New concepts and new devices for microelectronics

Maud Vinet, Marc Sanquer

2012

Quantum transport and Coulomb blockade in silicon nanodevices

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CEA-DSM-INAC-SPSMS

LaTEQS (Laboratoire de Transport Electronique Quantique et Supraconductivité)

http://www-drfmc.cea.fr/spsms/Phocea/Vie_des_labos/Ast/ast_visu.php?id_ast=205

Vendredi 23 mars 2012 M258 PHELMA Maud Vinet

14h00-15h15	Evolutions-Limites de la Microélectronique
15h15-15h30	Pause
15h30-16h15	Evolutions-Limites de la Microélectronique (suite)
16h15-16h30	Pause
16h30-17h30	Fabrication des SET, Multiple Tunnel Junctions, Few electrons memories, Digital and analog applications des SET …

Lundi 26 Mars 2012 M258 PHELMA Marc Sanquer

14h00-15h15	Scaling CMOS energy-time consideration; Shrinking the MOSFET: single dopant variability ; Single dopant –single electron transport
15h15-15h30	Pause
15h30-16h15	What is the physics involved in single dopant transport? Large mean level spacing, Resonant Tunnelling, confinement size close to Fermi wavelength ; Coupled dopants: electron interaction; Many dopants: impurity band, localization
16h15-16h30	Pause
16h30-17h30	Double occupation, Coulomb blockade, Orthodox theory ,Double dots,

Mercredi 28 Mars 2012 M258 PHELMA M. Sanquer

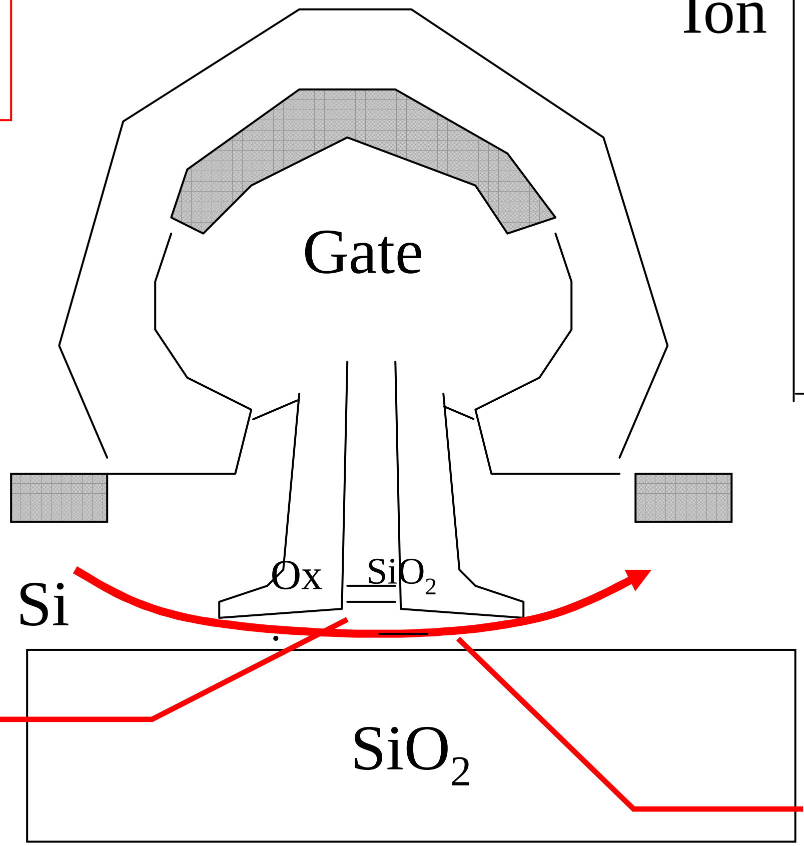
14h00-15h15	Qubits, orbital hybridization, ballistic transport, Landauer formula, transfer matrices,
15h15-15h30	Pause
15h30-16h15	SET versus FET MOS-SET devices
16h15-16h30	Pause
16h30-17h30	Room temperature SETs and quantum confinement effects

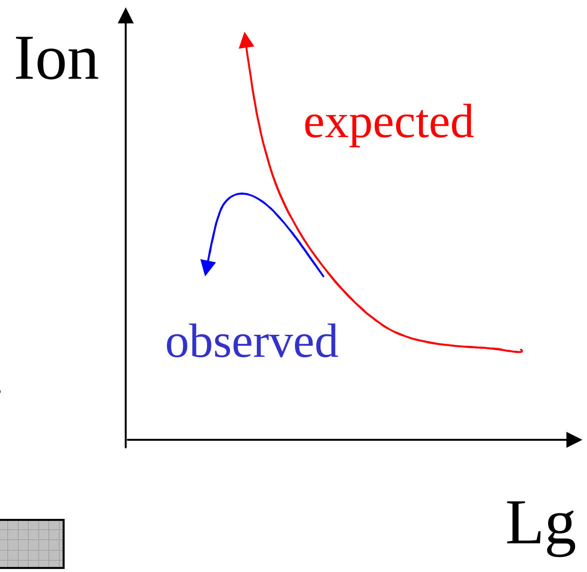
Down-Scaling CMOS:
energy-time considerations

ultimate MOSFET

Ion* α,
delay time/ α

Lgate/α
Wgate/α
Tox/α
Taccess/α..





B. Doris et al. IBM IEDM 2002

Gate length
6nm, Tsi=7nm

thin access

Ion saturation: ballisticity ? Ion decrease = parasitic access resistance ?

Lg=3.8nm MOSFET

Rsd~10^5 Ω

S-D Suk et al, VLSI Tech Symposium, Kyoto 2009,p142(Samsung)

- Active formation on SOI(or Poly Si on oxide)
- Damascene dummy deposition & photo
- Damascene dummy etch
- Self-limited inner spacer (gap formation)
- Implantation & Sicilidation after gate oxidation/TiN deposition & dummy removal on S/D

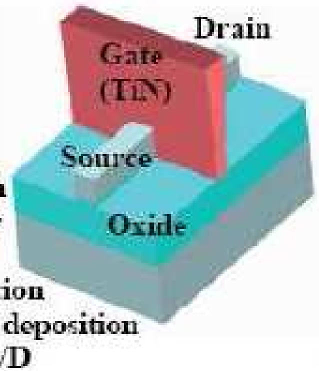


Fig. 1. Process flow sequence for sub 5 nm tri-gate nanowire MOSFET. L_G can shrink extreme size by newly developed self-limited spacer process in damascene region.

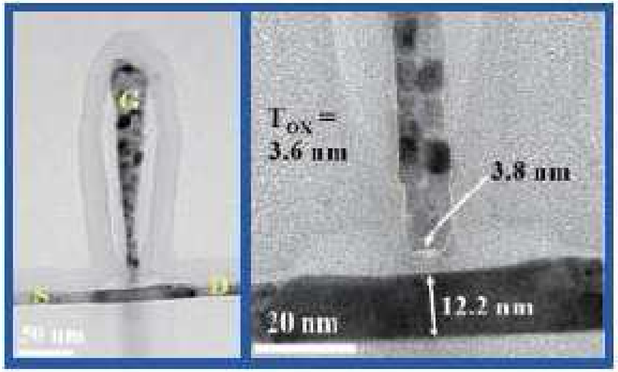


Fig. 2. L_G of 3.8 nm is achieved with well filled TiN gate. Nanowire height and gate oxide thickness are confirmed 12.2 nm and 3.6 nm, respectively.

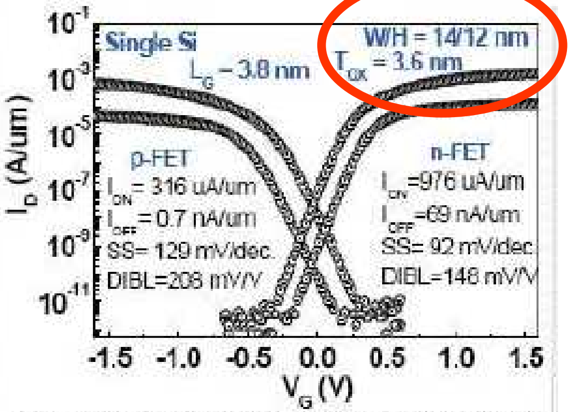


Fig. 3. I_D-V_G curves of single Si MOSFETs on SOI at L_G of 3.8 nm. n- and p-FET are well operated with good performance.

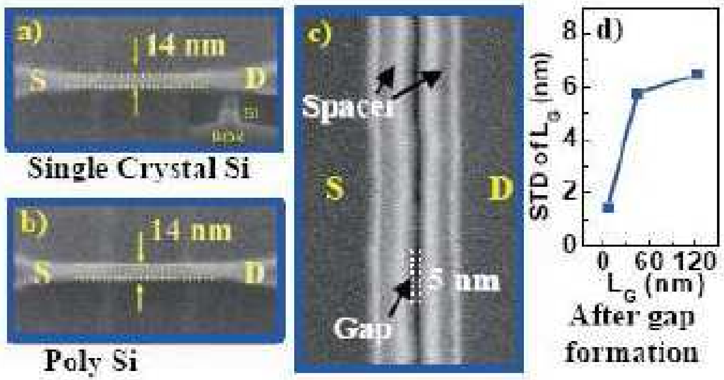


Fig. 4. a), b) SEM images of nanowires after active formation. c) SEM image of 5 nm damascene gap after new spacer process. Due to self-limited gap formation, smaller gap has better uniformity of L_G.

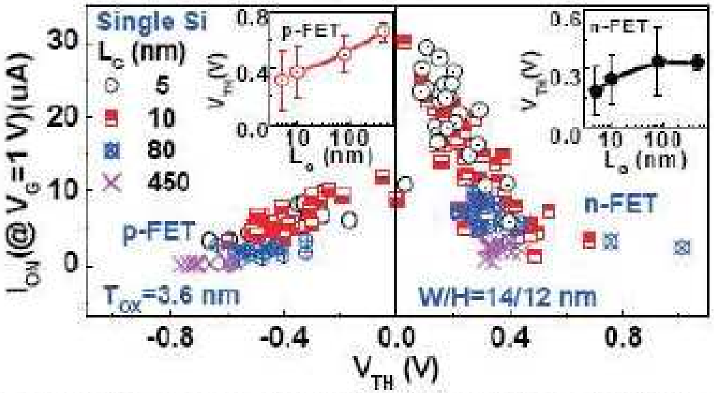


Fig. 5. V_{TH} is lowered about 0.15 V for n-FET and 0.3 V for p-FET as L_G reduces from 450 nm to 5 nm due to thin body tri-gate structure.

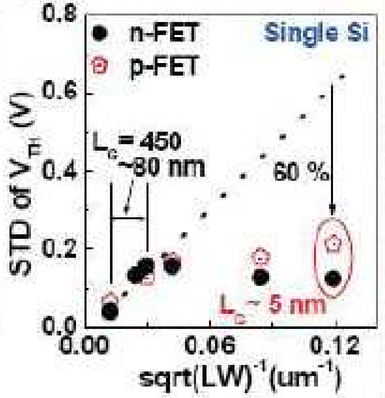


Fig. 6. Due to self-limited gap formation, variability improves 60 % in Pelgrom graph at L_G ~ 5 nm.

