOS-SET

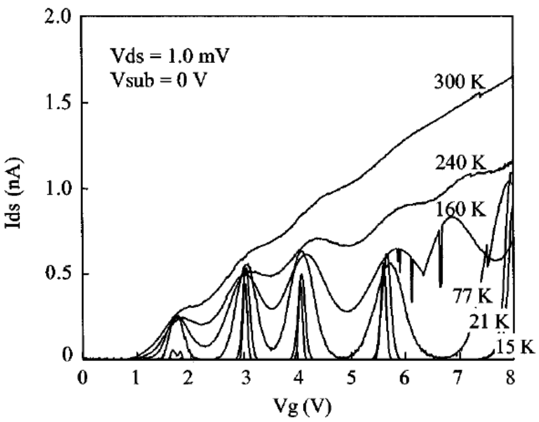
Enhanced Quantum Effects in an Ultra-Small Coulomb Blockaded Device Operating at Room-Temperature
S. J. Shin, et al. APL **97**, 103101 (*2010)*



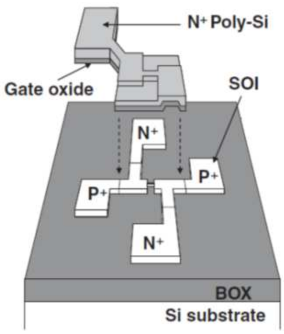
Coulomb Blockade in a SOI constriction

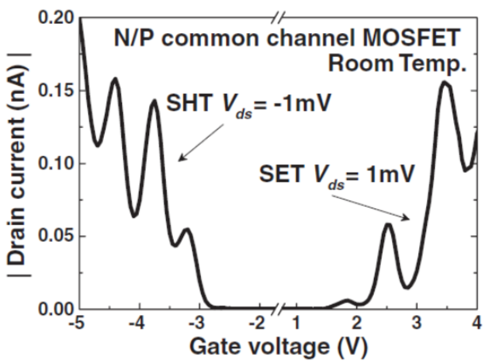
H. Ishikuro and T. Hiramoto
APL 71, 3691 (1997)





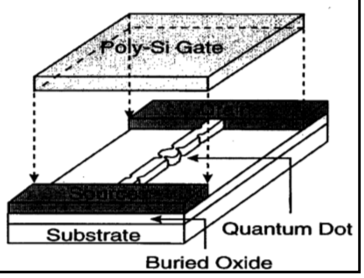
M. Kobayashi et al, Jpn. J. Appl. Phys., Vol. 47, 1813(2008)





Ex: CB in silicon devices at room temperature

•Hiramoto, Ishikuro (Tokyo) point contact MOSFETs
(Ishikuro et al. APL71, 3691 (1997).)
•Chou, Tsui (Princeton) MOSFET quantum dot
(L. Guo et al. APL70, 850 (1997).)
•Sakamoto, Baba (NEC) doped Silicon (SOI)
(Sakamoto et al. APL72, 795 (1998).)
•Peters, Dijkhuis (Utrecht) SOI MOSFET
(Peters et al. J of Appl. Phys. 84, 5052 (1998).)
•Kotthaus, Wharam (Munchen, Tubingen) SOI wires
(Tilke et al. APL75, 3704 (1999).)
•....

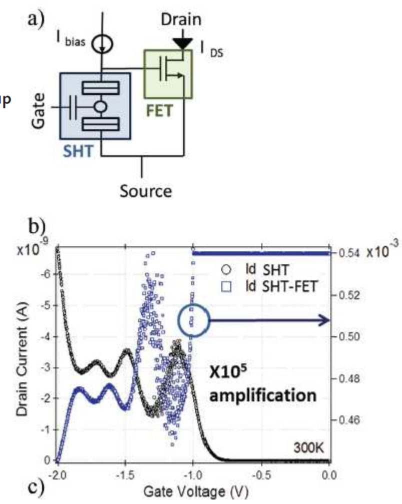


Hybrid SET-FET

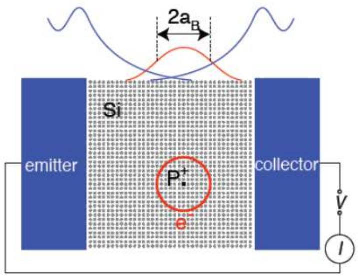
Demonstration of Single Hole Transistor and Hybrid Circuits for Multivalued Logic and Memory Applications up to 350 K Using CMOS Silicon Nanowires

Romain Lavieville, Sylvain Barraud, Christian Arvet, Christian Vizioz, Andrea Corna, Xavier Jehl, Marc Sanquer, and Maud Vinet*

Advanced electronic Materials2016



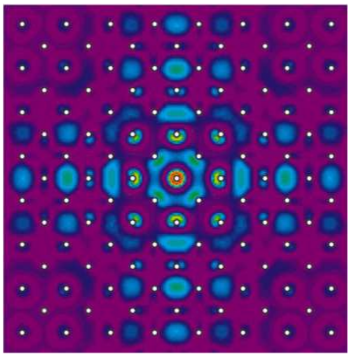
Confinement in dopants
vs
confinement in quantum dots



Scaling of the Bohr orbit:

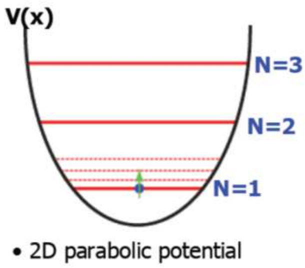
$$r_{\text{dopant}} = \frac{\epsilon_{\text{r}}}{m^*} \cdot r_{\text{Hydrogen}}$$
$$r_{\text{Hydrogen}} = 0.05\,\text{nm}$$
$$r_{\text{P:Si}} = 2.5\,\text{nm}$$
$$r_{\text{P:Ge}} = 6.4\,\text{nm}$$

Single dopant



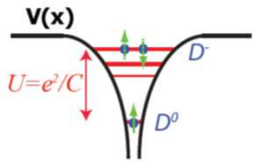
Shallow donor electronic orbital in
Silicon (Ground state,
after B. Koiler et al Phys. Rev. B 70, 115207 (2004)

Quantum dots

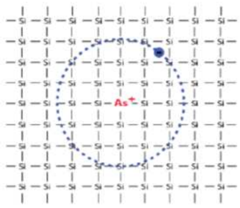


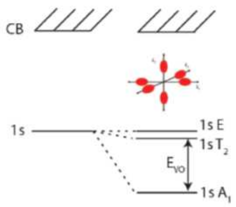
- 2D parabolic potential
- Constant charging energy
- Equidistant level spacing (excited states)

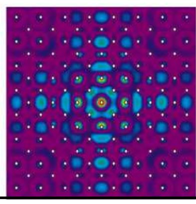
Dopant: shallow donors in Si



- 3D Coulomb potential
  - Can bind up to two electrons
  - Hydrogen-like level spectrum (D^0)
  - Valley-orbit →

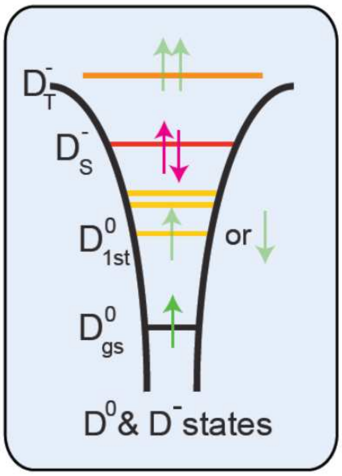






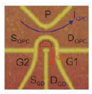
Shallow donor electronic orbital in
Silicon (Ground state, after B. Koiler et al Phys. Rev. B 70, 115207 (2004))

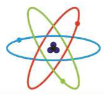
Double occupation of donors,
Coulomb blockade



eigen states

Relevant energy scales



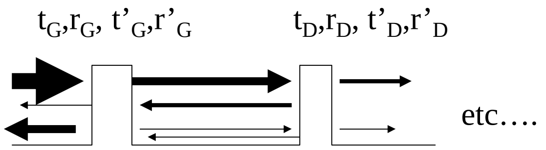


Coulomb interaction:	$E_C = \frac{e^2}{8\varepsilon\varepsilon_0 r}$	1-10 meV	1-10 eV
Size quantization:	$E_Q = \frac{\hbar^2}{m^* r^2}$	0.1-10 meV	1-10 eV
Orbital magnetic energy:	$E_L = \frac{\hbar eB}{m^*}$	0-20 meV	0-1 meV
Zeeman energy:	$E_Z = g^* \mu_B B$	0-1 meV	0-1 meV
Thermal energy:	$E_{th} = k_B T$	25° C: 25 meV -273°C: 15 μeV	

Large mean level spacing, Resonant Tunneling,
Confinement size close to Fermi wavelength
No interaction, no Coulomb blockade

Ex: a single shallow donor
an artificial atom (very small quantum dot)

For a better understanding of the single dopant case, we consider the (single channel) Resonant tunneling



$$t_{total} = t_G t_R e^{ikL}\left(1 + r'_G r_D e^{2ikL} + (r'_G r_D e^{2ikL})^2 + \ldots\right) = \frac{t_G t_R e^{ikL}}{1 - r'_G r_D e^{2ikL}}$$

$$T = \frac{T_G T_D}{1 + R_G R_D - 2\sqrt{R_G}\sqrt{R_D}\cos(2kL + \pi)}$$

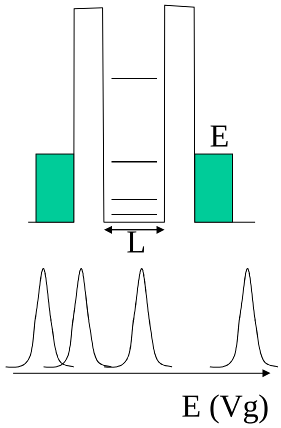
$\pi = \phi'_G + \phi_D$ dephasing inside (infinite) tunnel barriers

(far from any Rayleigh (elastic) resonance , the transmission is purely imaginary)

M. Buttiker IBM Journal Res. Dev. 32,63 (1988)

maximum transmission if 2kL=π, i.e. for an infinite double well cavity:

$$E_n = \left(\frac{\hbar^2\pi^2}{2m}\right)\left(\frac{n}{L}\right)^2$$



then:

$$T_{coherent} = \frac{4T_G T_D}{(T_G + T_D)^2}$$

$T_{coherent} = 1$ if $T_G = T_D$

Far from resonance (2kL=2nπ):

$$T_{coh,HR} = \frac{1}{4} T_G T_R$$

incoherent (sequential) transmission does not depend on energy:

$$T_{incoh.} = \frac{T_D T_G}{T_D + T_G}$$

$$T = \frac{T_G T_D}{1 + R_G R_D - 2\sqrt{R_G}\sqrt{R_D}\cos(2kL + \pi)}$$

$$\cong \frac{T_G T_D}{\left(\frac{T_G + T_D}{2}\right)^2 + 2\left(1 - \cos(\theta(E))\right)}$$

If T $_{D,G}$ << 1

weak transmission

$$1 - \cos(\theta(E)) \approx \frac{1}{2}\left(\theta(E) - 2n\pi\right)^2 \approx \frac{1}{2}\left(\frac{d\theta(E)}{dE}\right)^2 \left(E - Eres\right)^2$$

Near a resonance

(Taylor series development)

$$T = \frac{\left(\Gamma_D \Gamma_G\right)}{\left(E_F - E_N\right)^2 + \frac{1}{4}\left(\Gamma_D + \Gamma_G\right)^2} \qquad \Gamma_{G,D} = \frac{dE}{d\theta} T_{G,D}$$

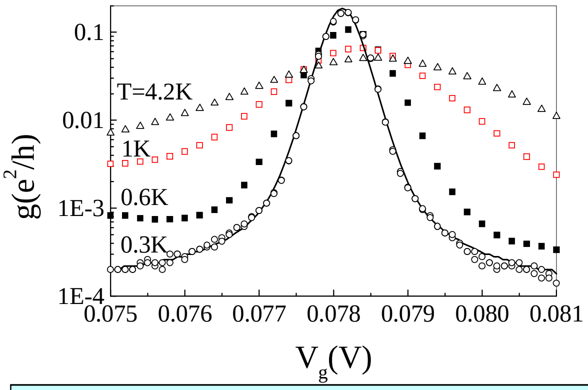
(Breit-Wigner formulae)

$$\Gamma_{G,D} = \frac{dE}{d\theta} T_{G,D} = \frac{dE}{dk}\frac{dk}{d\theta} T_{G,D} = \hbar v_F \frac{1}{2L} T_{G,D}$$

$$\Gamma = \omega\left(T_G + T_D\right) = \frac{\Delta}{\hbar}\left(T_G + T_D\right) = \frac{1}{\upsilon_{1D}}\left(T_G + T_D\right)$$

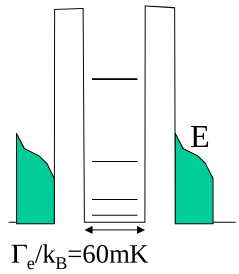
quality factor: $Q = \frac{\Delta}{\hbar \Gamma} = \left(T_G + T_D\right)^{-1}$

Thermal broadening of the resonant tunneling :



energy level separation Δ:

$\Delta > k_B T > \Gamma_{in}, \Gamma_e$



$$G(Vg,T) = \frac{e^2}{h} \int \frac{\Gamma_e^{\,2}}{(e\alpha Vg - E\;)^2 + \Gamma_e^{\,2}} \frac{df(E,Vg)}{dE} dE$$

$$f(E) = \frac{1}{1+exp(\frac{E-E_F}{k_B T})} \qquad -\frac{\delta f(E)}{\delta E} = \frac{1}{4k_B T} cosh^{-2}(\frac{e\alpha Vg - E}{2k_B T})$$

Thermal broadening of the resonant tunneling (2):

If $\Delta > k_B T >> \Gamma_{in}, \Gamma_e$

$$G(Vg,T) = \frac{e^2}{h} \int \frac{\Gamma_e^{\,2}}{(e\alpha Vg - E\;)^2 + \Gamma_e^{\,2}} \frac{df(E,Vg)}{dE} dE$$