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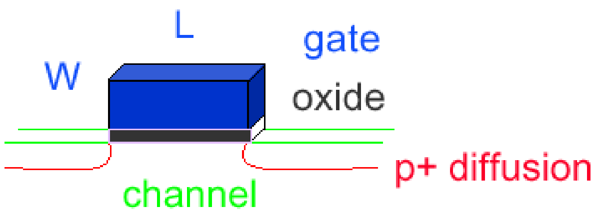
Shrinking down the MOSFET:



$L \Rightarrow L/\alpha$

$W \Rightarrow W/\alpha$

$d_{oxide} \Rightarrow d_{oxide} / \alpha$

$V \Rightarrow V/ \alpha$

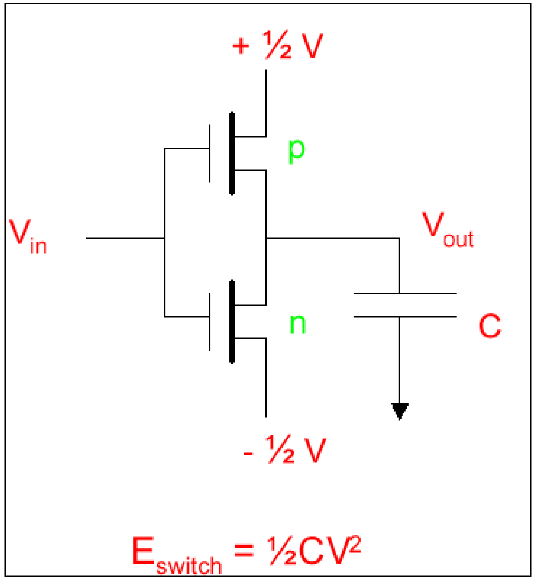
gate capacitance	$C = \varepsilon\, W\, L\, /\, d_{oxide}$	$\propto 1/\alpha$
switching energy	$E = ½\, C\, V^2$	$\propto 1/\alpha^3$
switching time	$\tau \propto L^2/V$	$\propto 1/\alpha$
switching power	$P = E/\tau$	$\propto 1/\alpha^2$

Power per unit area remains constant

Henk van Houten, LATSIS symposium, June 2000

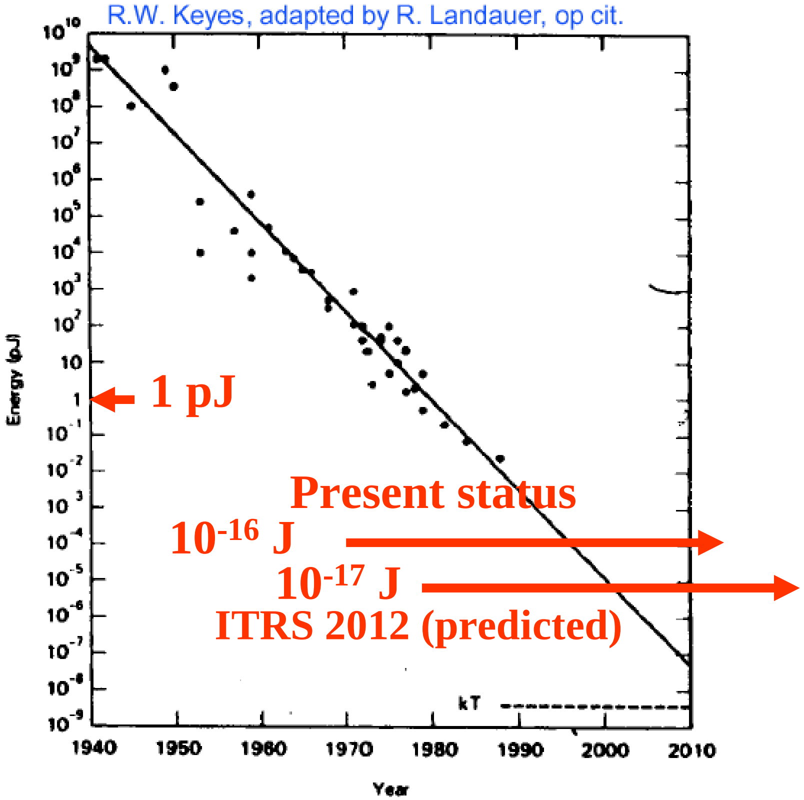
Scaling of switching energy

(Landauer '88)



Scaling : $E = ½\, C\, V^2 \propto 1/\alpha^3$

in10 years $\alpha \approx 5$ (Moore's law)

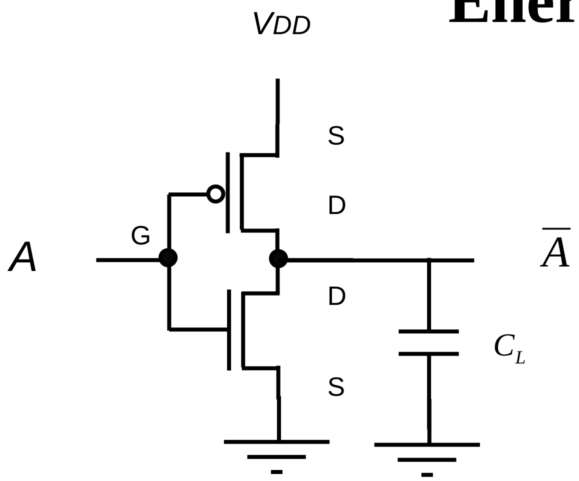


Henk van Houten, LATSIS symposium, June 2000

Today (energy switching, device level)
10^{-16} J

τ=Cg Vdd / I = 1 ps (low power)

Energy-time product ~10^{-27} Js >> h = 10^{-34} Js



PHYSICAL LIMITATIONS / energy considerations

physical limitations for digital calculation:

switching energy larger than temperature and should obey Heisenberg uncertainty principle

$$\boxed{E_{sw}=1/2 \times CV^2 > kT \ \text{ et } \ E_{sw} \times \tau > h/2\pi}$$

10nm/10nm ballistic FET: Gate capacitance (Tsi=1nm): 10^{-18} F

Channel resistance 30kΩ

➔Ballistic Transit time =10nm/v_F~10^{-13}s Power: 10µW

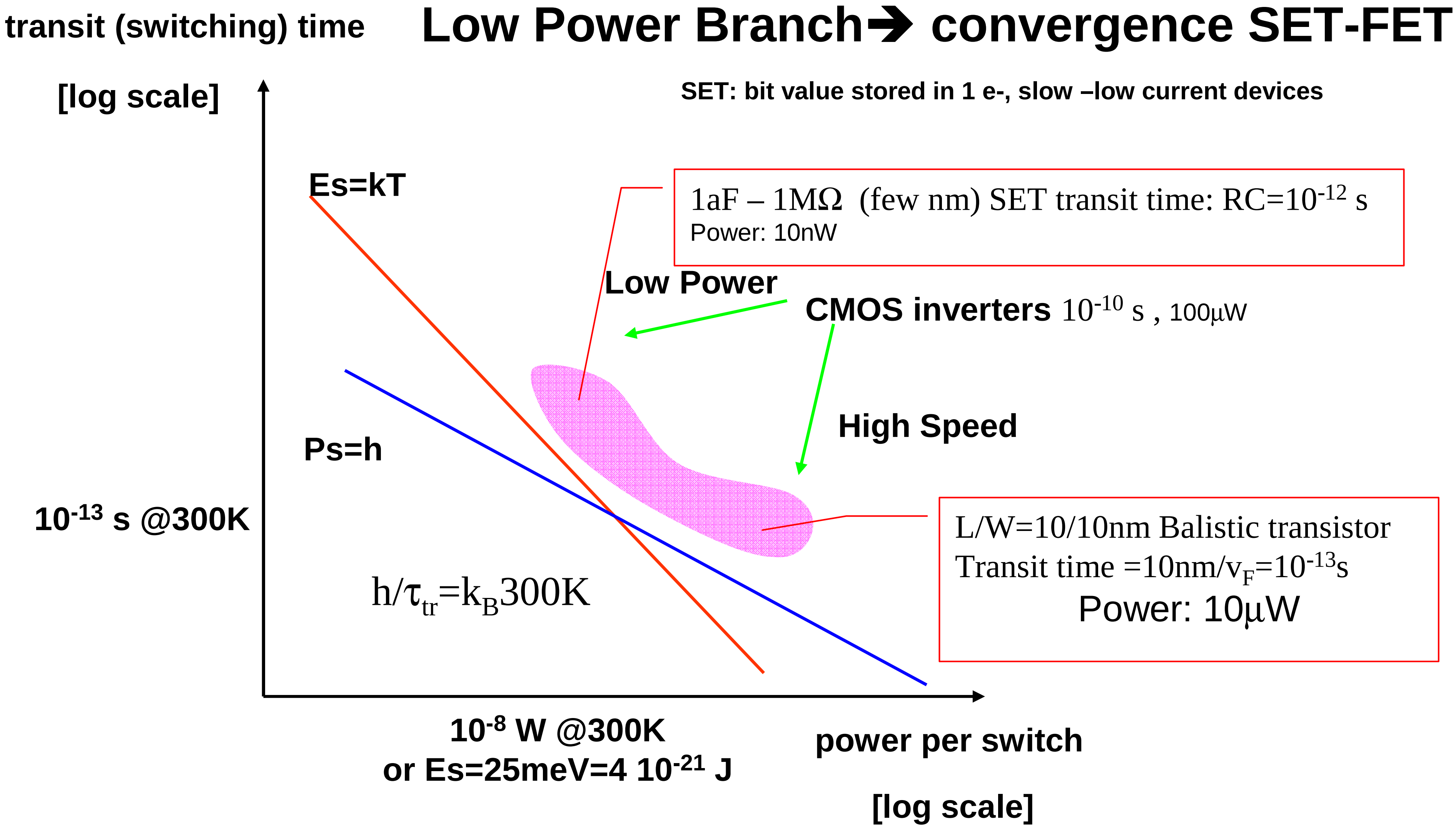
➔electronic circuit time constant :RC~ 10^{-13}s

➔Esw (@V=1V)= 1eV >>25meV=k_B 300K

➔Esw×transit time=10^{-31} >>h = 10^{-34} Js

still safe ....

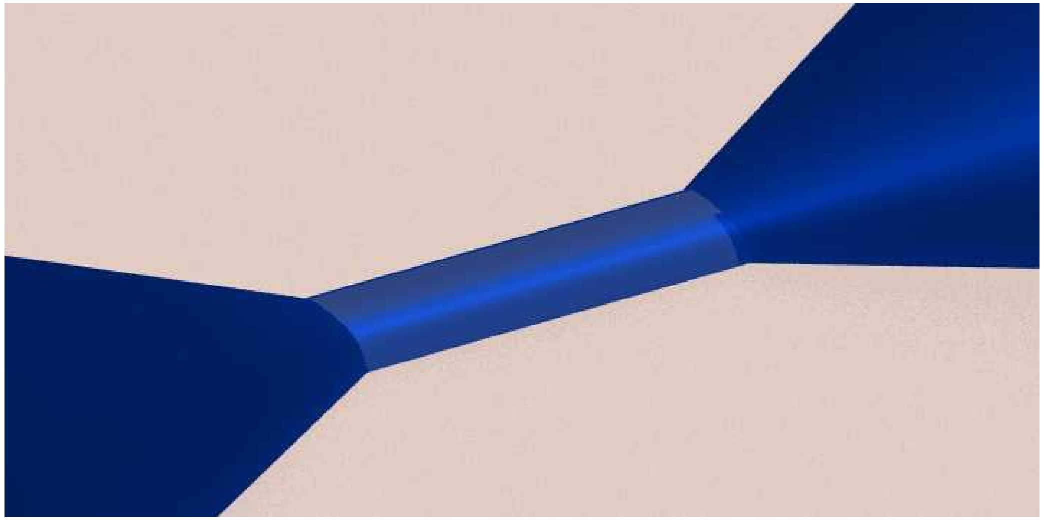
Low Power Branch➔ convergence SET-FET



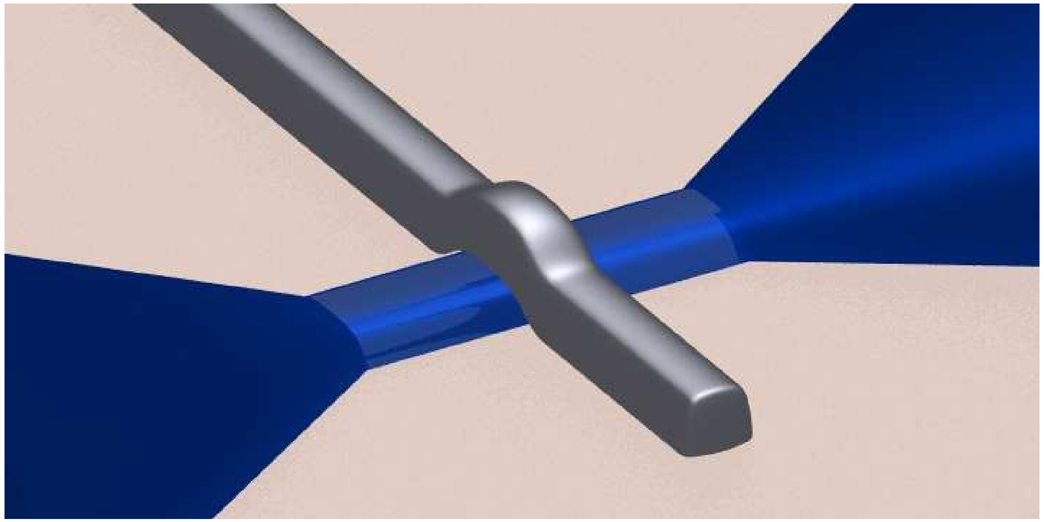
Short channel effect and dopant variability:
An exemple: the Single Atom Transistor

See also Fuechsle et al. Nature Nano 2012 for a bottom-up approach

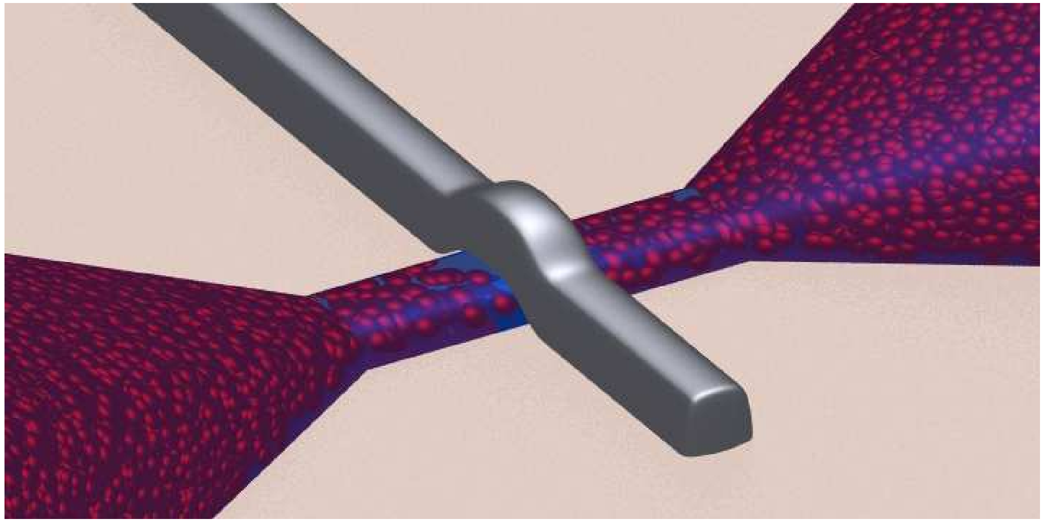
Sample fabrication: 200 mm wafers
8-20 nm Thick Silicon-on-Insulator (SOI)
e-beam litho of active layer (down to 20nm), Thermal oxidation (5nm SiO_2)



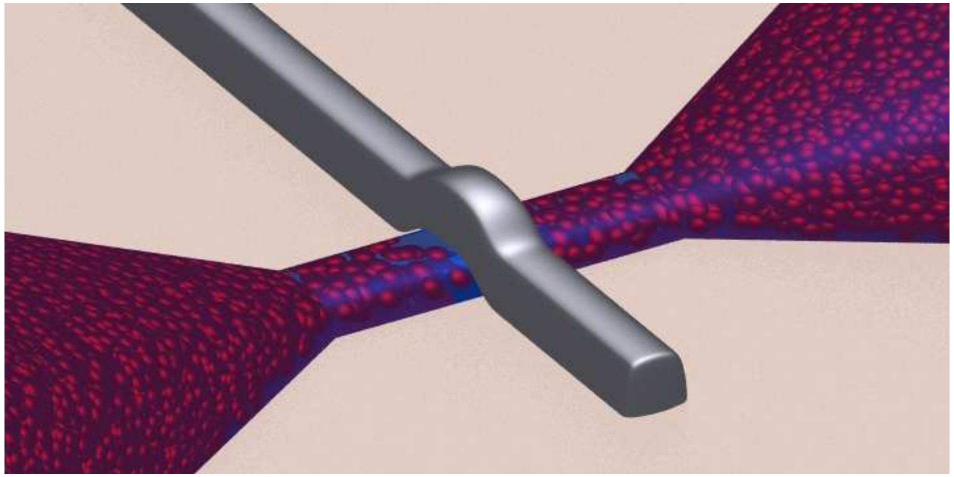
e-beam litho of poly Si gate layer gate length down to 20nm
Eventually multigate design= Pitch 70 nm

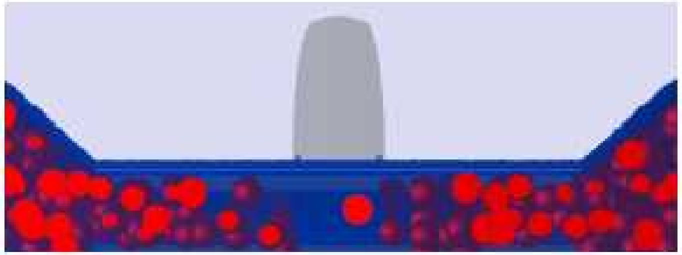


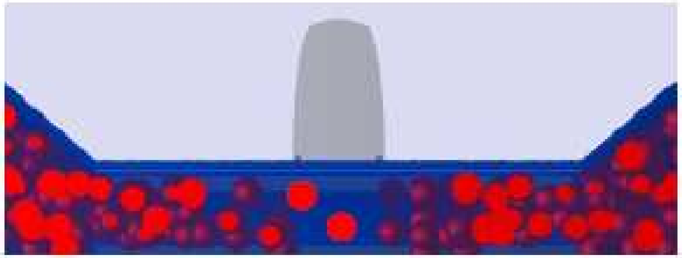
Arsenic implantation of highly doped Source and Drain
(channel masked by the gate stack)

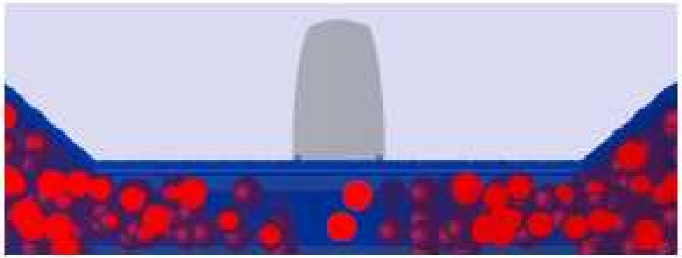


OVERLAP GATE/SOURCE-DRAIN
No (or thin) nitride spacers







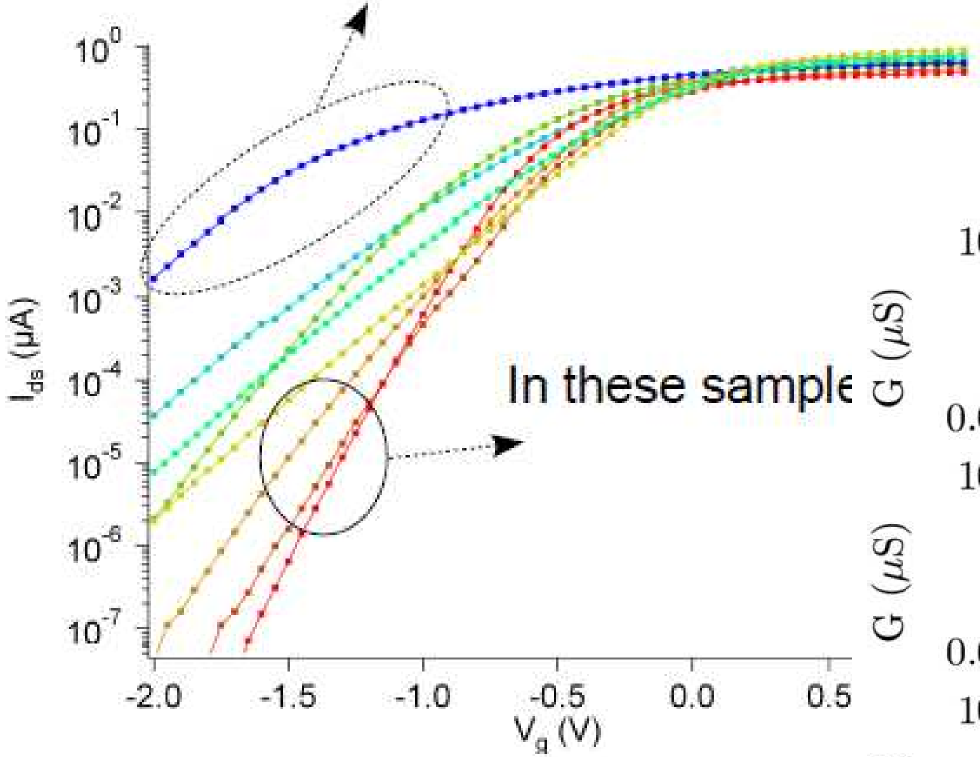


Without nitride spacers

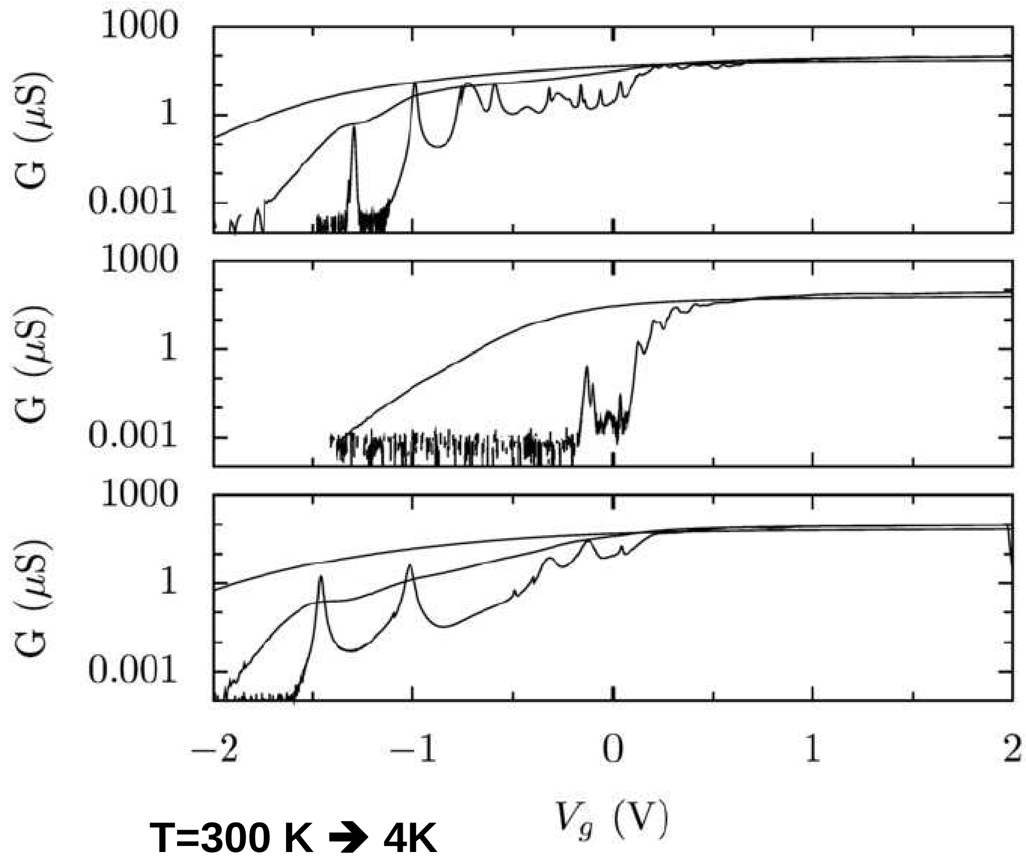
Lateral dopant diffusion:
➔« Short channel effects »
+ Variability