➔

$$G(Vg,T) = \frac{e^2}{h}\left(\frac{1}{4kT}\right) A\cosh^{-2}\left(\frac{e\alpha Vg - Eres}{2kT}\right)$$

Classical Coulomb blockade,
Fermi wavelength <<size

General case for a small numbers of electrons

(« artificial atom»)

N=0 E=0

N=1 $E_1-e\phi_{ext}$

$e\phi_{ext}= E_1$

N=2 $2E_1+U_{11}-2e\phi_{ext}$ $E_1+ E_2 +U_{12}-2e\phi_{ext}$

$e\phi_{ext}= E_1 +U_{11}$

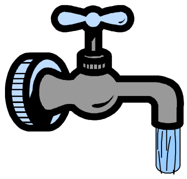
$e\phi_{ext}= E_2 +2U_{12}$

N=3 $2E_1+ E_2 +U_{11} +2U_{12} -3e\phi_{ext}$ etc…

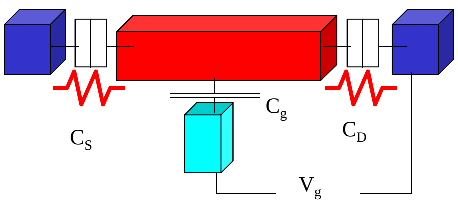
$$U_{ij} \approx \iint \psi_i^*(r_1)\psi_j^*(r_2)V(r_1-r_2)\psi_i(r_1)\psi_j(r_2)dr_1dr_2 - \acute{e}change$$

Generally $U_{ii} > U_{ij}$, spin polarization (Hund's rules)

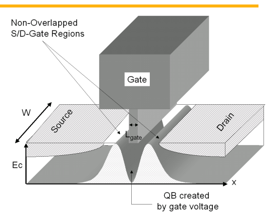
- Conventional devices (MOSFET)
  Continuous flow of charge carriers
  - Modelled by hydrodynamic equations



- Single Electron Devices
  - Operation based on the charge quantization
  - One-by-one flow of electron
  - Localization of electron wave function



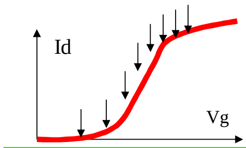
$$E(N)=(Ne)^2/2C-Ne\phi_{ext}(V_g)$$



Depending on the access resistance: (T=0K)

FET ($R<h/e^2$) or SET ($R>h/e^2$)

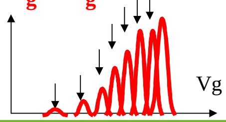
« The more electrons, the more current »



« e-periodic current: E(N)=E(N+1)»

$(N+1/2)e/C_g= V_g$

$kT<Ec=e^2/C$



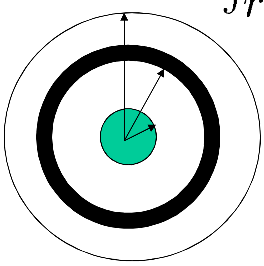
	Gate	S/D barriers	Advantages	Drawbacks
metal		*Metal oxide Al2O3*[7]*, NbOx*[8]		*Offset charges Medium size*
		Thin films resistors[18]	*Minimizes co-tunneling*	*Offset charges Medium size*
	Thin metallic film[17]		*gain*	
Metallic clusters[19]	*No gate*	*Metal oxide*	*Small size*	*No control gate*
Semiconducting clusters	*Lateral*[14]*, front or back gate*	*oxide*	*Small size*	*Poor control*
Vertical dot[20]	*Lateral gate*	*Epitaxially grown*	*Few electron regime, disorder free*	
Lateral multigates cavities	*Lateral, front or back gate*	*Split gate*[21]*, lateral gate*[13]*, top gate* [9,22] *Schottky gate or local oxidation (AFM)*	*versatile*	*Medium or large size, crosstalk*
Lateral single gate dot	*Front or back gate*	*Oxide (PADOX)*[10,11]*, etched constriction*[1,2,3,5,6,12,15,16,25]*, doping modulation*[4]	*Small size One control gate*	*Poor control of the D/S barriers*
Dynamic Quantum dots[24]		*Moving electrostatic potential (surface acoustic waves..)*		

Partial list in a very large literature:

1 H. Ishikuro et al. APL71, 3691 (1997)
2 L. Guo et al. APL70, 850 (1997)
3 Sakamoto et al. APL72, 795 (1998)
4 Peters et al. J of Appl. Phys. 84, 5052 (1998)
5 Tilke et al. APL75, 3704 (1999)
6 L . Zhuang et al. APL72, 1205 (1998)
7 Nakamura APL76 (2000)
8 Shirakashi APL72 (1998)
9 A. Fujiwara et al. APL 88, 053121 (2006).
10 H. Namatsu et al, J. Vac. Sci. Technol. B21, 2869 (2003).
11 S. Horiguchi et al. Jpn. J. Appl. Phys. Part 2, 40 , L29 (2001).
12. J. Gorman et al. PRL95, 090502 (2005).
13 P. A. Chain, H. Ahmed and D. A. Williams, J. Appl. Phys. 92, 346 (2002).
14 Y. T. Tan et al. J. Appl. Phys. 94,633 (2003)
15 E. Leobandung et al. APL67, 2338 (1995).
16. M. Saitoh et al. APL84,3172 (2004).
17 Yu. A. Pashkin et al. APL74, 132 (1999).
18 V. A. Krupenin et al., J. Appl.. Phys., 90, 2411 (2001)
19 D.C. Ralph et al. PRL74, 3241 (1995).
20 S. Tarucha et al. PRL77,3613 (1996)
21 J. A. Folk et al. PRL76, 1699 (1996).
22 D. H. Kim et al. APL79, 3812 (2001)
23 C. Single et al. APL78, 1421 (2001).
24 J.A.H. Stotz et al. Nat. Materials 4, 585 (2005)
25 R. Augke et al. APL76, 2065 (2000)

electron-electron interaction on a disk:

$$\int_{r_0}^{R} \frac{2\pi r dr}{a^2}\frac{e^2}{\epsilon r} = \frac{2\pi e^2}{\epsilon}\left(\frac{R}{a^2} - \frac{r_0}{a^2}\right)$$

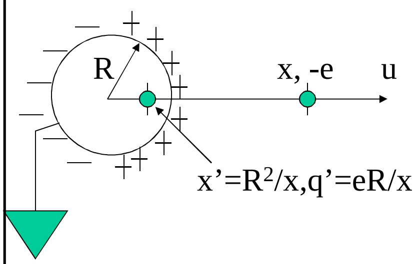


Electrostatic energy for $N=(R/a)^2$ electrons, i.e.:

$$E_1 = (Ne)^2\frac{2\pi}{\epsilon R} = \frac{Q^2}{2C} \qquad E_2 = N\frac{2\pi e^2}{\epsilon a}$$

Charging energy + correlation energy (r_0=a)

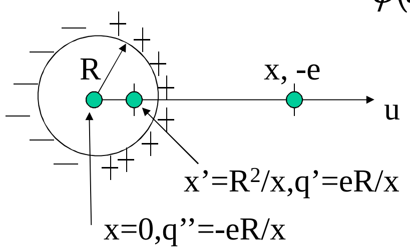
Charging energy for a metallic sphere: $E_c=e^2/2C$



$$\varphi_{im}=\frac{q'}{x-x'}\approx\frac{e}{2(x-R)}\Big|_{x\to R}$$

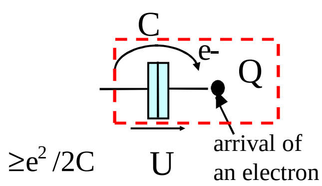
$$\epsilon_0 = 1$$

$$\varphi(u,x)=\frac{-e}{|x-u|}+\frac{eR/x}{|u-R^2/x|}-\frac{eR/x}{u}$$

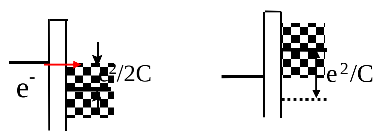


For u=R, the third term varies from 0 to e/R=e/C. Work needed to charge the sphere:

$$W=\int_{+\infty}^{R} dq''\frac{q''}{R}=\frac{1}{2}\frac{e^2}{C}$$



tunneling junction



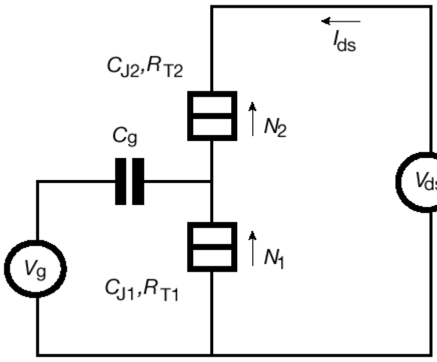
before tunneling after tunneling

$$\boxed{\Delta F=e\left(U-\frac{e}{2C}\right)\ge 0} \quad (\equiv \Delta W)$$

⇒ **critical junction voltage for the arrival of an electron:** $\boxed{U > e/2C}$

The Coulomb repulsion opens a gap for the arrival or departure of electrons

K. Likharev, FED Journal
Vol.6 Suppl.1, pp.5-14, 1995



single-electron transistor

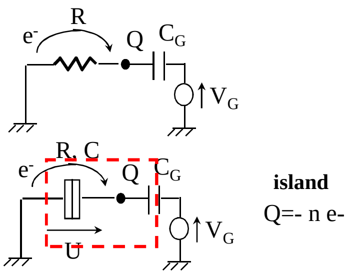
$$E_C = e^2/C_\Sigma$$

$$E_{el} = E_C[N_2 - N_1 - (C_g V_g/e) - (C_2 V_{ds}/e) + q_0]^2 - eN_2 V_{ds}$$

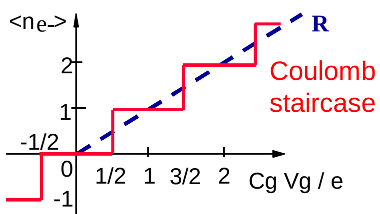
NB: sequential ($\Delta N_2 \neq \Delta N_1$) prevented, correlated events ($\Delta N_2 = \Delta N_1$) favorable

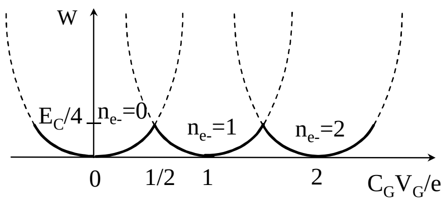
- Capacitor charged through a resistor
- Capacitor charged through a tunnel junction

$$kT \ll E_C = \frac{e^2}{2C_\Sigma}$$

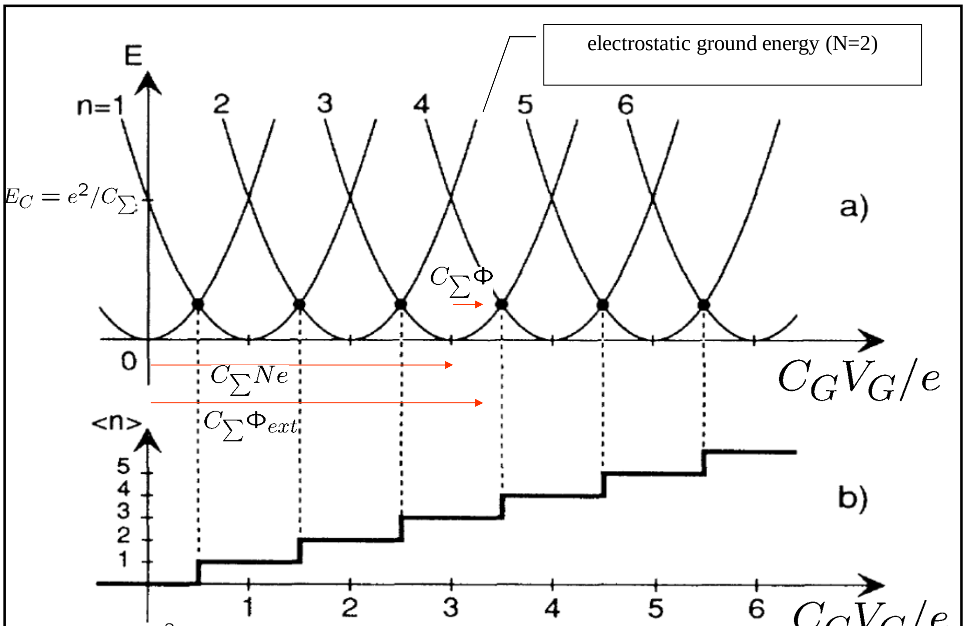


Electron Box





H. Grabert and M. Devoret
NATO ASI Series B, Vol.294, Plenum Press, New-York, physics edition, 1992

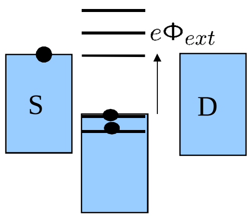


$E_N = \frac{e^2}{2C_\Sigma}(N - C_G V_G/e)^2 = N^2 e^2/2C_\Sigma - Ne\Phi_{ext} + C_\Sigma \Phi_{ext}^2/2$

$\Phi_{ext} = \frac{C_G V_G}{C_\Sigma}$ $\quad \Phi(N) = -eN/C_\Sigma + \Phi_{ext}$

electrochemical potential= energy to add one electron:
μ(N)=E(N)-E(N-1)

$$\mu(N) = E(N) - E(N-1) = (N-1/2)\frac{e^2}{C_\Sigma} = \phi_{ext}$$

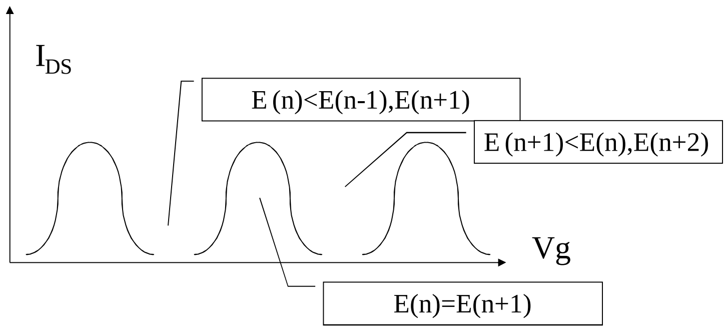


$E_{elec}(N) = E_{elec}(N+1)$

$N^2e^2/2C_\sum - Ne\Phi_{ext} + C_\sum\Phi_{ext}^2/2 = (N+1)^2e^2/2C_\sum - (N+1)e\Phi_{ext} + C_\sum\Phi_{ext}^2/2$