Bad for transistors but…
Opportunity to study transport thru
Single shallow donors

sub-threshold current due to thermally broadened transport via dopant states

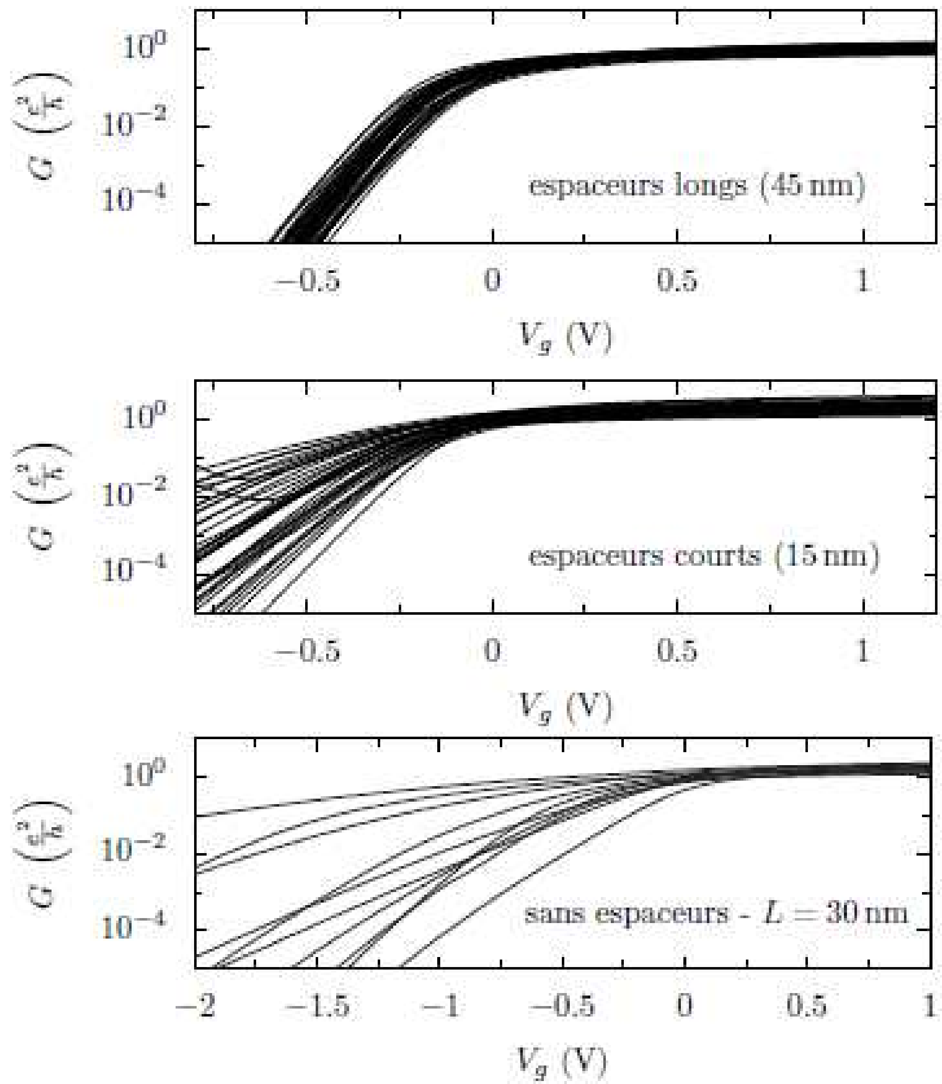


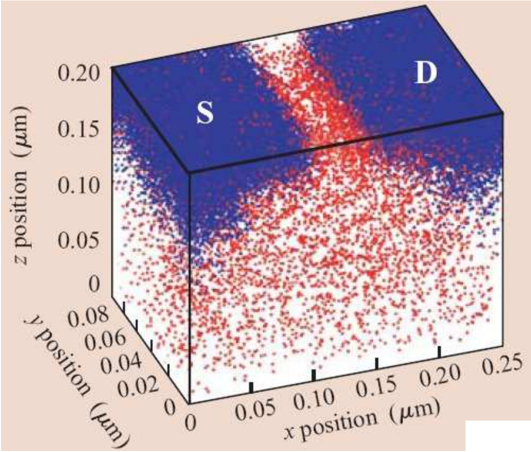
T=300 K



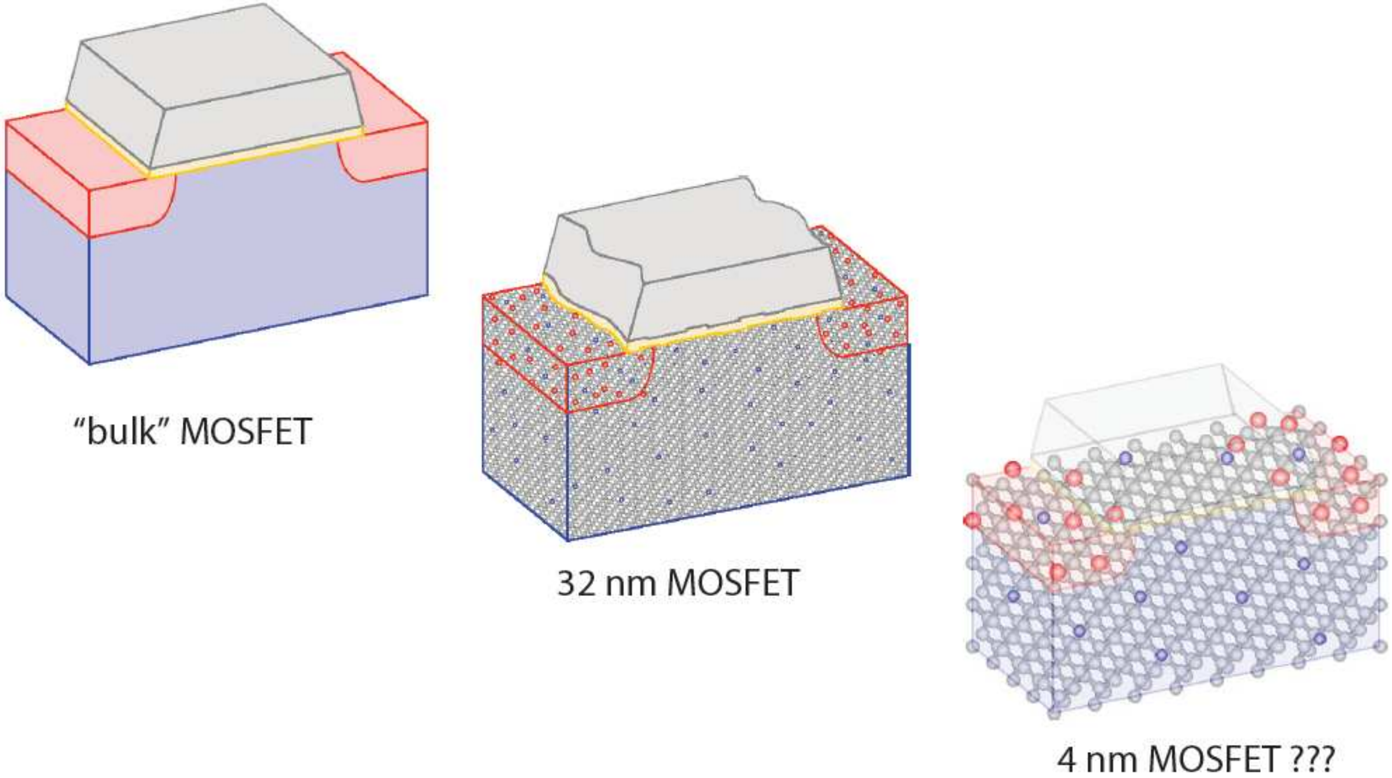
T=300 K ➔ 4K

Shrinking the mosfet: single dopant variability





Atomic-scale electronics



[Asenov IEEE Trans. Elec. Dev. 50, 1837, 2003]

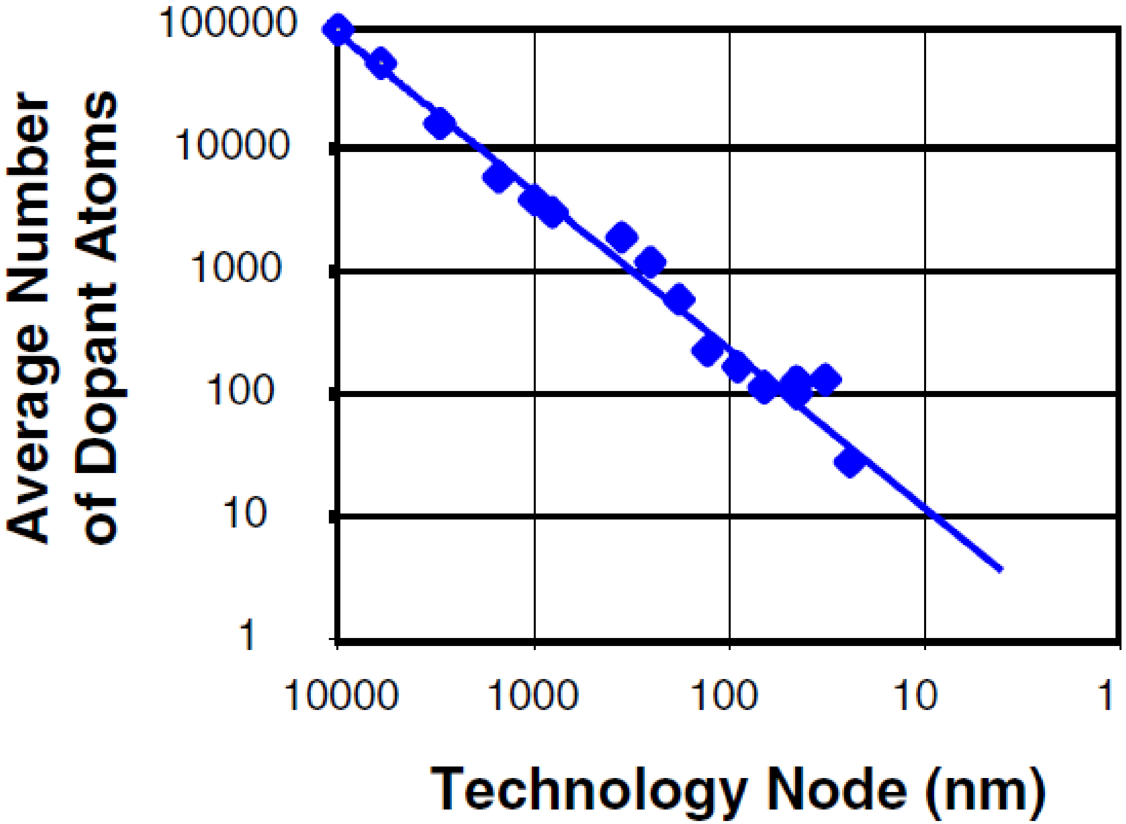
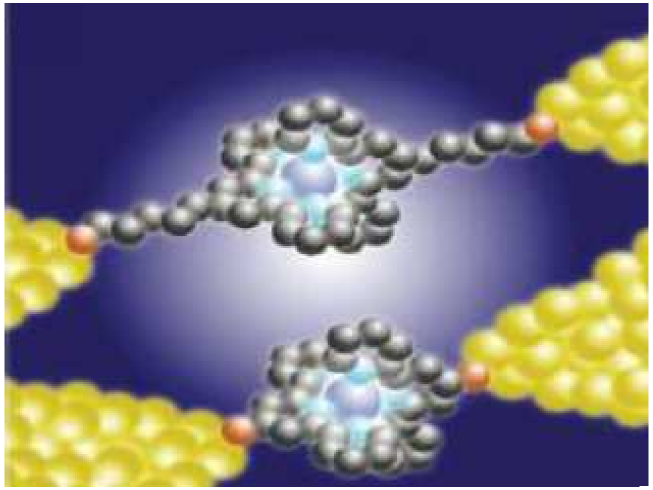
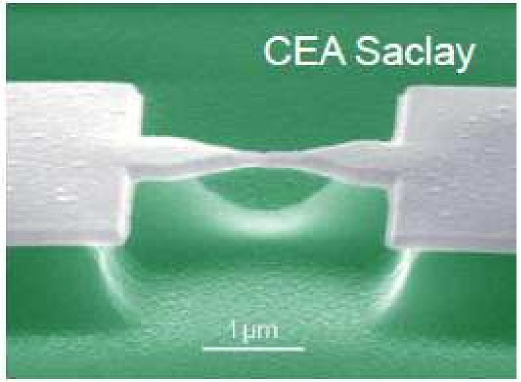