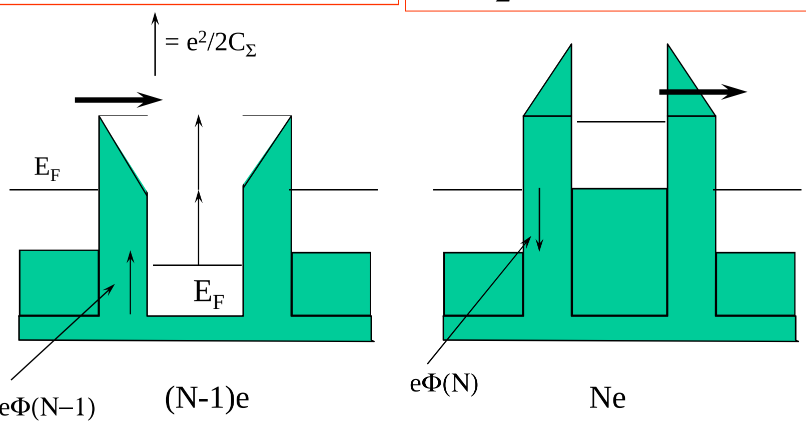
$\Phi_{ext} = \frac{C_G V_G}{C_\sum}$

$$e(N+1/2)/C_G = V_G$$



$\frac{e^2}{2C_\Sigma} + E_F = E_F + e\phi(N-1)$

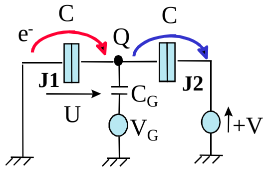
$-\frac{e^2}{2C_\Sigma} + E_F = E_F + e\phi(N)$

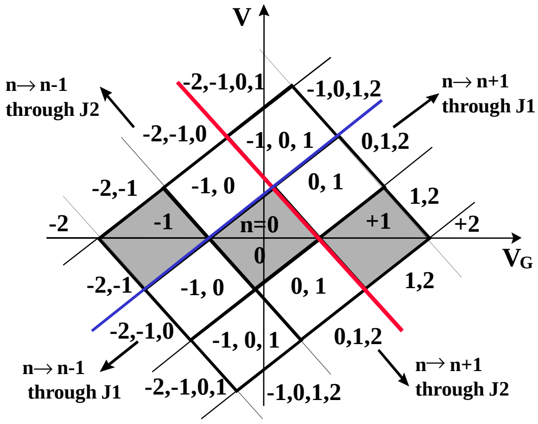


$$\Phi(N) = -eN/C_\sum + \Phi_{ext}$$

(e>0) $\Phi_{ext} = \frac{C_G V_G}{C_\sum}$

Charge states of SETs



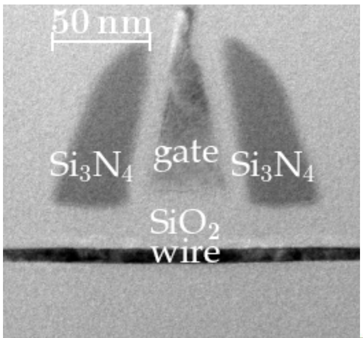


SET off → CB

SET on
(sequential arrival and departure of electrons)

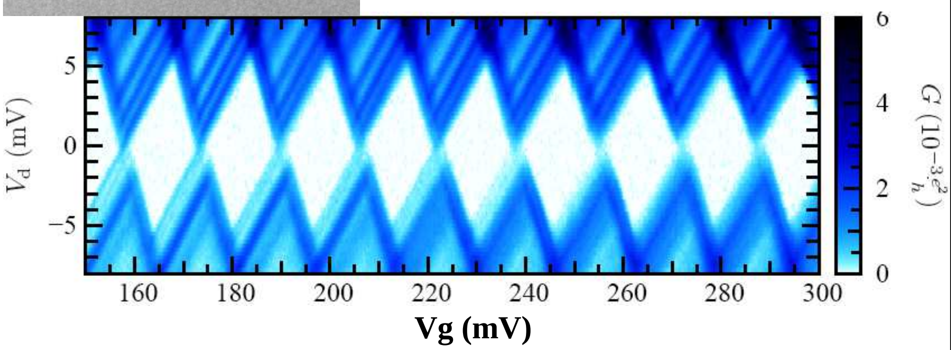
n→ n+1 through J1

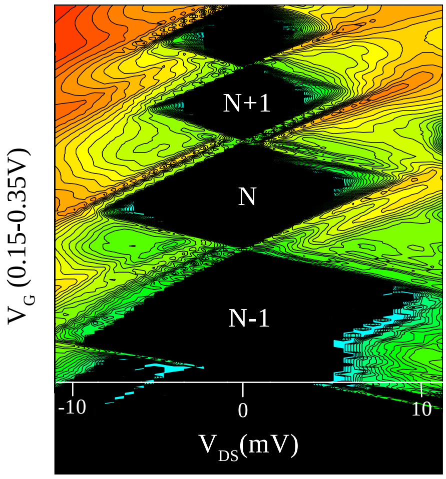
$$\Delta W = \frac{e}{C_\Sigma}\left(-\frac{e}{2} - ne + CV + C_G V_G\right) \geq 0$$



M. Hofheinz, X. Jehl , M. Sanquer , G. Molas, M. Vinet and S. Deleonibus ,
"A simple and controlled single electron transistor based on doping modulation in silicon nanowires",
Applied Physics Letters vol.89, 143504 (2006).

(Vg, Vd) Stability diagram





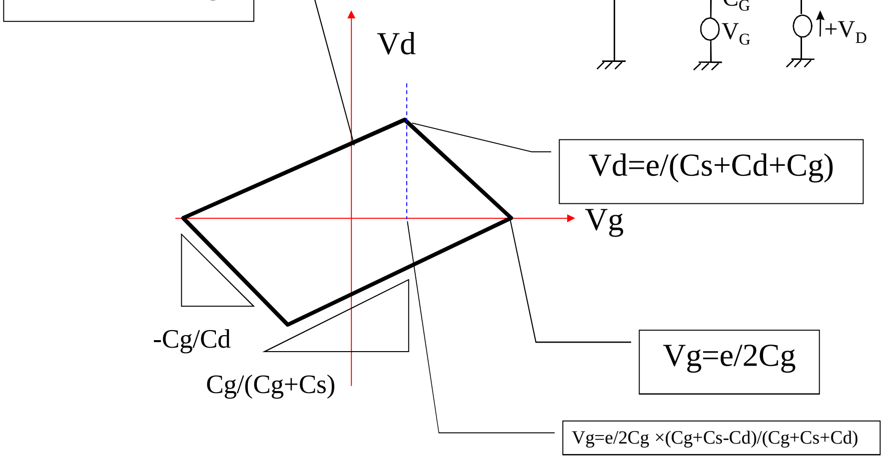
Stability diagram:

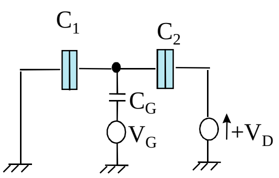
Si-MOSFET

gate length
= 100nm
W=400nm
T=0.1 K

$C \cong 80aF$
$E_C \cong 2meV$

SET parameters extraction





- CBO period → C_G
- Voltage gain $G_{SET}=C_G/C_2$
- $V_{th} = e/2(C_1+C_G)$

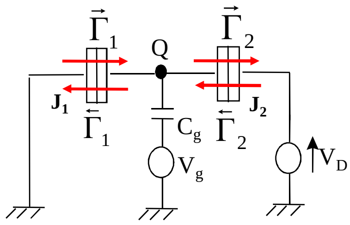
Schema for Cg=Cs=Cd

Calculation of the currents:

Hypothesis: stationary case, markovian process (no history),
No cotunneling (only single electron tranfer, inelastic process: "simultaneous" tunneling of e- through two junctions,elastic process: same electron tunnels through both junctions)
electrostatic problem treated (outside the electron tranfer, constant external potential)

Step 1: count all the possible single electron transfers

The **state** of the device is characterized by the number n of electrons on the island. Let p_n the probability to find the device in the state n.



Q=-ne

P_n may change:
- -by leaving state n to n-1 or n+1
- -or by coming into state n from n-1 or n+1

$$\Rightarrow \boxed{\dot{p}_n = \overbrace{\Gamma_{n,n+1}\, p_{n+1} + \Gamma_{n,n-1}\, p_{n-1}} - \overbrace{(\Gamma_{n+1,n} + \Gamma_{n-1,n})\, p_n}} \quad (1)$$

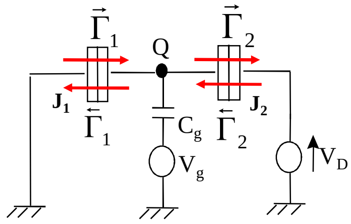
Where $\Gamma_{k,l}$ is the rate for a transition from state l to state k

$$\boxed{\Gamma_{n+1,n} = \vec{\Gamma}_1(n) + \vec{\Gamma}_2(n)} \qquad \boxed{\Gamma_{n-1,n} = \overleftarrow{\Gamma}_1(n) + \overleftarrow{\Gamma}_2(n)} \quad (2)$$

Where $\overleftrightarrow{\Gamma}_i$ are the electron tunneling rates

Step 2: Derivate the current (for the stationary case)

$$\dot{p}_n = 0$$



Current conservation

$$\boxed{I(V) = e\left(\sum_{n=-\infty}^{+\infty} p_n \left(\vec{\Gamma}_1(n) - \overleftarrow{\Gamma}_1(n)\right)\right) = e\left(\sum_{n=-\infty}^{+\infty} p_n \left(\vec{\Gamma}_2(n) - \overleftarrow{\Gamma}_2(n)\right)\right)}$$

I=e Γ

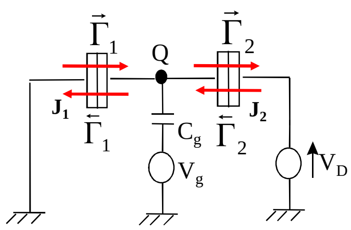
0.16 attoAmp= 1electron/sec

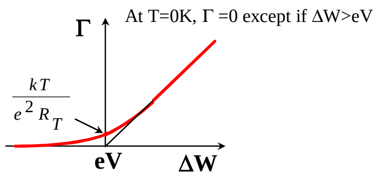
Step 3: calculate the tunneling rates (function of electrostatic energy before and after each electron transfer)

the rate of tunneling events is given by the "orthodox" theory, for the weak tunneling regime ($R_T >> R_Q$):

$$\boxed{\Gamma = \frac{\Delta W - eV}{e^2 R_T \left(1 - \exp\left(\frac{-(\Delta W - eV)}{kT}\right)\right)}}$$

- ΔW: drop of electrostatic energy
- R_T: tunneling resistance
- eV: work to be done by the voltage source





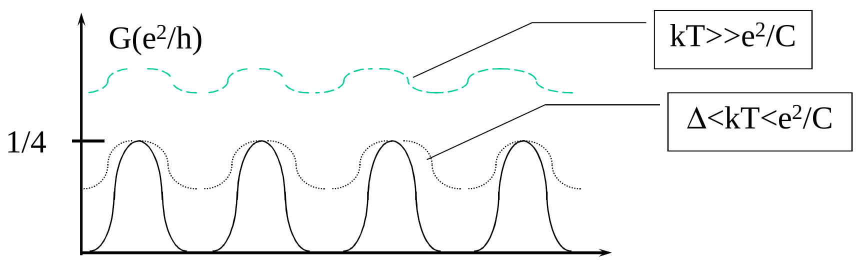
I. Kulik and R. Shekhter 1975, D. Averin and K. Likharev 1985

Results for the orthodox model in the linear regime (V~0)

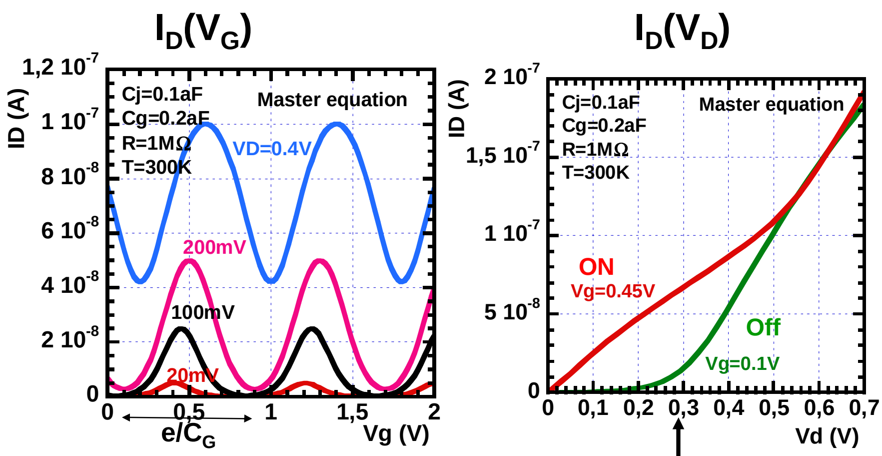
$$G_{\max} = \frac{e^2\rho}{2}\frac{\Gamma^l\Gamma^r}{\Gamma^l+\Gamma^r} \quad \text{if } \Delta E \ll kT \ll e^2/C,$$

$$G = e^2\rho\frac{\Gamma^l\Gamma^r}{\Gamma^l+\Gamma^r} \equiv G_\infty \quad \text{if } \Delta E, e^2/C \ll kT \ll \bar{\mu}, E_F$$

(means 2 barriers in series)



Exemple for a SET working at room temperature



Coulomb Blockade Oscillations (CBO) **Coulomb Blockade threshold**

"on" → $<n_e>$ half-integer

"off" → $<n_e>$ integer

Low V_D operation

Complex Coulomb blockade when :

ι) Δ is non negligible (« quantum capacitance »)

ii) when the size becomes comparable to the mean distance between carriers.

General case for a small numbers of electrons

(« artificial atom»)

N=0 E=0

N=1 $E_1 - e\phi_{ext}$

$e\phi_{ext} = E_1$

$e\phi_{ext} = E_1 + U_{11}$

N=2 $2E_1 + U_{11} - 2e\phi_{ext}$ $E_1 + E_2 + U_{12} - 2e\phi_{ext}$

$e\phi_{ext} = E_2 + 2U_{12}$

N=3 $2E_1 + E_2 + U_{11} + 2U_{12} - 3e\phi_{ext}$ etc…

$$U_{ij} \approx \iint \psi_i^*(r_1)\psi_j^*(r_2) V(r_1 - r_2)\psi_i(r_1)\psi_j(r_2) dr_1 dr_2 - \text{échange}$$

Generally $U_{ii} > U_{ij}$, spin polarization (Hund's rules)

Δ varies like L^{-d} , E_c varies like L^{-1}

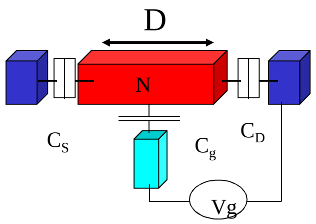
system	estimated size	Charging energy	mean level spacing
doped SiDot (baba)	20nm	1.4meV (56 aF)	0.5 meV
SOI FET wire (Tilke)	70nm	15meV	-----
SOI FET wire (Rokhinson)	10-40nm	13meV (12.3aF)	-----
SOI FET wire (Zhuang APL72, 98)	12nm	96meV	17meV
SOI point contact (Ishikuro)	6nm	58 meV (1.4aF)	30meV
Niobium island (Shirakashi APL72,98)	<10-20nm	1000 meV(0.16aF)	negligible (orthodox theory)
Aluminum island (Nakamura APL76,2000)	2-4nm	45-115meV	25-100meV
SOI FET wire (Augke)	20nm	4.9meV (32aF)	1.4meV

Large
(e^2/C) means small size

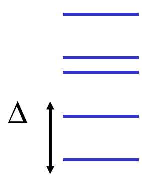
small size + semiconductors means large Δ

SET at room temperature means size of few nm.

Room temperature SET: D ≈1-2nm



kT<<Ec



- Size of the box >> λ_F : many electrons in the box

$Ec=e^2/2(C_g+C_S+C_D)$ >> Δ mean level spacing due to confinement

- Decrease of the size:

Ec increases as D^{-1}, Δ increases as D^{-2}

- Small size: large Ec ≈ Δ, Coulomb blockade + resonant tunneling:

$D \approx \lambda_F$

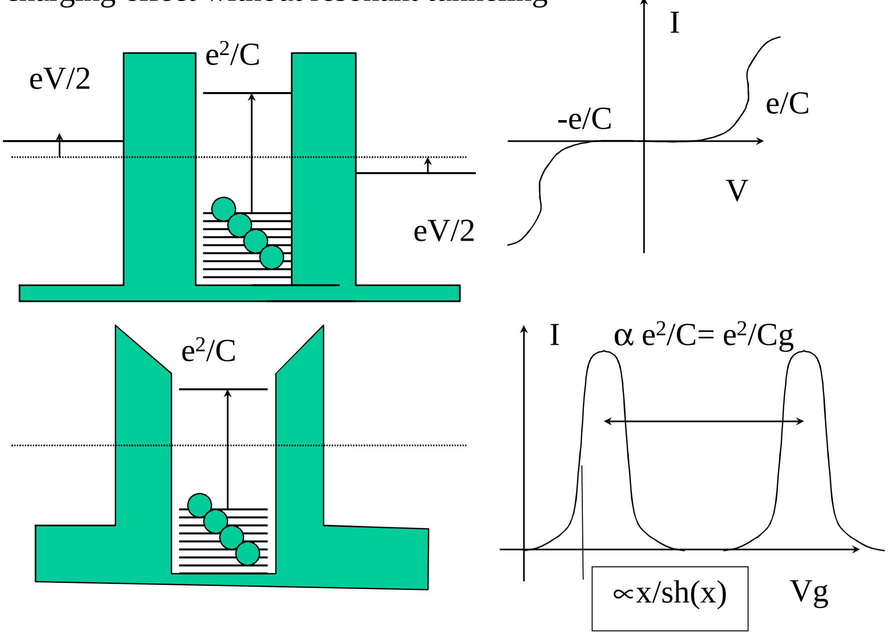
How robust are charging and quantum effects at room temperature ?

$e^2/2C$ (charge effect) and Δ (resonant tunneling) (much) larger than k_BT=25meV at T=300K

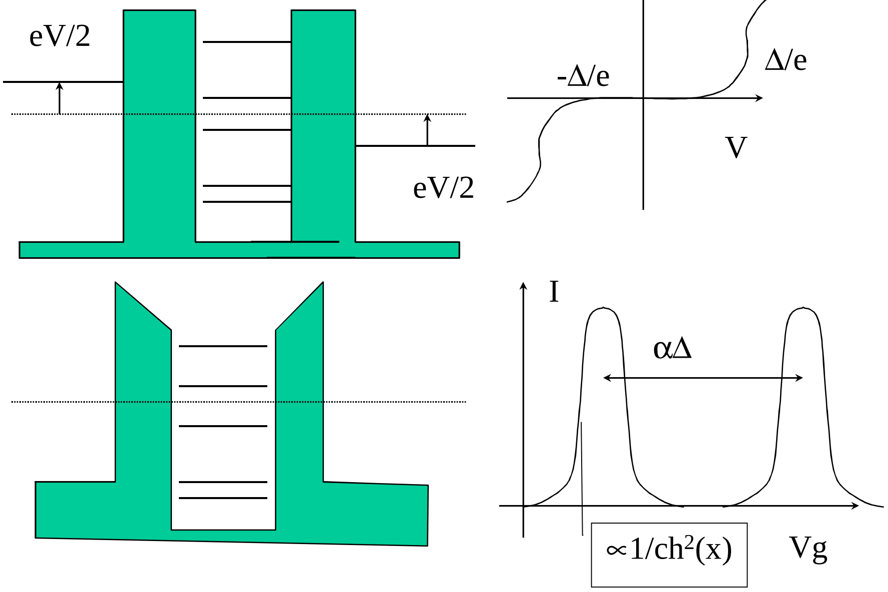
Also larger than eVd : small Vd, small current (eventually high current density)

Should be verified for 2-5 nm size

charging effect without resonant tunneling

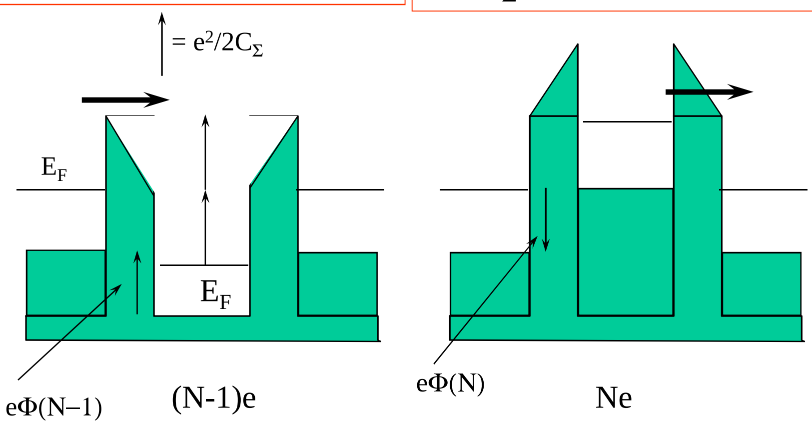


resonant tunneling without charging effect



$$\frac{e^2}{2C_\Sigma} + E_F = E_F + e\phi\,(N-1)$$

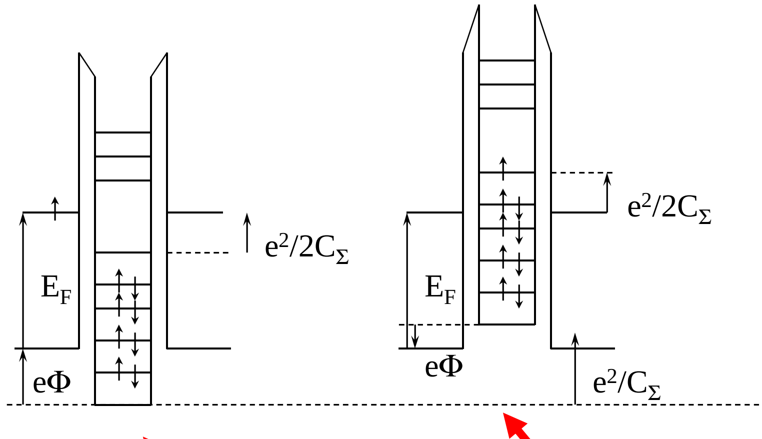
$$-\frac{e^2}{2C_\Sigma} + E_F = E_F + e\phi\,(N)$$



$$\Phi(N) = -eN/C_\sum + \Phi_{ext}$$

$$\Phi_{ext} = \frac{C_G V_G}{C_\sum}$$

(e>0)



$$\boxed{\frac{e^2}{2C_\Sigma} + K_N = E_F + e\phi\,(N-1)}$$

$$\boxed{-\frac{e^2}{2C_\Sigma} + K_N = E_F + e\phi\,(N)}$$

$$\Phi(N) = -eN/C_{\Sigma} + \Phi_{ext}$$ e>0

orthodox model:

$$E(N) - E(N-1) = 0 \qquad \left(N - \frac{1}{2}\right)\frac{e^2}{C_\Sigma} = e\phi_{ext}$$

with an E_N spectrum

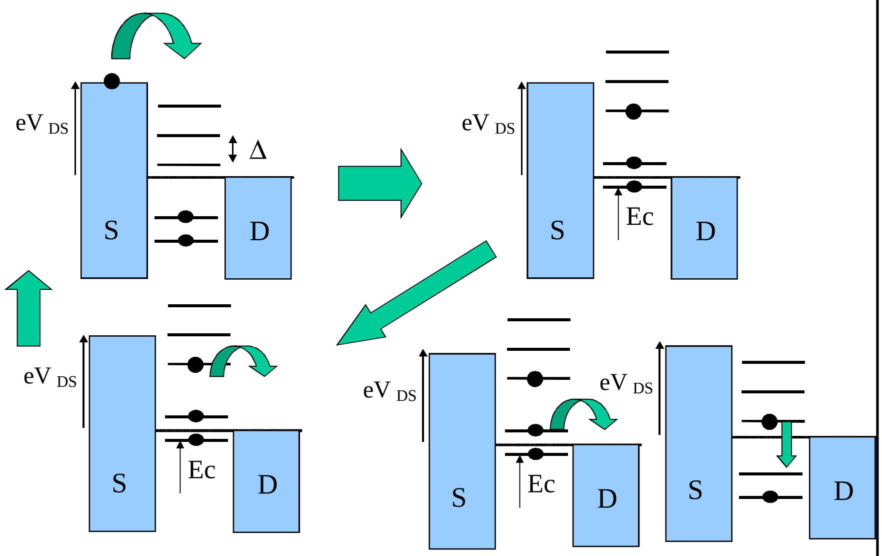
$$E(N) - E(N-1) = E_F - K_N$$

$$\left(N - \frac{1}{2}\right)\frac{e^2}{C_\Sigma} + K_N = E_F + e\phi_{ext}$$

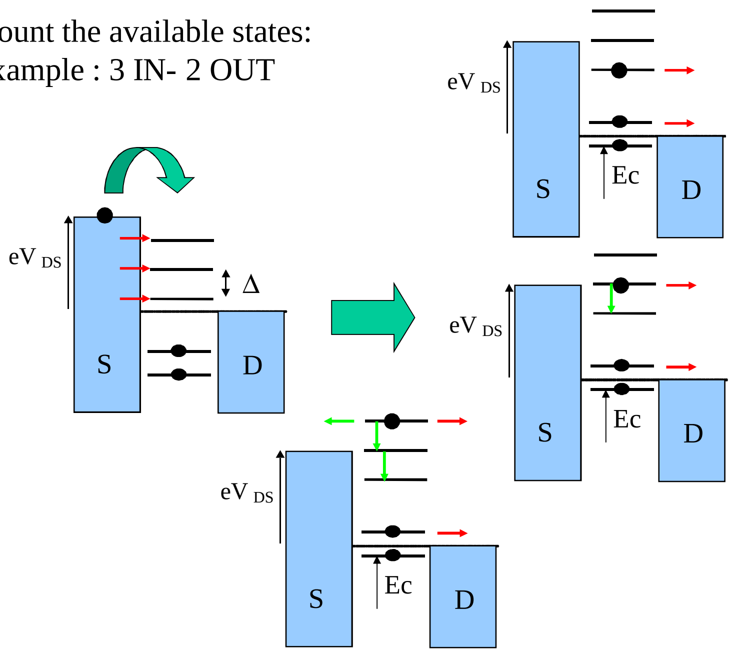
Hypothesis: E_F does not vary with ϕ_{ext}

$$\frac{e^2}{C_\Sigma} + \Delta K_N = e\Delta\phi_{ext} = e\frac{C_G}{C_\Sigma}\Delta V_G$$

Some effects of finite Δ (at finite bias)