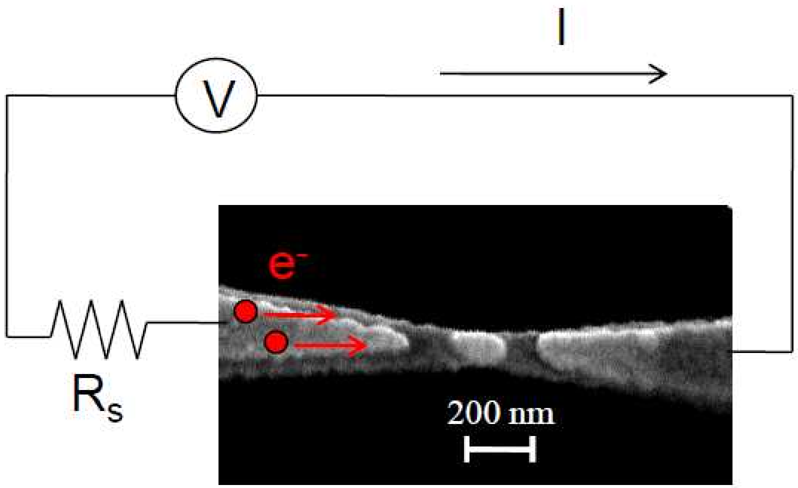
Figure 2: Average number of dopant atoms in the channel as a function of technology node



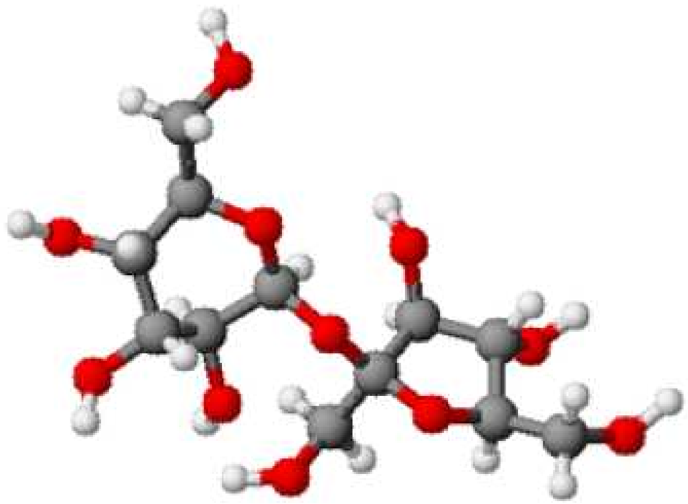


Mechanically controllable break junctions (MCBJ)

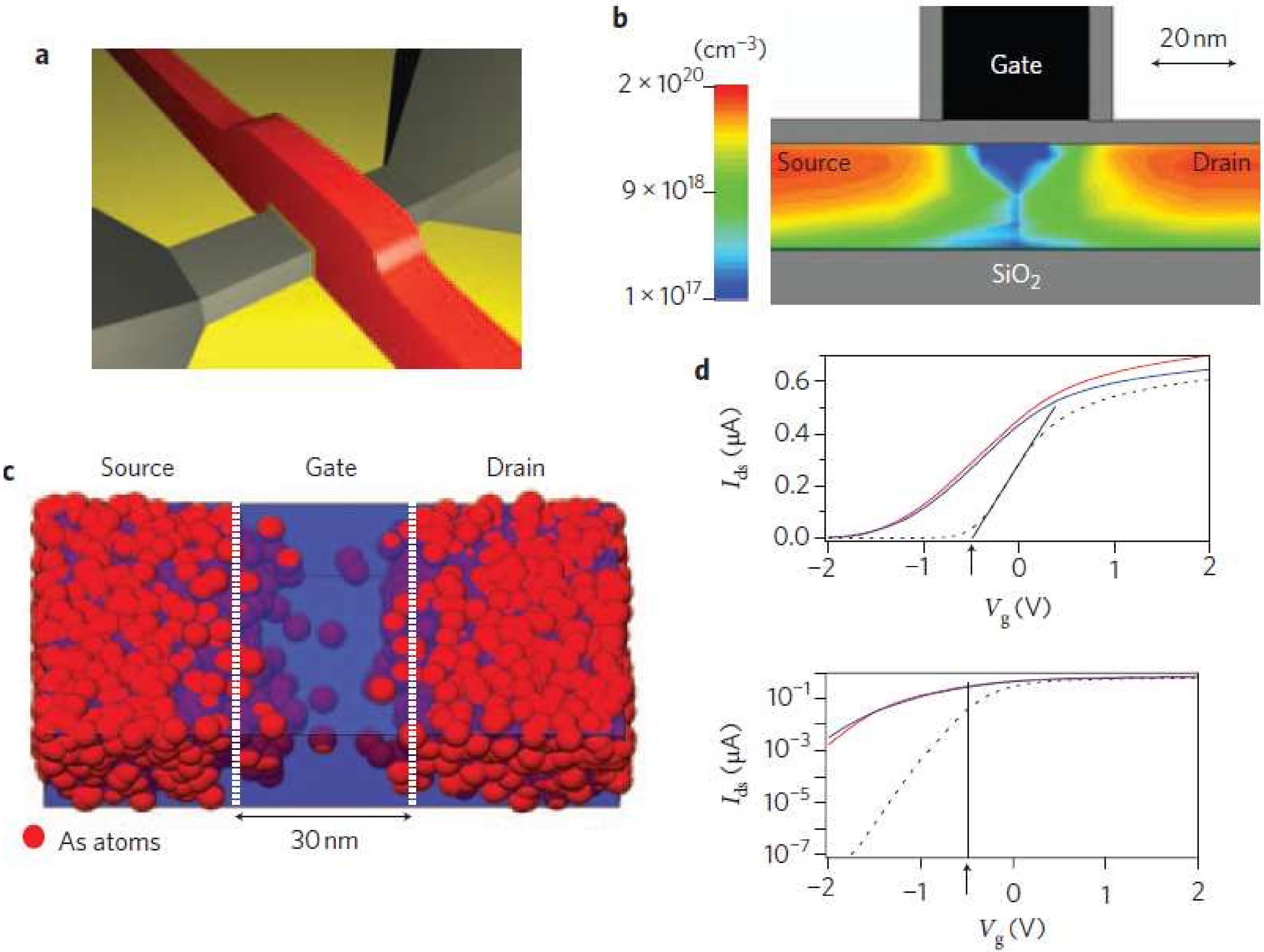
... circuit inside the molecule



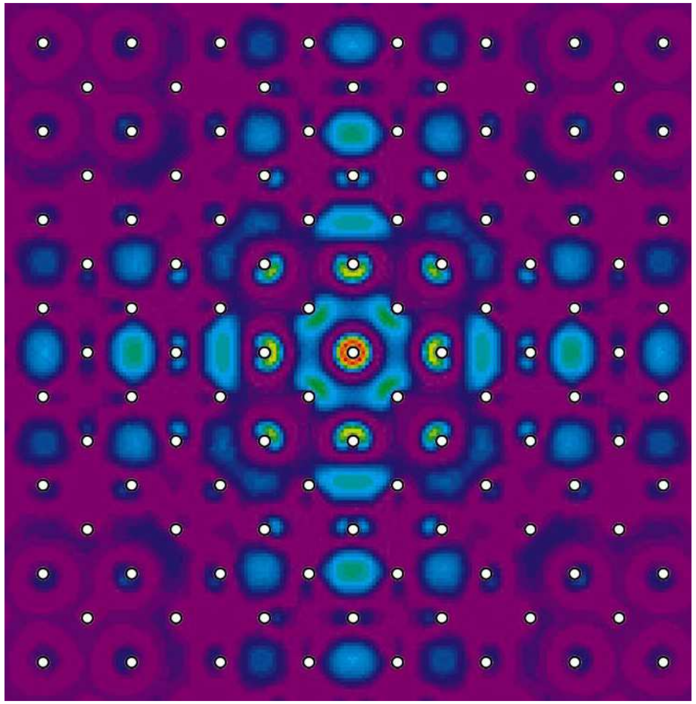
momentum transfer from the electrons



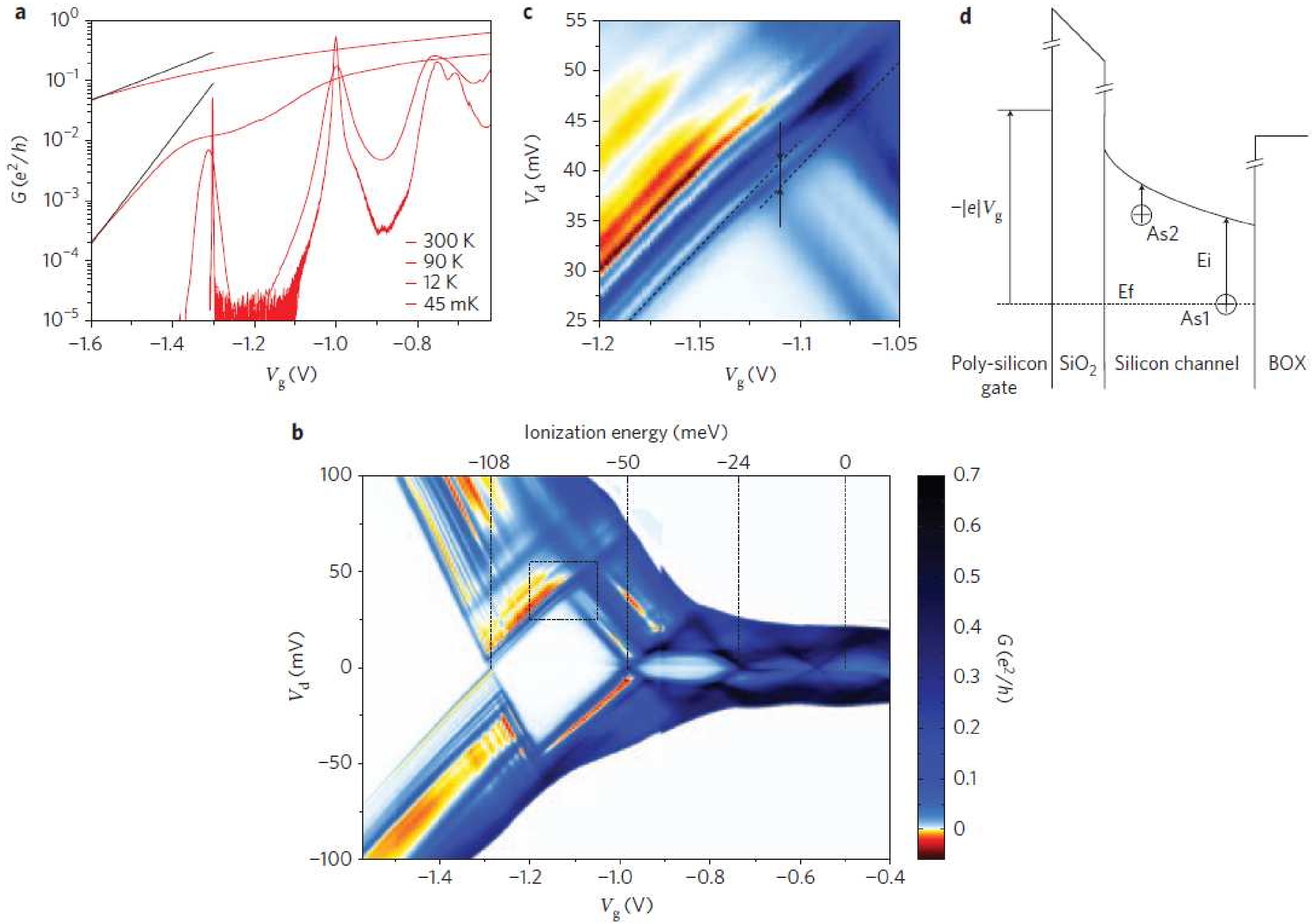
M. Pierre *et al.*, Nat. Nanotechnol. **373**, 10.1038, (2009)