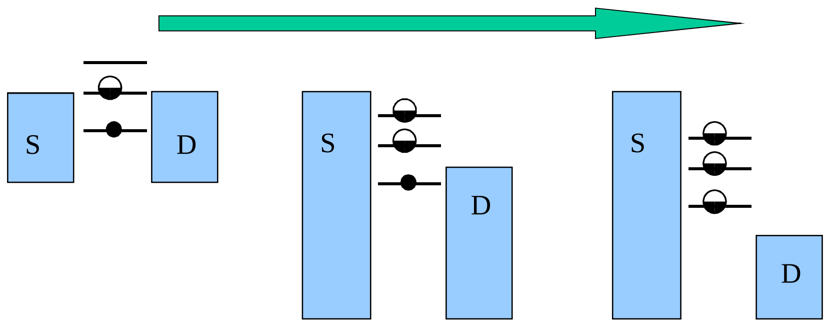


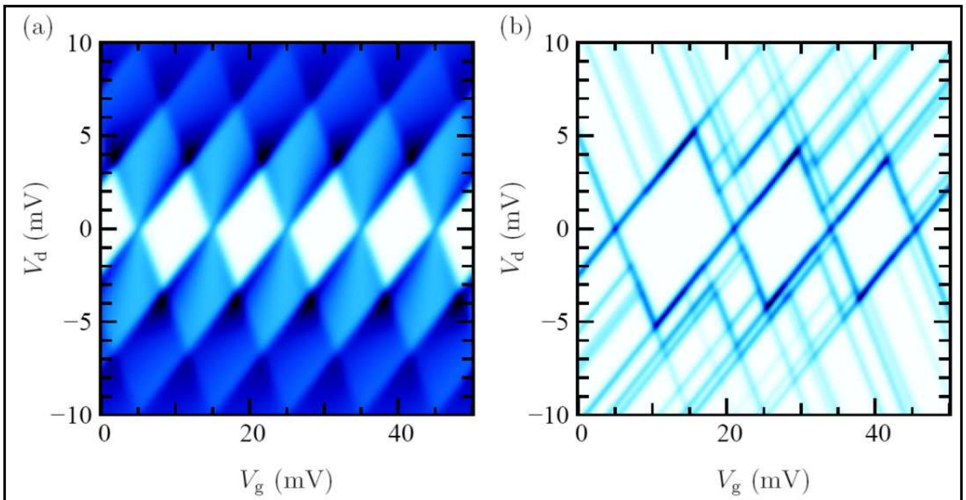
Count the available states:
example : 3 IN- 2 OUT



1-2 electrons

Id increases

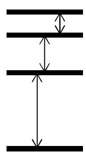




calculated spectrum : T=1K, Cs=Cd=Cg=100 e/V for both cases

left =continuum

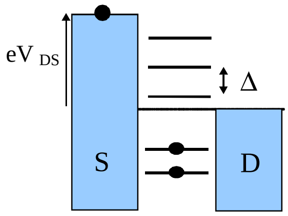
Δ=2, 1, 0.5 meV

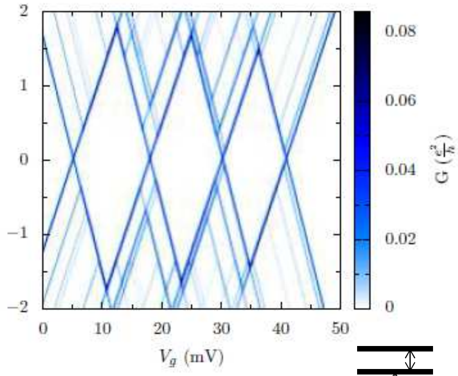


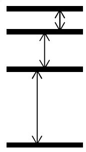
PhD Max Hofheinz 2006

*M. Pierre et al. Eur. Phys. J. B **70**, 475–481 (2009)*

Excitation spectrum of the dot

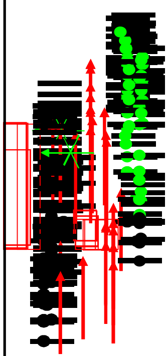


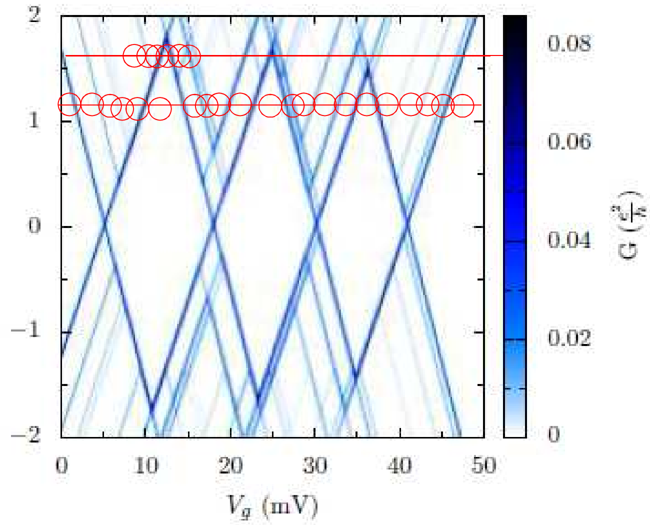




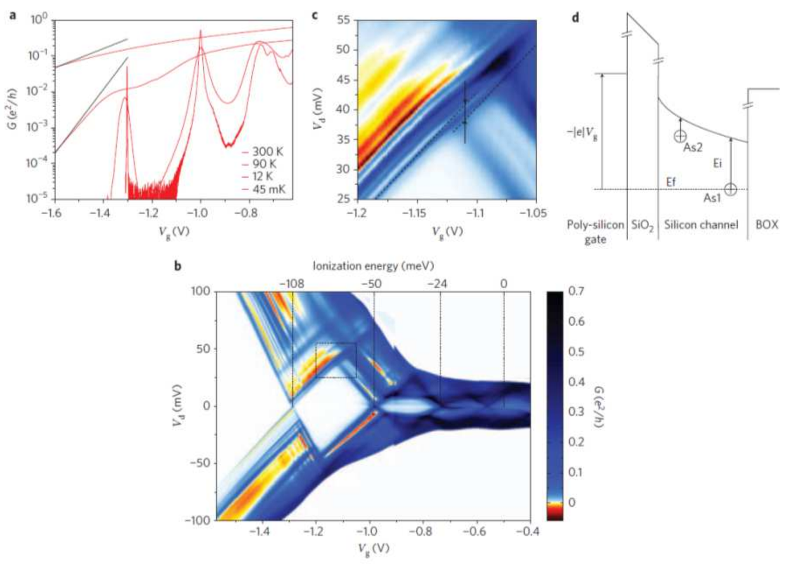
Ec increases as D^{-1}, Δ increases as D^{-2}

Δ= 0.4,0.3,0.1meV
Ec=1.38meV
T=0.1 K

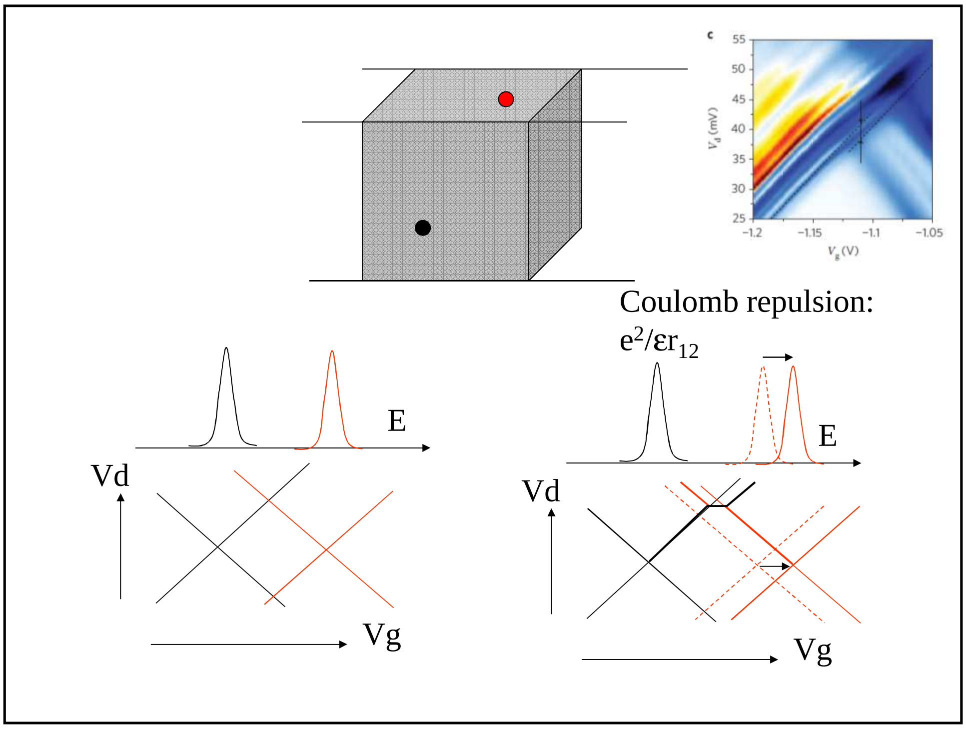


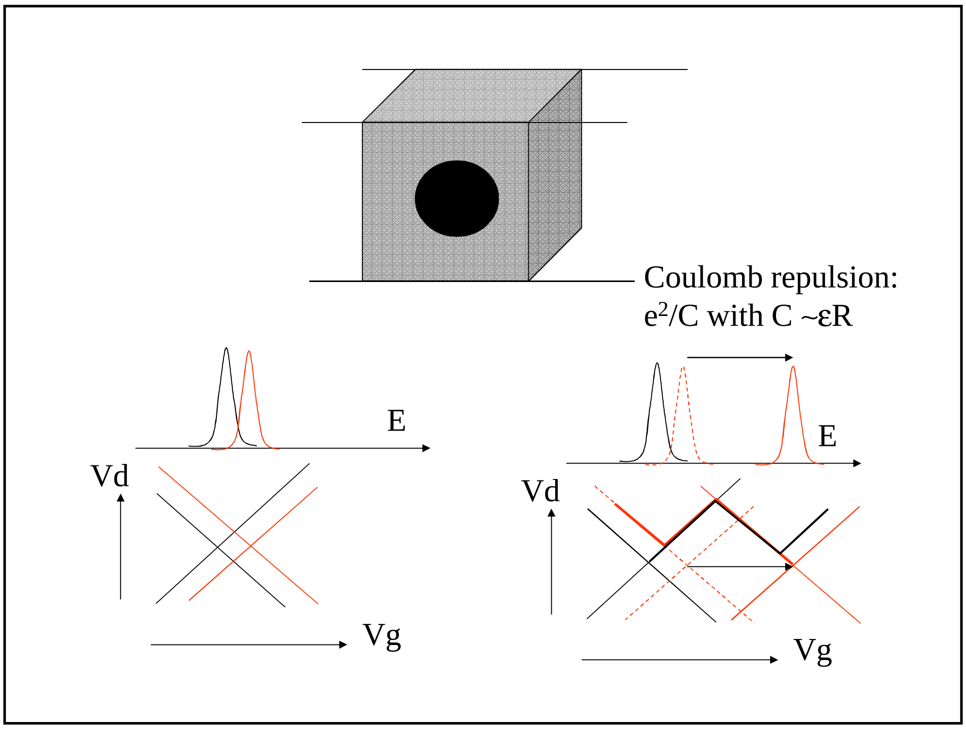


Coupled dopants: electron Coulomb repulsion

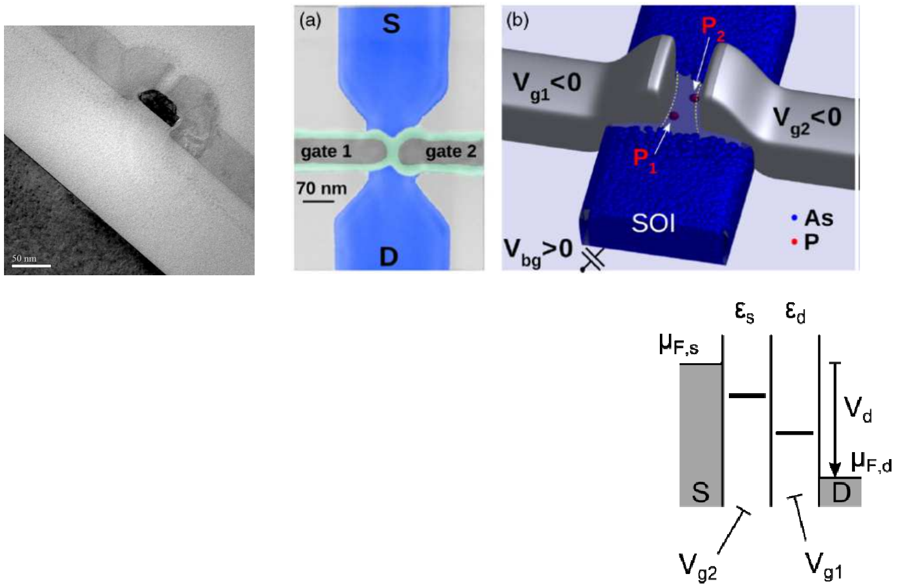


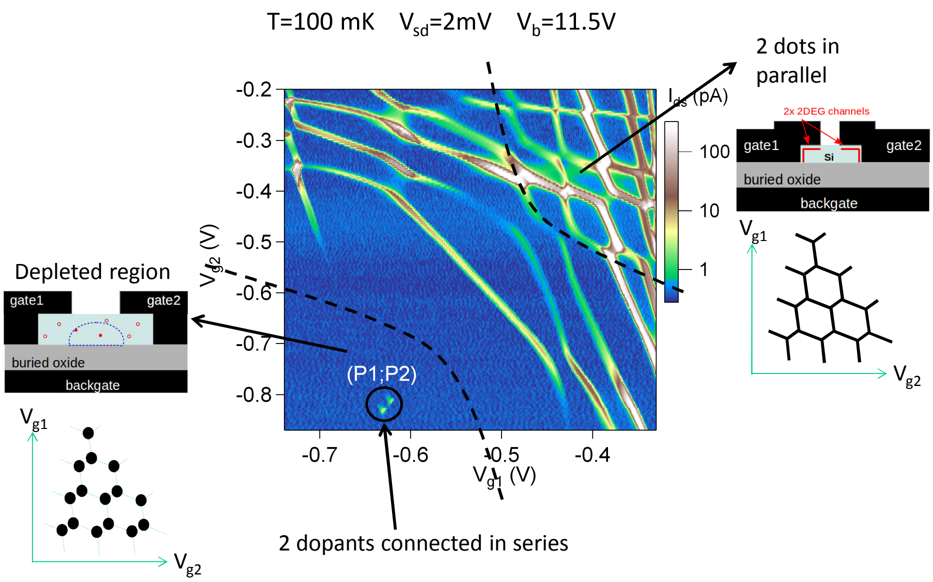
M. Pierre *et al.*, Nat. Nanotechnol. **373**, 10.1038, (2009)


c
55
50
45
40
35
30
25
V_d (mV)
−1.2
−1.15
−1.1
−1.05
V_g (V)
Coulomb repulsion:
$e^2/\varepsilon r_{12}$
E
Vd
Vg
E
Vd
Vg


Coulomb repulsion:
e^2/C with C ~εR
E
Vd
Vg
E
Vd
Vg

The coupled atom transistor (B. Roche et al. PRL2012)



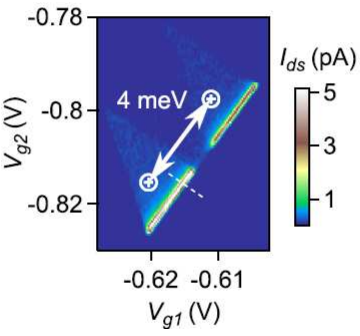


114

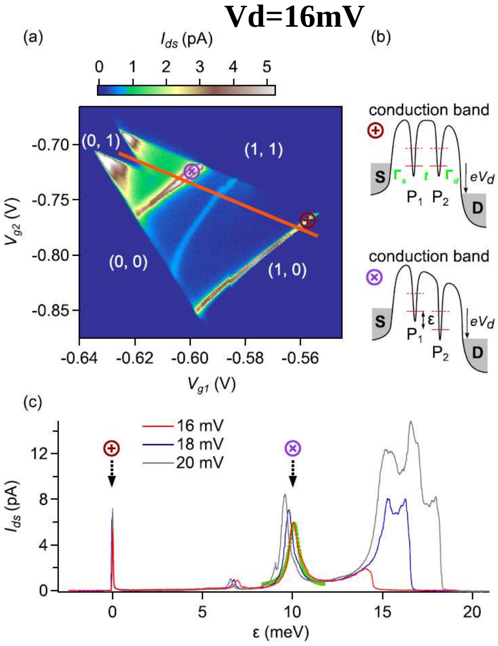
Finite bias triangles

Roche et al PRL 2012

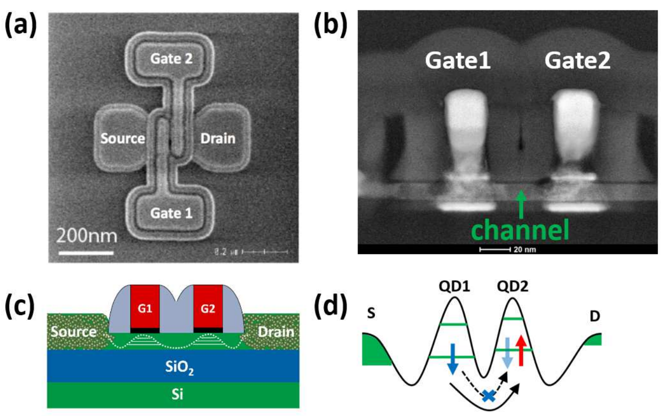
Vd=16mV



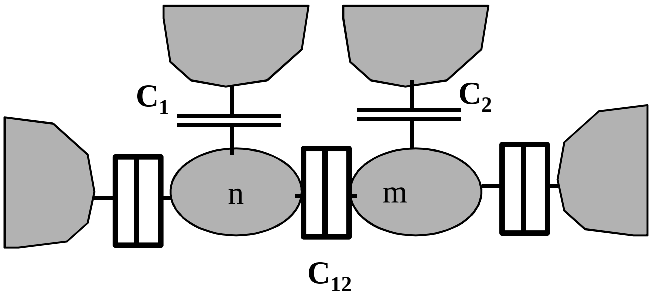
Vd=3mV



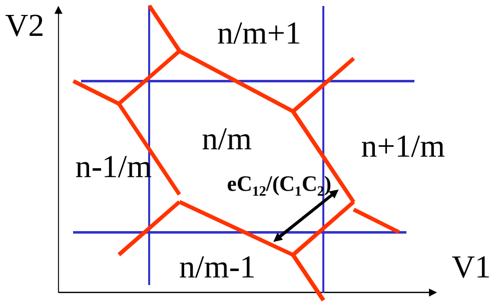
Coupled quantum dots



(Holes)

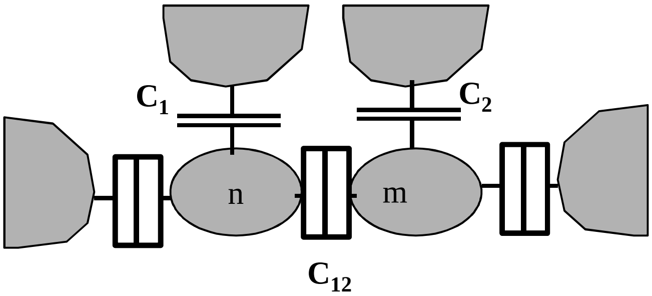


Doubled quantum dot

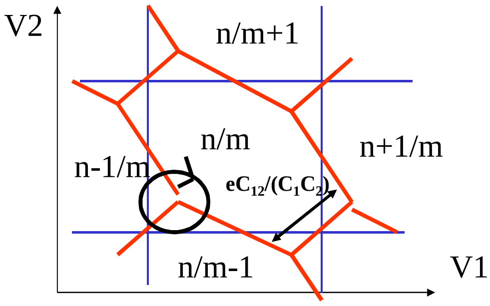


Van der wiel et al.
Rev. Modern Phys.75,1 ('03)

Double dots & their applications as pumps

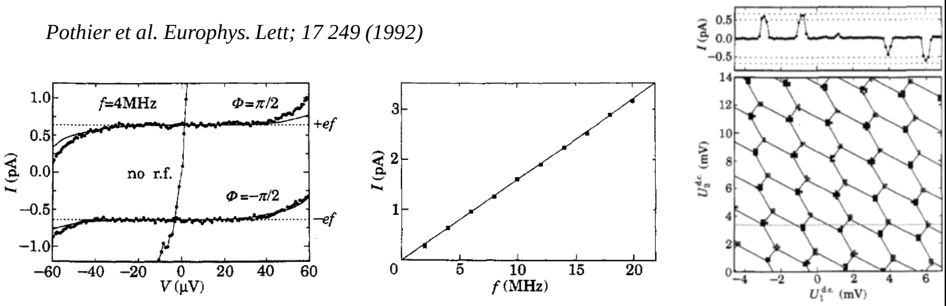


Doubled quantum dot= electron pump



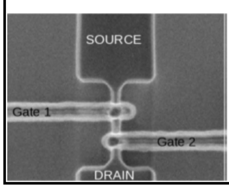
Van der wiel et al.
Rev. Modern Phys.75,1 ('03)

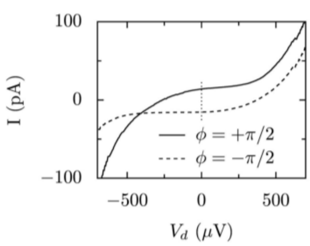
Pothier et al. Europhys. Lett; 17 249 (1992)

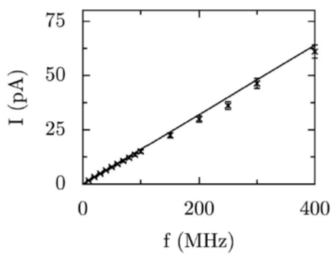


See also: A. Fujiwara et al., "Nanoampere charge pump by
single-electron ratchet using silicon nanowire MOSFET ", *Appl. Phys. Lett.*, vol. 92, 042102, 2008.

M. Pierre et al. CPEM2010





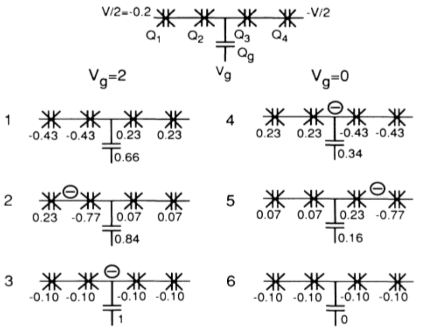


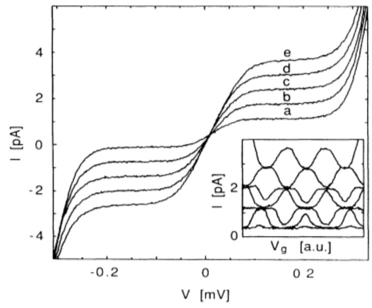
« electron turnstile », (non stochastic) current given by the frequency :

$$I = ef$$

f=4,8,12,16,20 Mhz

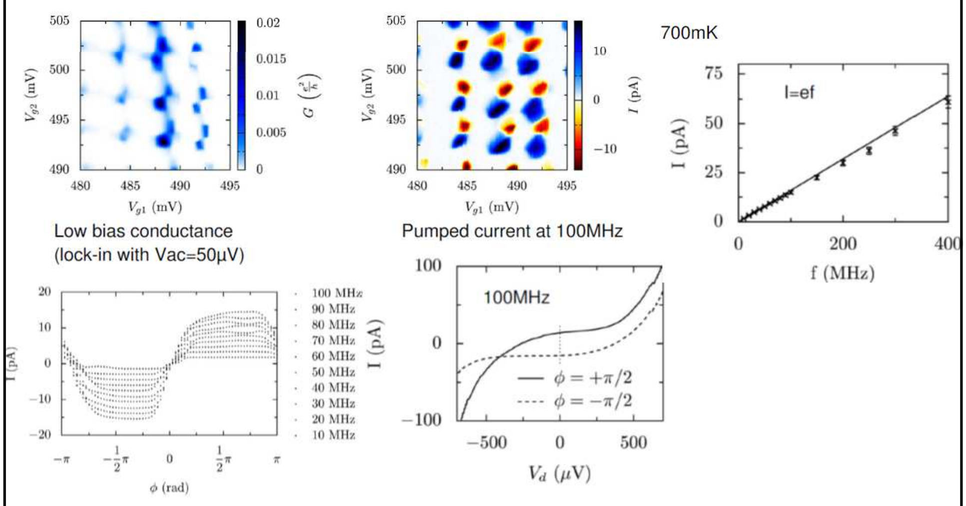
Pothier et al. 1991





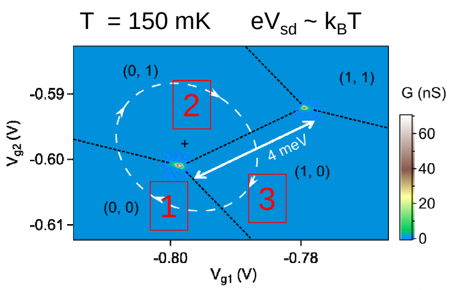
electron pump (see later on)

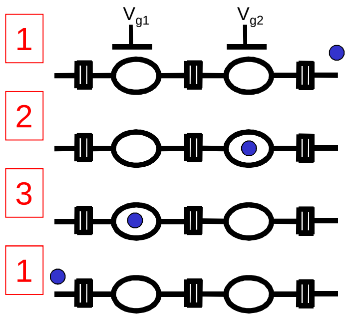
M. Pierre et al. CPEM2010



Electron pumping (V_{sd} = 0)

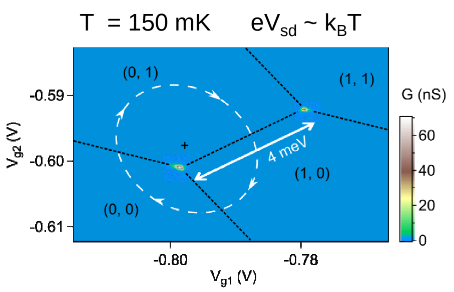
If one electron is transfered each period: I_{ds} = ef
(= 0,16 pA / MHz)

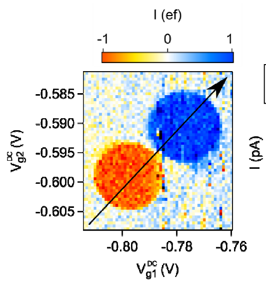




Pothier et al.
Europhysics Letters **17**, 249 (1992)

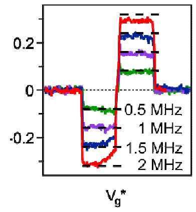
Electron pumping – adiabatic regime





f = 1 MHz

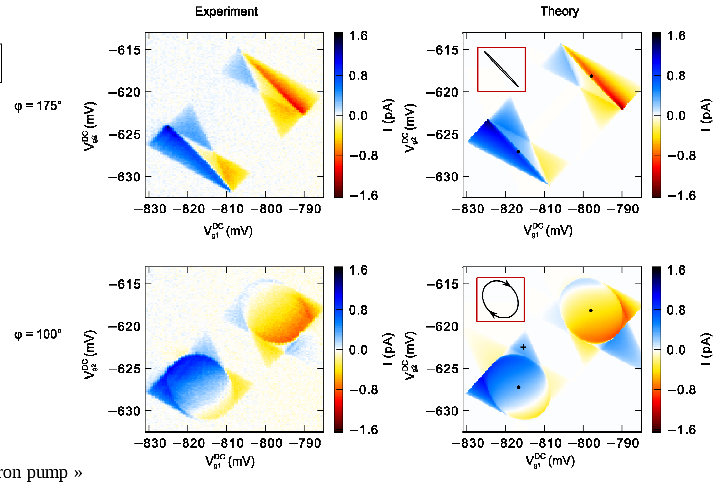
Increasing frequency:
(cut along the arrow)



« A two-atom electron pump »
B. Roche, R.-P. Riwar, B. Voisin, E. Dupont-Ferrier, R. Wacquez,
M. Vinet, M. Sanquer, J. Splettstoesser & X. Jehl
Nature Communications 2013