Single dopant –single electron transport



Shallow donor electronic orbital in Silicon (Ground state, after B. Koiler et al Phys. Rev. B 70, 115207 (2004)

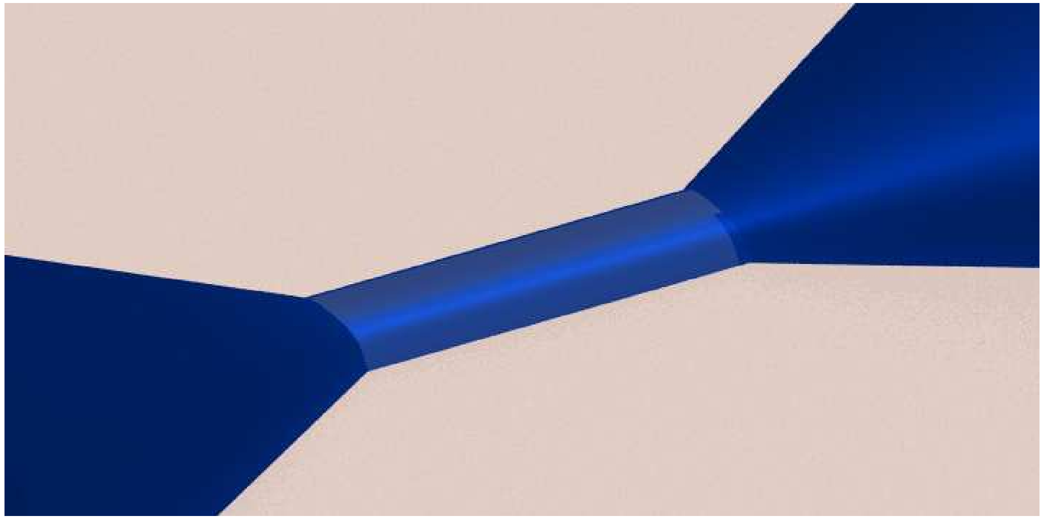


M. Pierre *et al.*, Nat. Nanotechnol. **373**, 10.1038, (2009)

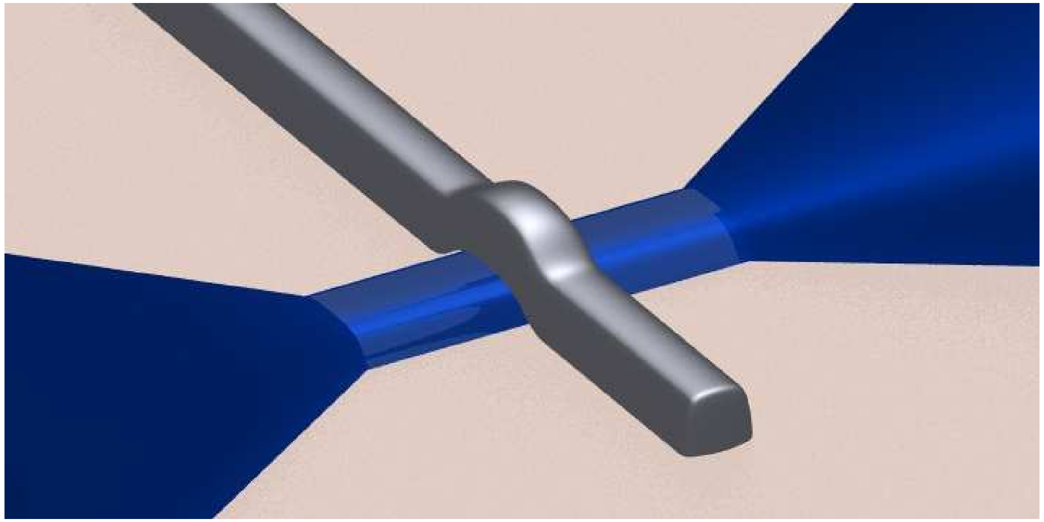
Convergence SET-FET: the MOS-SET

(the underlap analog of the preceding exemple)

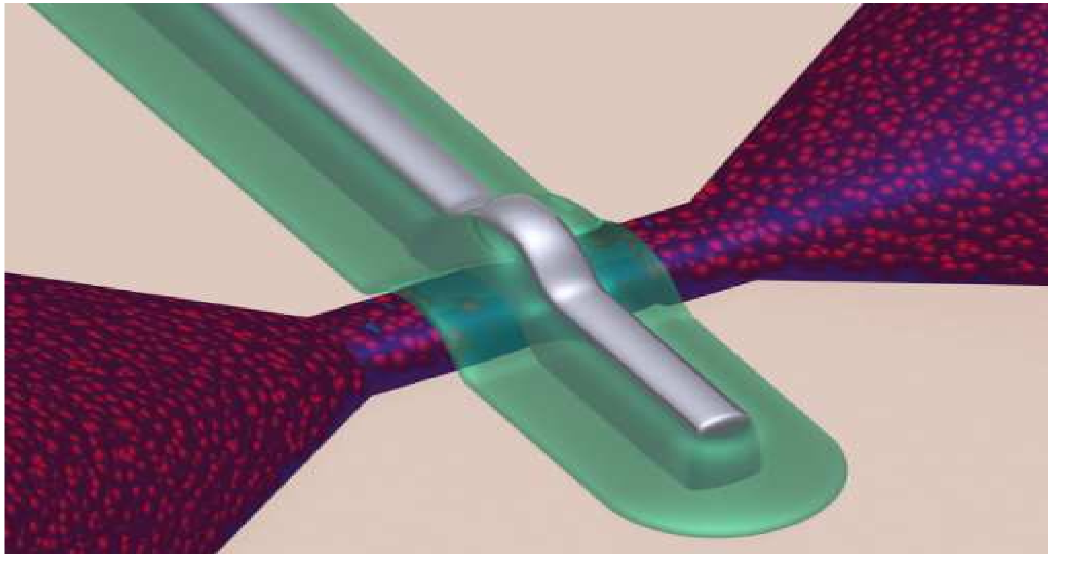
Sample fabrication: 200 mm wafers
8-20 nm Thick Silicon-on-Insulator (SOI)
e-beam litho of active layer (down to 20nm), Thermal oxidation (5nm SiO_2)



e-beam litho of poly Si gate layer gate length down to 20nm
Eventually multigate design= Pitch 70 nm

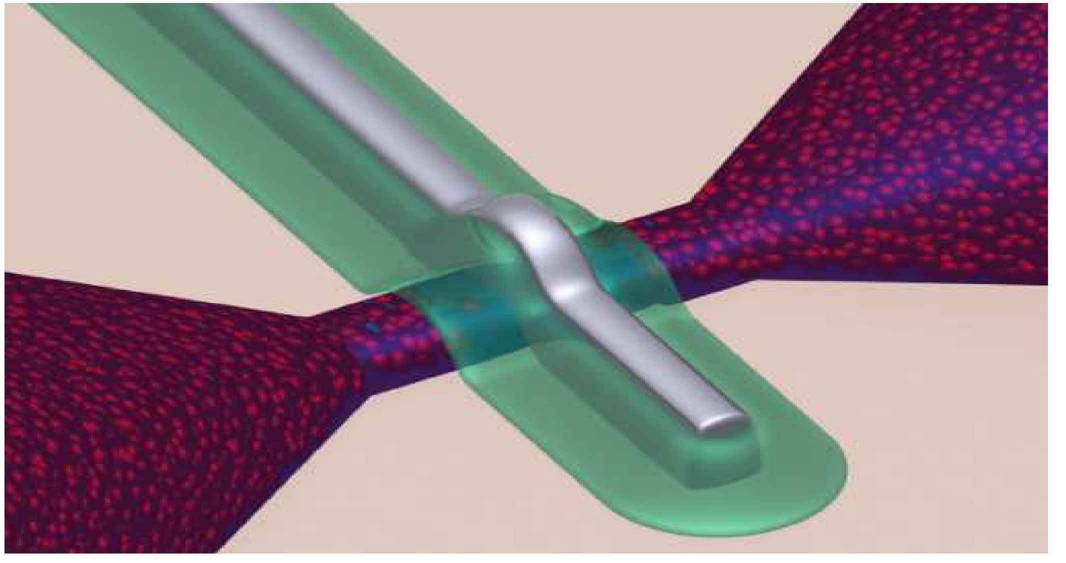


formation of self aligned Si_3N_4 spacers before Arsenic implantation of highly doped Source and Drain