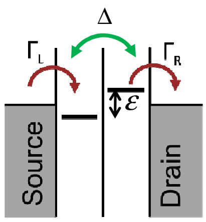
Electron pumping – nonadiabatic regime

f = 10 MHz



« A two-atom electron pump »
B. Roche, R.-P. Riwar, B. Voisin, E. Dupont-Ferrier, R. Wacquez,
M. Vinet, M. Sanquer, J. Splettstoesser & X. Jehl
Nature Communications 2013

Model



Δ/h = 1.2 GHz
Γ_L = 20 MHz
Γ_R = 700 MHz

- Master equation:

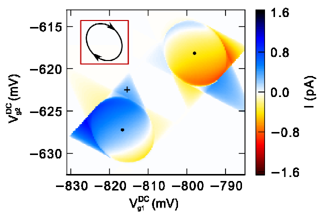
$$\frac{d}{dt}\mathbf{P}(t) = \mathbf{W}(t)\cdot\mathbf{P}(t)$$

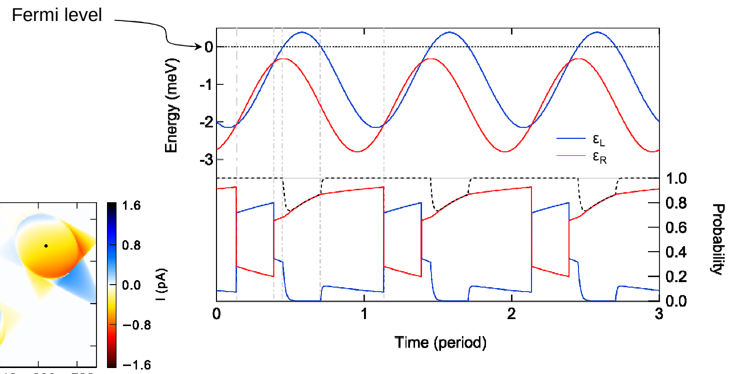
- When the levels cross
  → Landau-Zener transition

Probability for the electron to hope on the other donor:

$$P_{LZ} = 1 - \exp\left(-\frac{\pi}{2}\frac{\Delta^2}{hv}\right) \quad ; \quad v = \frac{d\varepsilon}{dt}$$

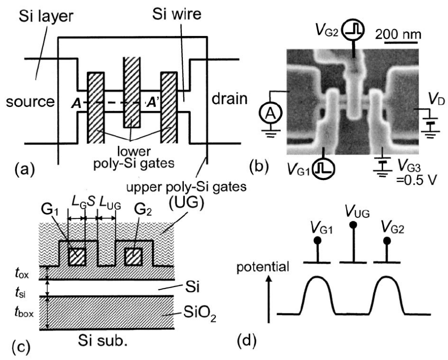
Levels & probabilities during pumping

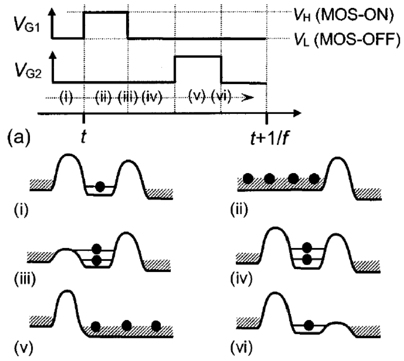


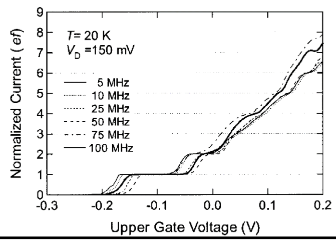


CB vs CCD electron pump

Fujiwara et al. APL2004







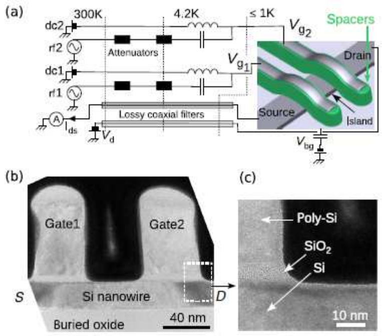
$$I = Nef$$

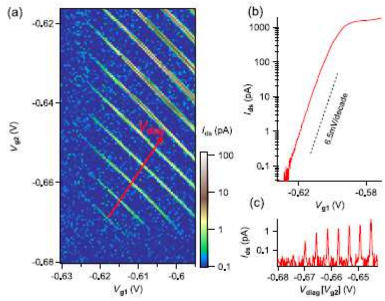
f= pump frequency

(not eV_d/h) ;N depends on potentials

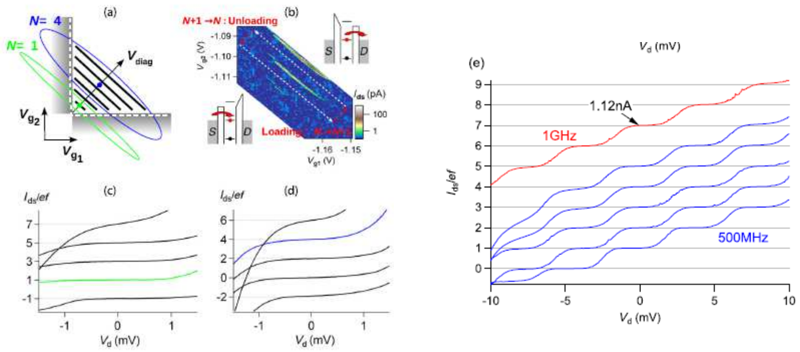
X. Jehl et al. PHYS. REV. X 3, 021012 (2013)

Hybrid Metal-Semiconductor Electron Pump for Quantum Metrology



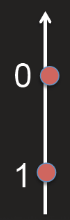


X. Jehl et al. PHYS. REV. X 3, 021012 (2013)



Silicon quantum bits

Quantum Logic



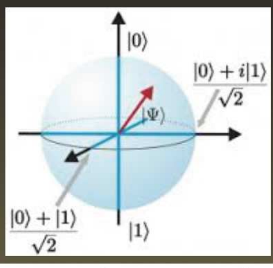
Ordinary computer with binary logic:

Classical bit: 0,1
N-bit state: defined by a single combination of N bits (e.g. 01001…01)

Quantum computer with two-level qubits:

Quantum bit: $|\psi\rangle = a|0\rangle + b\,|1\rangle$

2-qubit state: $|\psi\rangle = a|00\rangle + b|10\rangle + c|01\rangle + d|11\rangle$
3-qubit state: $|\psi\rangle = a|000\rangle + b|100\rangle + c|010\rangle + \ldots$
….



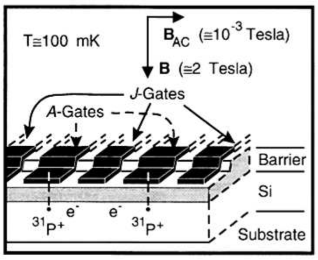
N-qubit state defined by 2^N complex numbers!

Naturally built-in parallelism

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

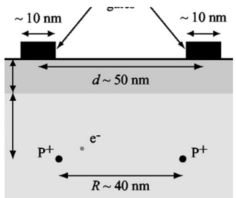
Silicon Spin qubit

Spin qubit

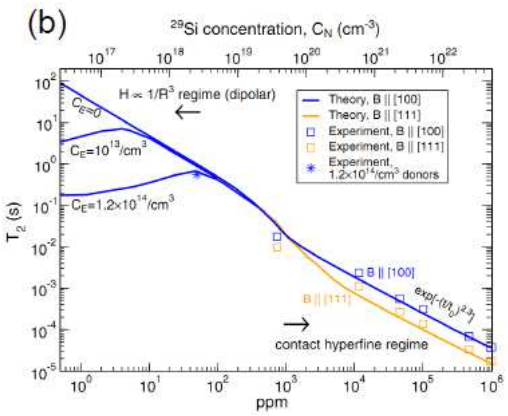


B. E. Kane, *Nature* 393, 133 (1998)

Charge qubit



Advantage of silicon: few nuclear spins/ isotopic purification= « spin vacuum »



Electron spin on donor in isotopically pure Si

Witzel et al. PRL105 187602 (2010)

Nuclear spin on donor P:T_{2N}=192s @T=1,8K in ^{28}Si:P (5 10^{11}cm^{-3})

Science 336, 1280 *Muhonen et al., arXiv:1402.7140*

T_{2e}*=0,12ms T_{2e}=28ms electron spin on quantum dot in^{28}Si (T_{2e}=40µs , T_{2e*}=1µs in QuDot ^{nat}SiGe)

Veldhorst et al., Nature Nano. 2014 arXiv: 1407.1950

Setup for isotopes separation at LLC „Electro-Chemical Plant“ (ECP) in Zelenogorsk, Krasnoyarsk region (Siberia)

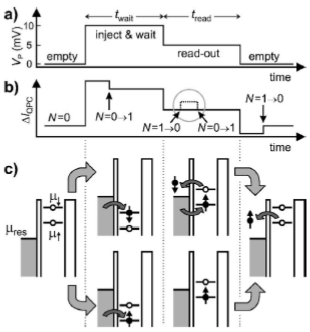
(Courtesy P. Sennikov IChHPS, Nizhny-Novgorod (RF)

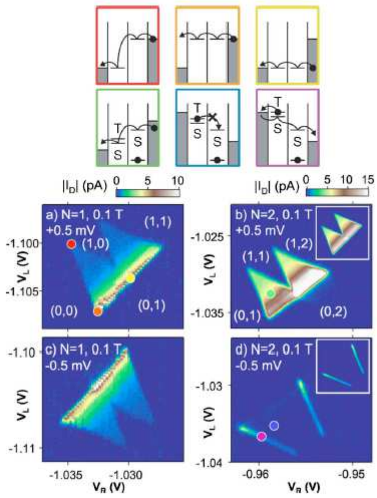
Si species	28	29	30
nat	92,23	4,67	3,1
^{28}Si	99,9986	0,00082	0,00058

« Silicon Vacuum »

(Courtesy P. Sennikov IChHPS, Nijni-Novgorod (RF)

Pauli spin blockade in Double Quantum Dots
« Spins in few-electron quantum dots”
R. Hanson et al Rev. Mod. Phys. **79**, 1217



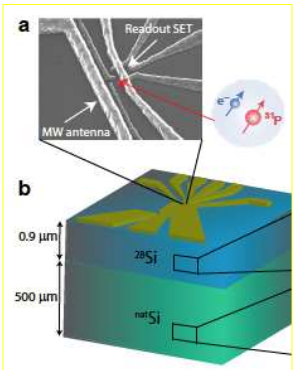


Detection of single spin
In QDot

Silicon spin qubit (magnetic manipulation)

<u>Electron bound to ^{31}P</u>

Muhonen et al., Nature Nanotechnology **9**, 986 (2014).

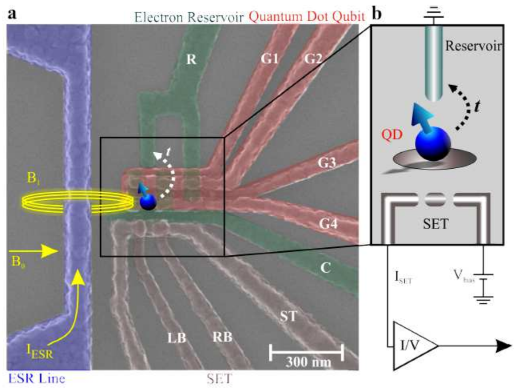


$T_2^* \sim$ 270 μs
$T_2^{echo} \sim$ 1 ms

$T_\pi \sim$ 150 ns

<u>Electron in Quantum Dot</u>

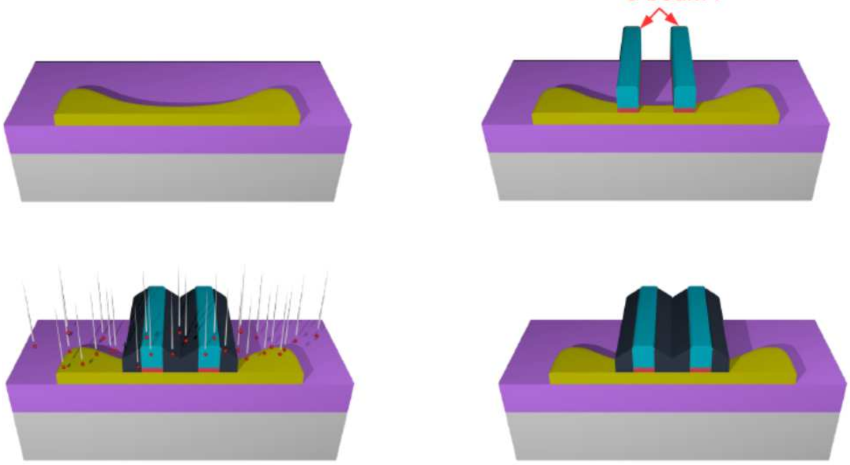
Veldhorst et al., Nature Nanotechnology **9**, 981 (2014).

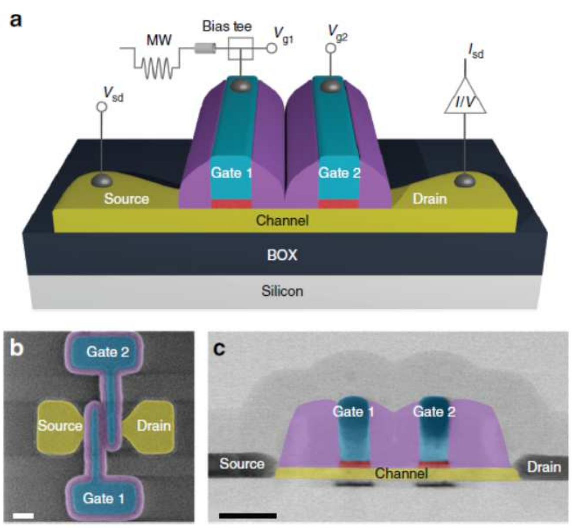


$T_2^* \sim$ 120 μs
$T_2^{echo} \sim$ 30 ms

$T_\pi \sim$ 1 μs

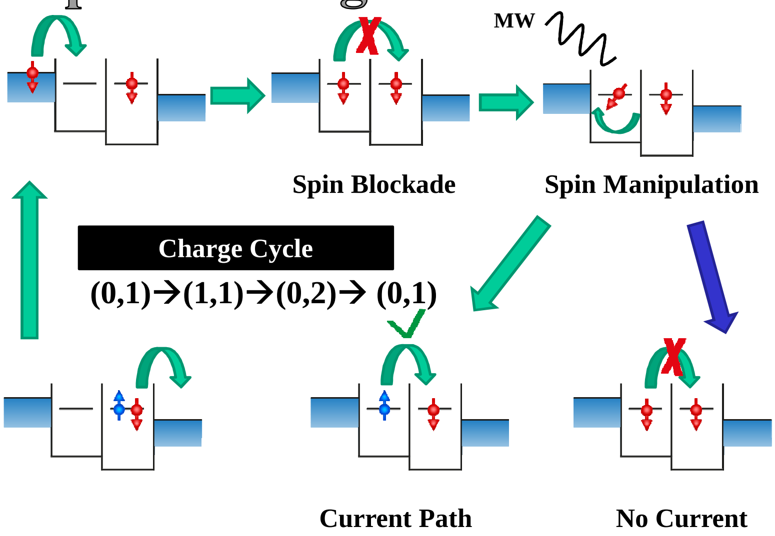
Fabrication



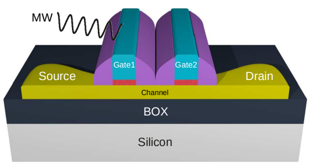


Scale bar 50 nm

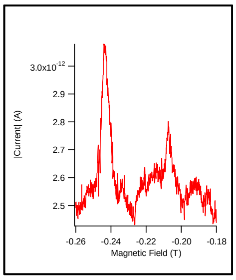
Spin to charge conversion

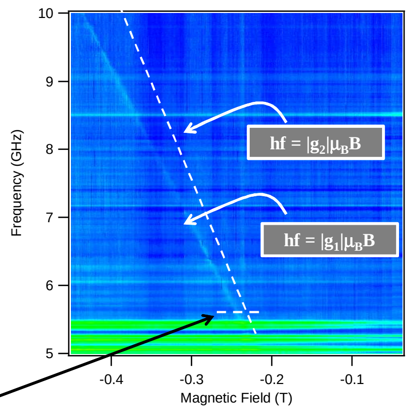


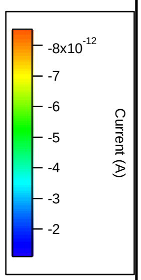
Hole spin resonance



Applying a MW excitation on Gate1





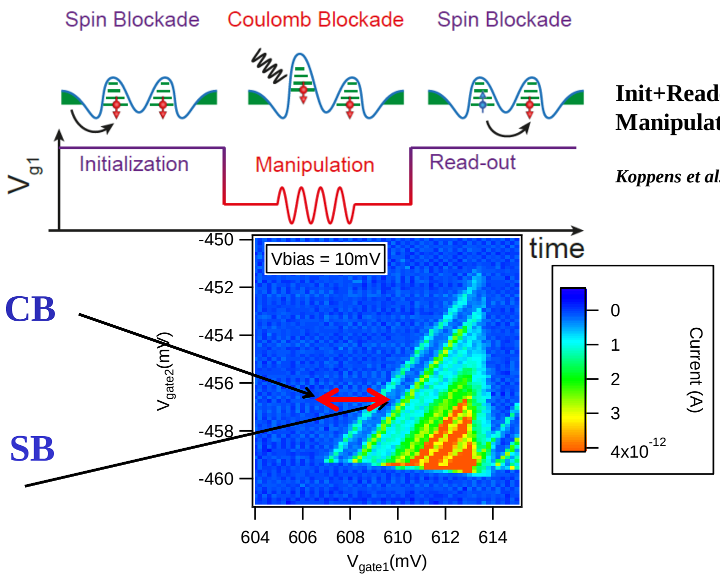


QD1 $|g_1| = 1{,}63$ QD2 $|g_2| = 1{,}92$

Hole spin resonance in InSb NWs, Pribiag et.al. Nat. Nano. 8, 170 (2013)

142

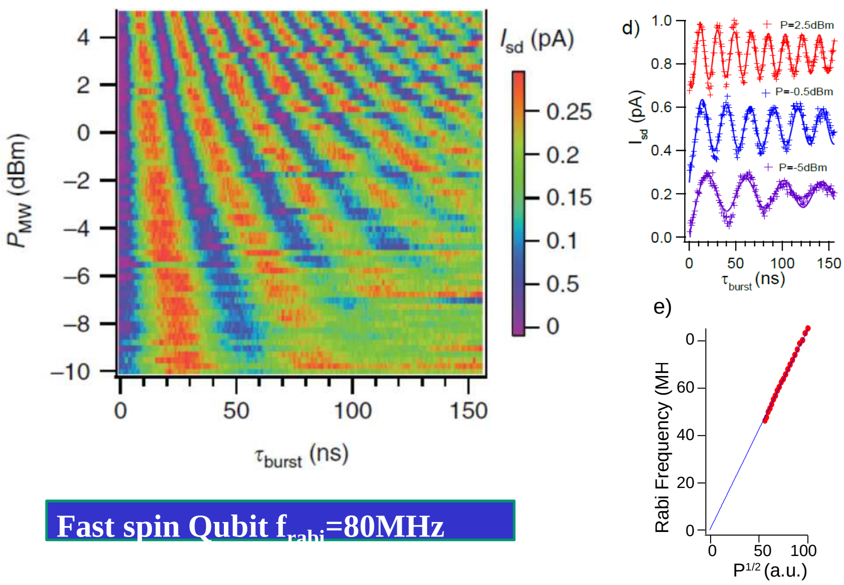
Gate voltage cycle



Init+Readout = 270ns
Manipulation = 175ns

Koppens et al. Nature 2006

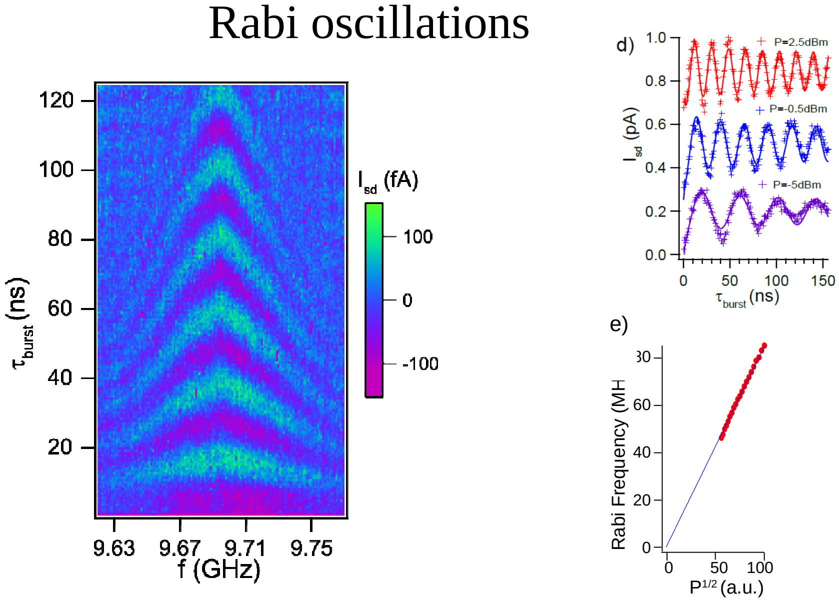
Rabi oscillations



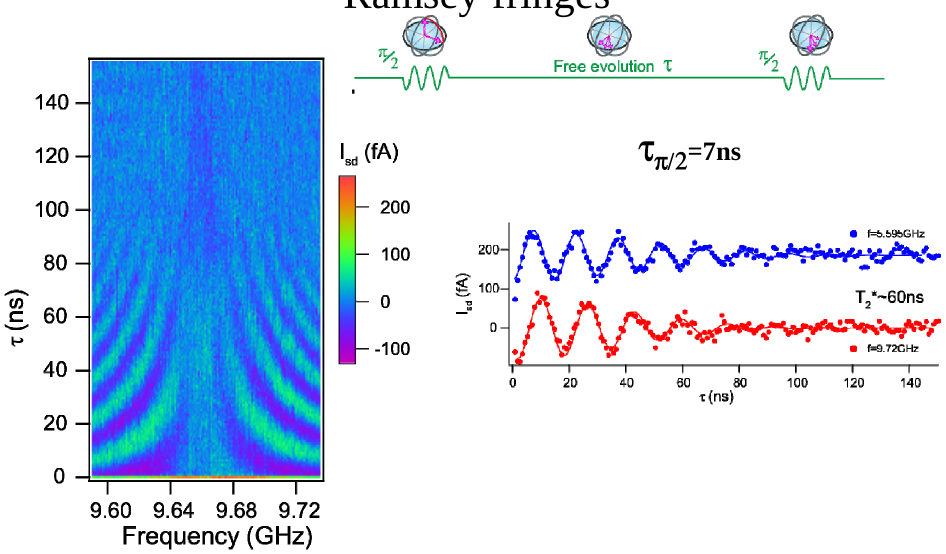
Fast spin Qubit f_{rabi}=80MHz

144

Rabi oscillations

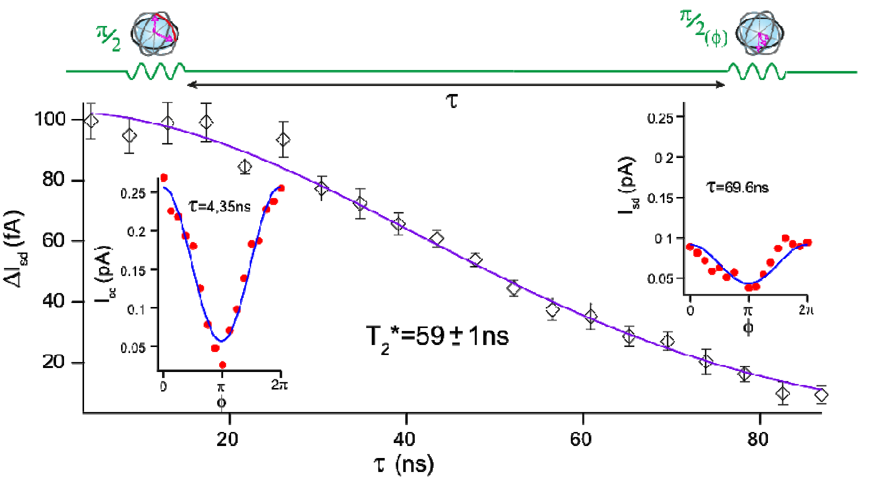


Ramsey fringes

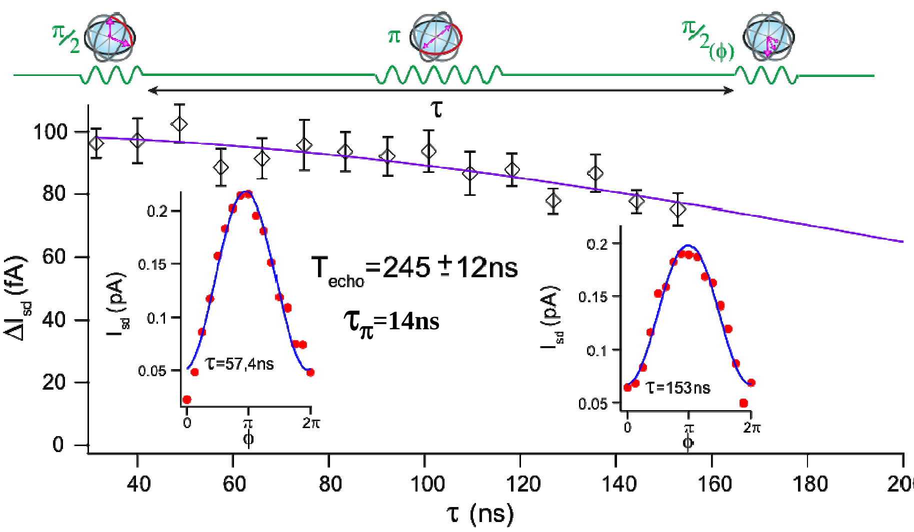


Ramsey fringes/ phase pulse control

Dephase the two bursts by Φ



Hahn Echo Sequence



Bibliography:

C. W. J. Beenakker ”Theory of Coulomb-blockade oscillations in the conductance of a quantum dot”, Phys. Rev. B44, pp 1646-1656, 1991.

D. V. Averin, A.N. Korotkov and K. K. Likharev ”Theory of single-electron charging of quantum wells and dots”, Phys. Rev. B44, pp 6199-6211, 1991.

Y. Alhassid “The statistical theory of quantum dots” , Rev. of Modern Physics 72, 895 (2000)

E. Akkermans et G. Montambaux “Physique mésoscopique des électrons et des photons “ Collections Savoir actuels EDP Sciences 2005

S. Datta: Electronic « Transport in Mesoscopic Systems »
Cambridge University Press 1995

T. Ando, A.B. Fowler and F. Stern,“Electronic properties of 2D systems” Review of Modern Physics 54, 437 (1982).

H. Grabert, M. Devoret, "Single charge tunneling Coulomb blockade phenomena in nanostructures", Vol. 294, NATO ASI Series B, Plenum Press, New York and London, physics edition , 1992

“Silicon Quantum Electronics “ Floris A. Zwanenburg et al. cond-mat1206.5202 (Rev. Modern Phys. 2013)

“Single dopants in semiconductors “Paul M. Koenraad and Michael E. Flatté, Nature Materials 2011 doi: 10.1038/nmat2940

« Spins in few-electron quantum dots” R. Hanson, L. P. Kouwenhoven, J. R. Petta, S. Tarucha, and L. M. K. Vandersypen Rev. Mod. Phys. **79**, 1217