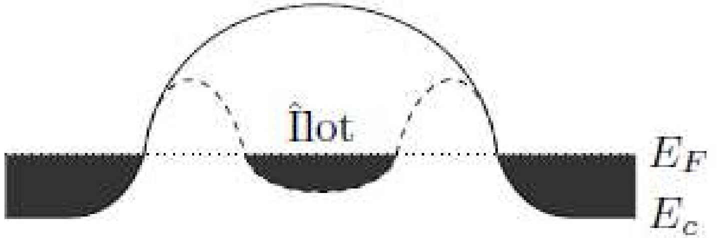
Leti

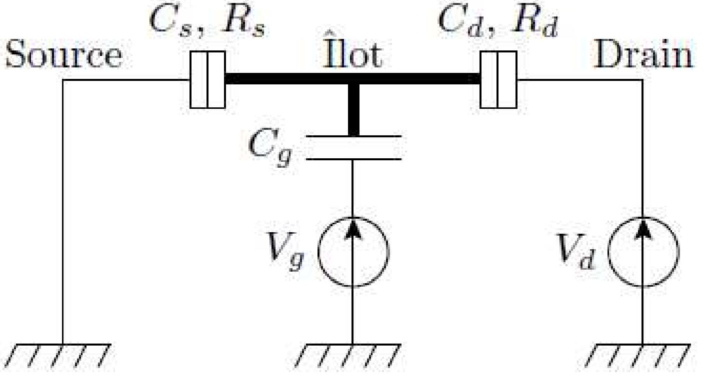
UNDERLAP GATE/SOURCE-DRAIN



UNDERLAP GATE/SOURCE-DRAIN

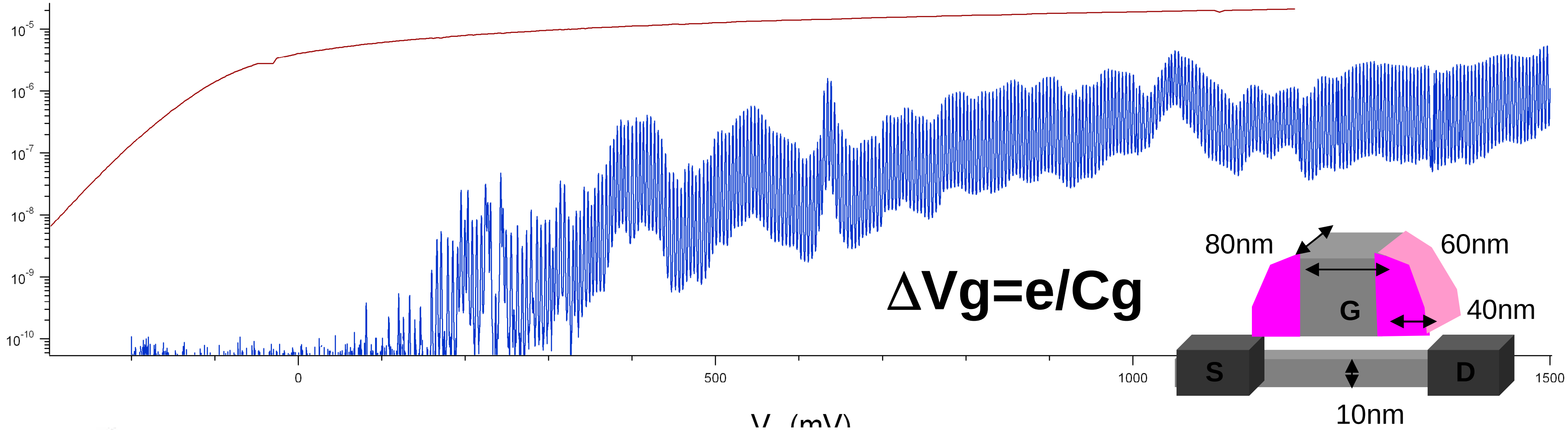
The MOS-SET

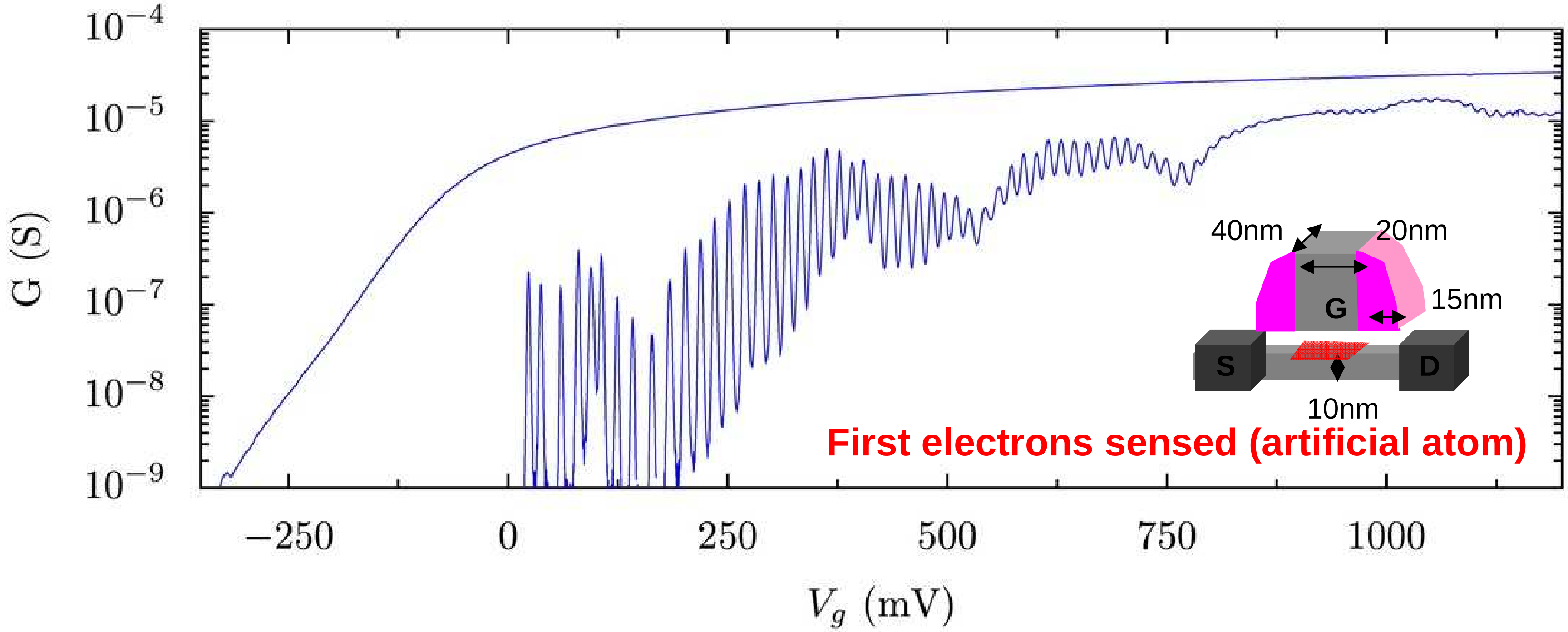


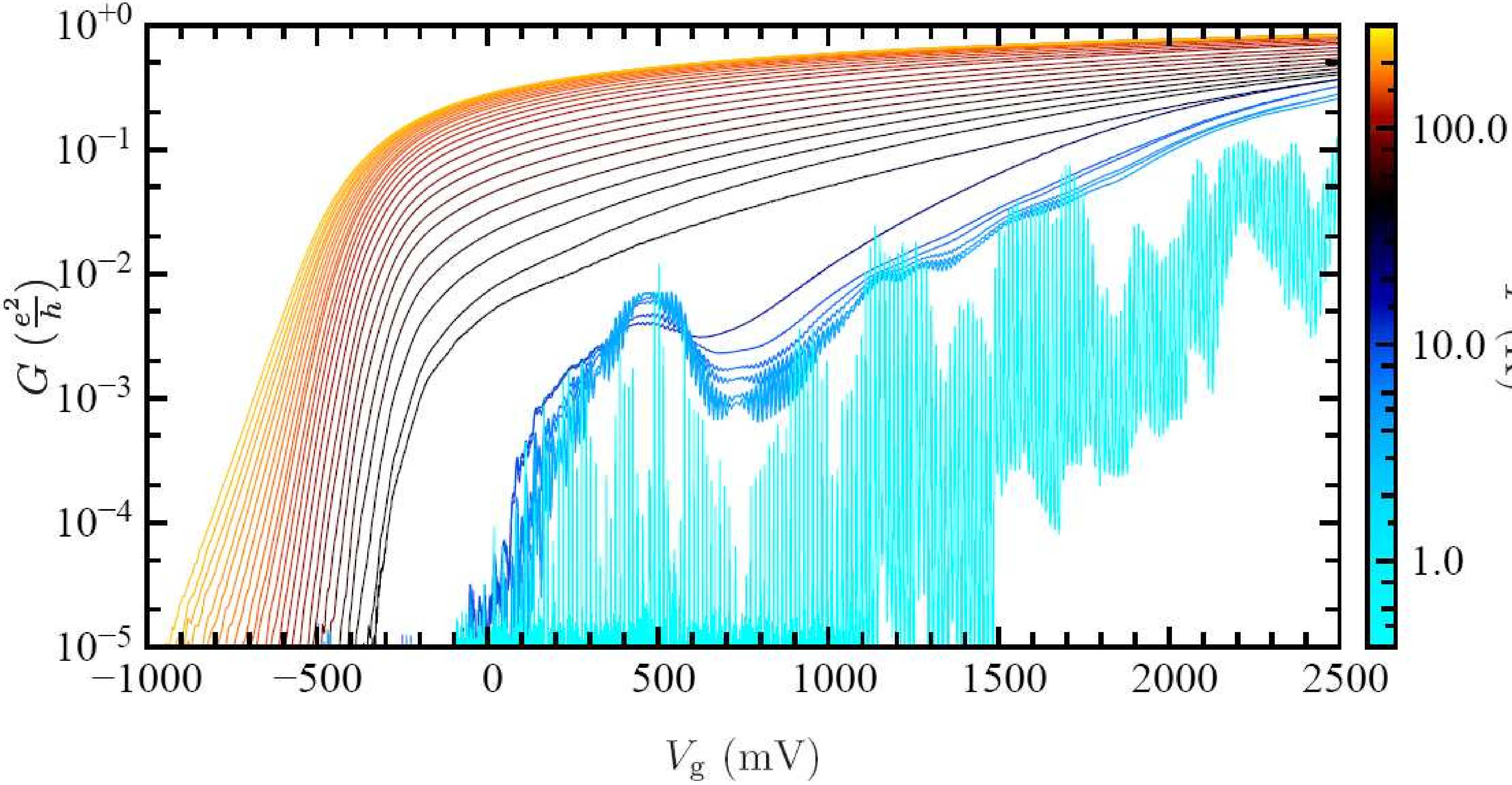


Excellent MOS-SETs - Scaling Rules

Counting electrons one-by-one up to 300!



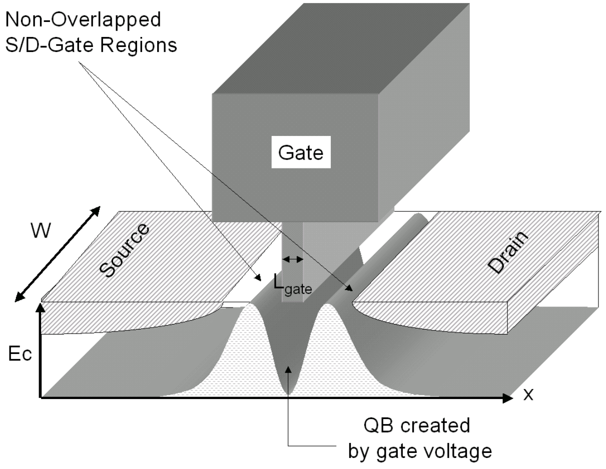


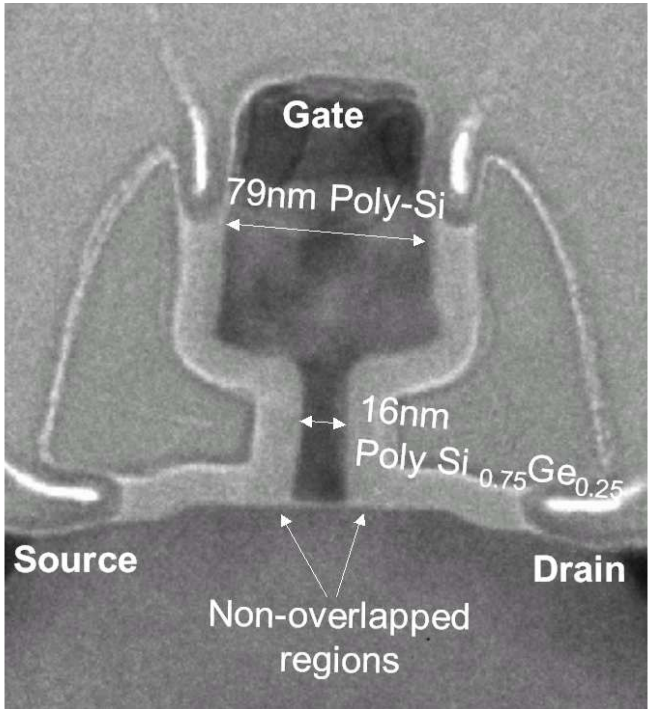


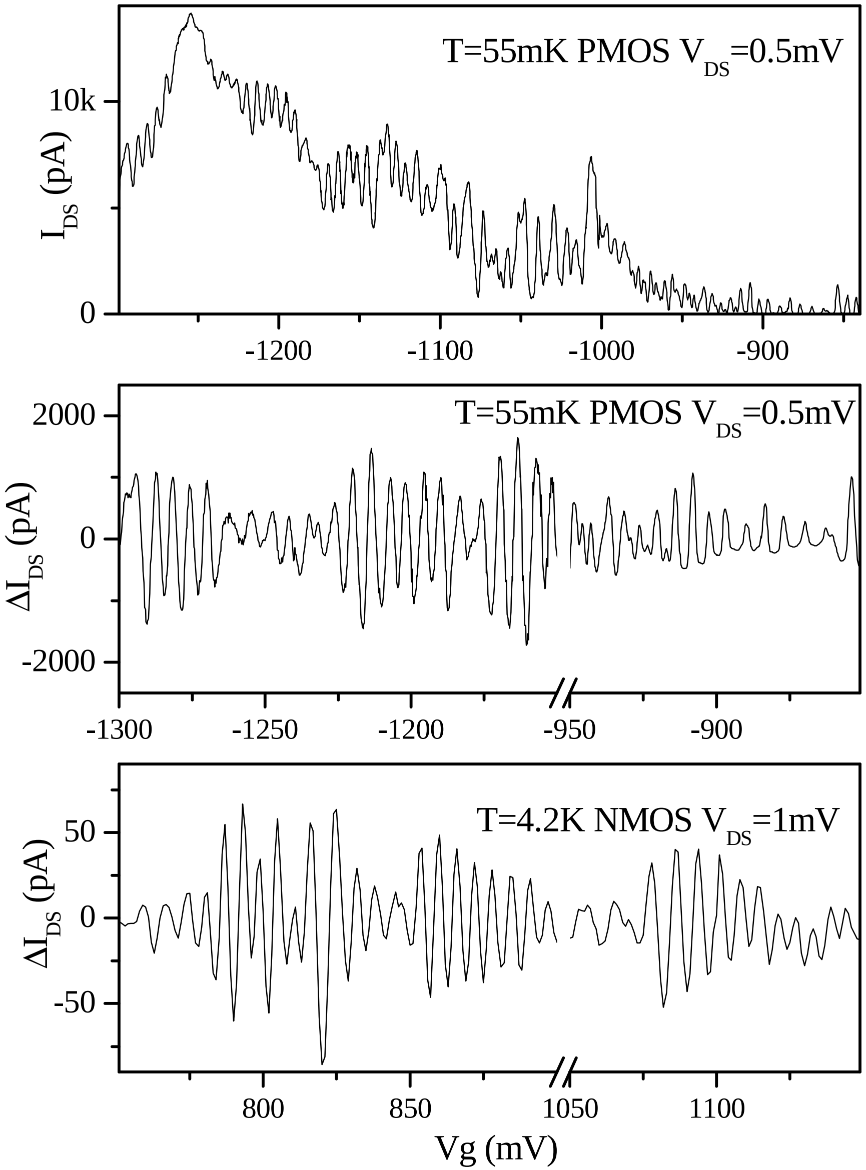
« Trigate » MOSFET

M. Hofheinz PhD, U. J. Fourier 2006

access resistances ex: MOS STMicro Non-Overlapped geometry

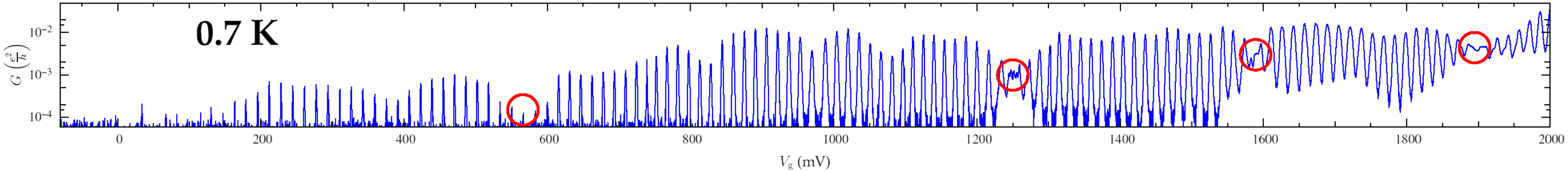






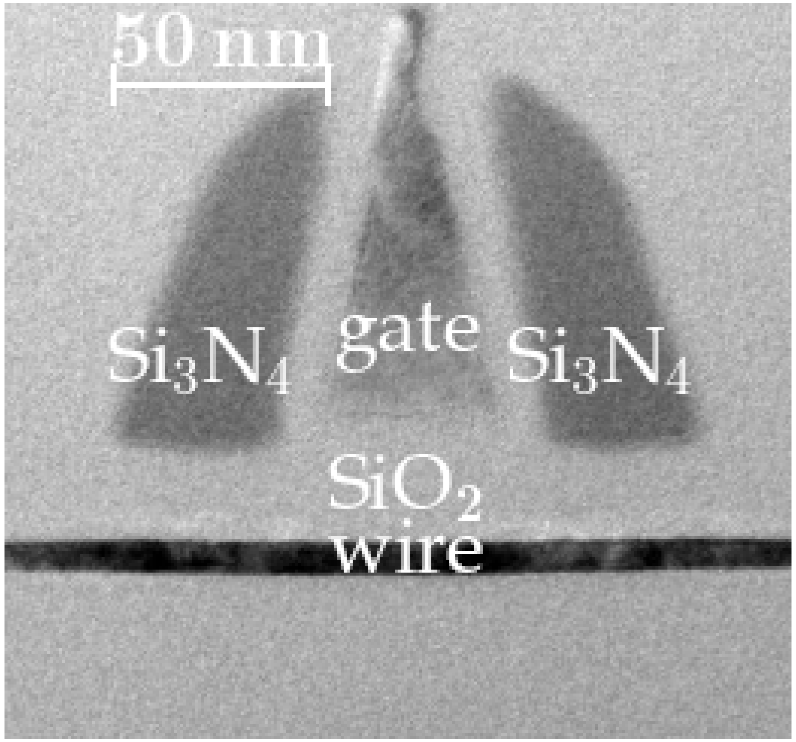
F. Bœuf et al. IEEE transactions on Nanotechnology, vol2, No3, p144 (2003).

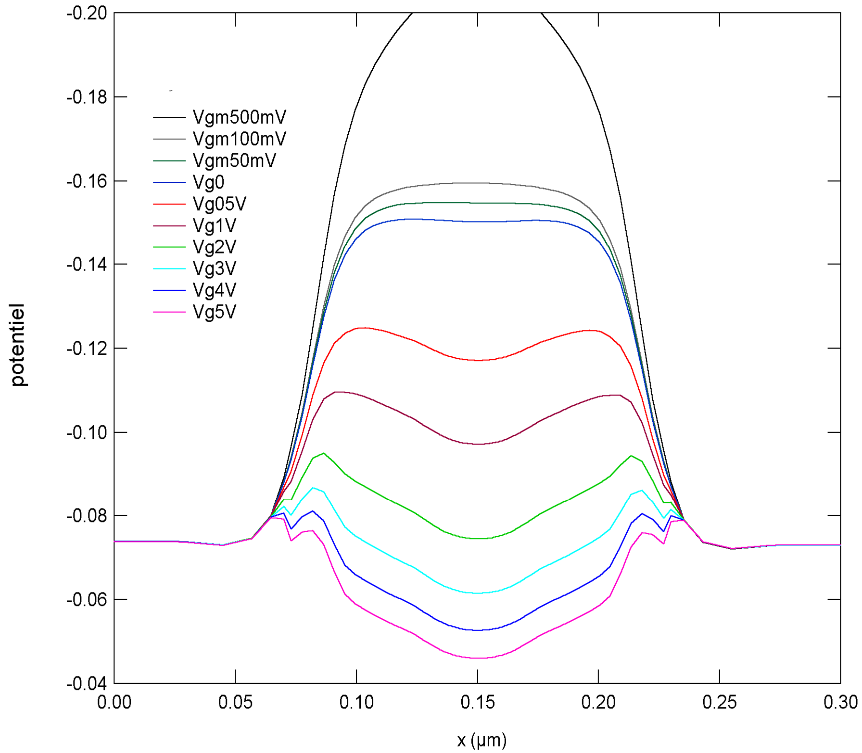
access resistances. ex: SOI Non-Overlapped nanowire FET

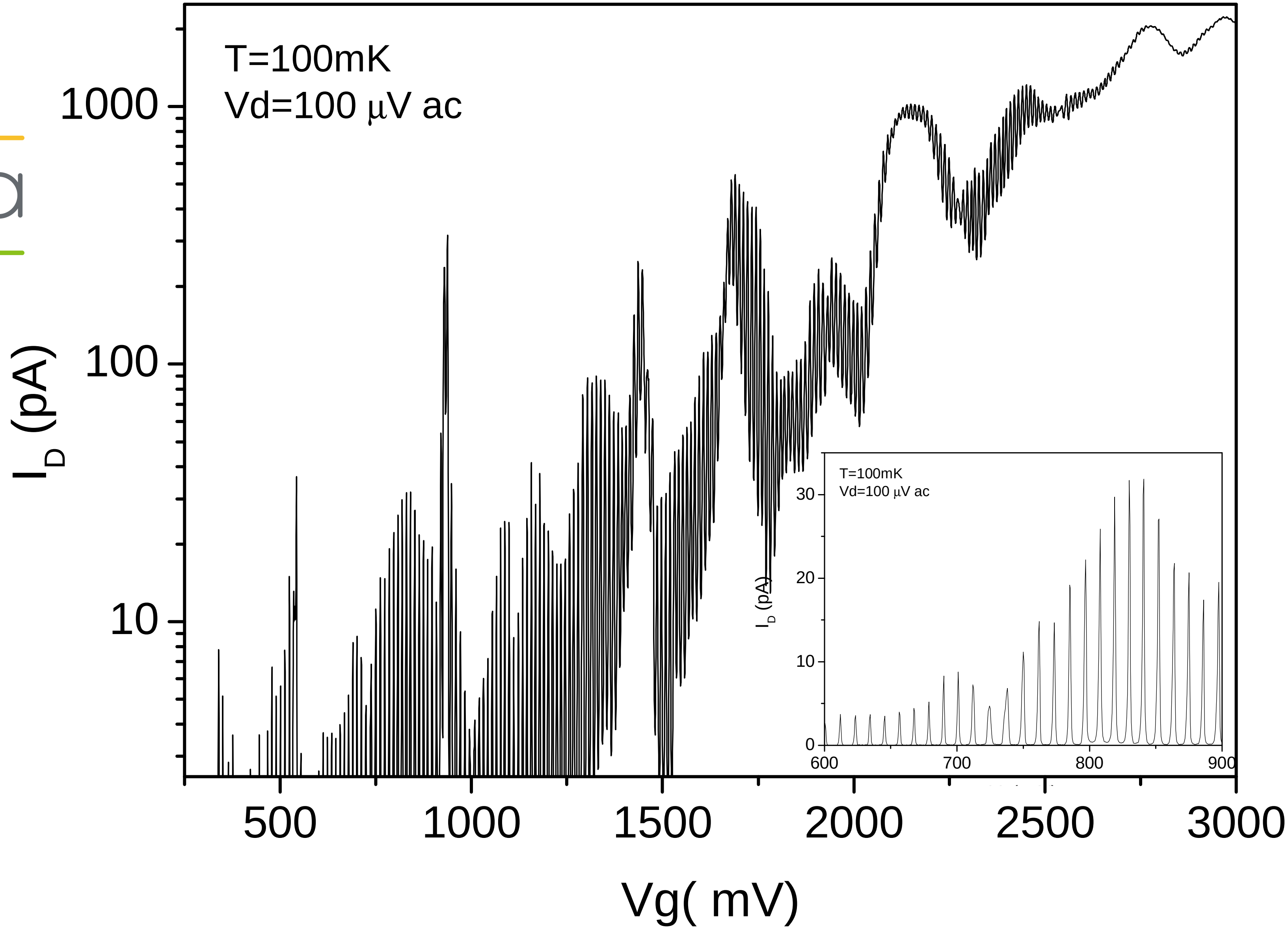


Max Hofheinz PhD 2006

fabrication: **cea-leti**

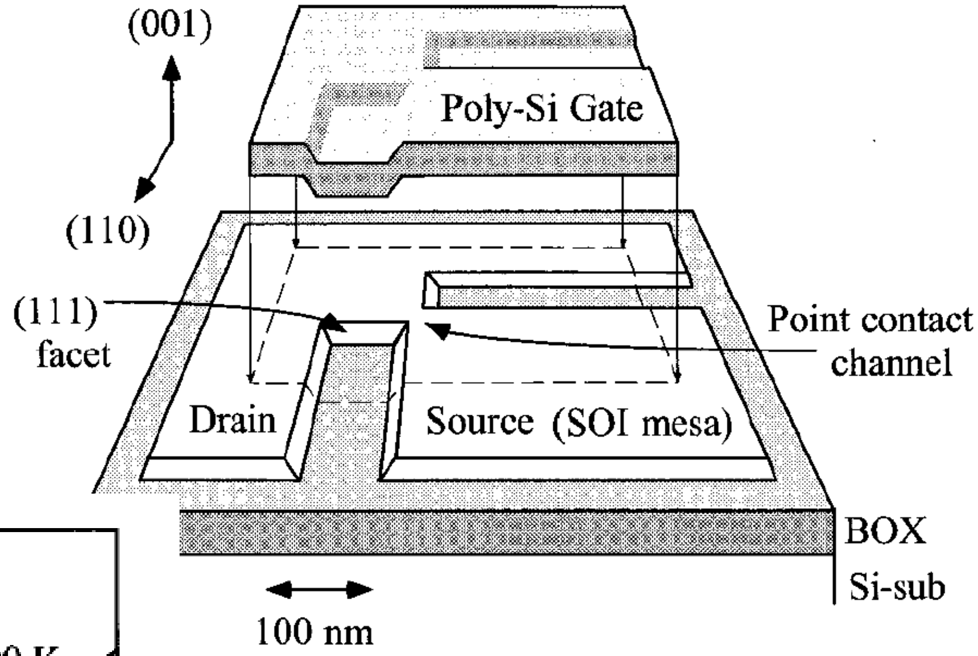


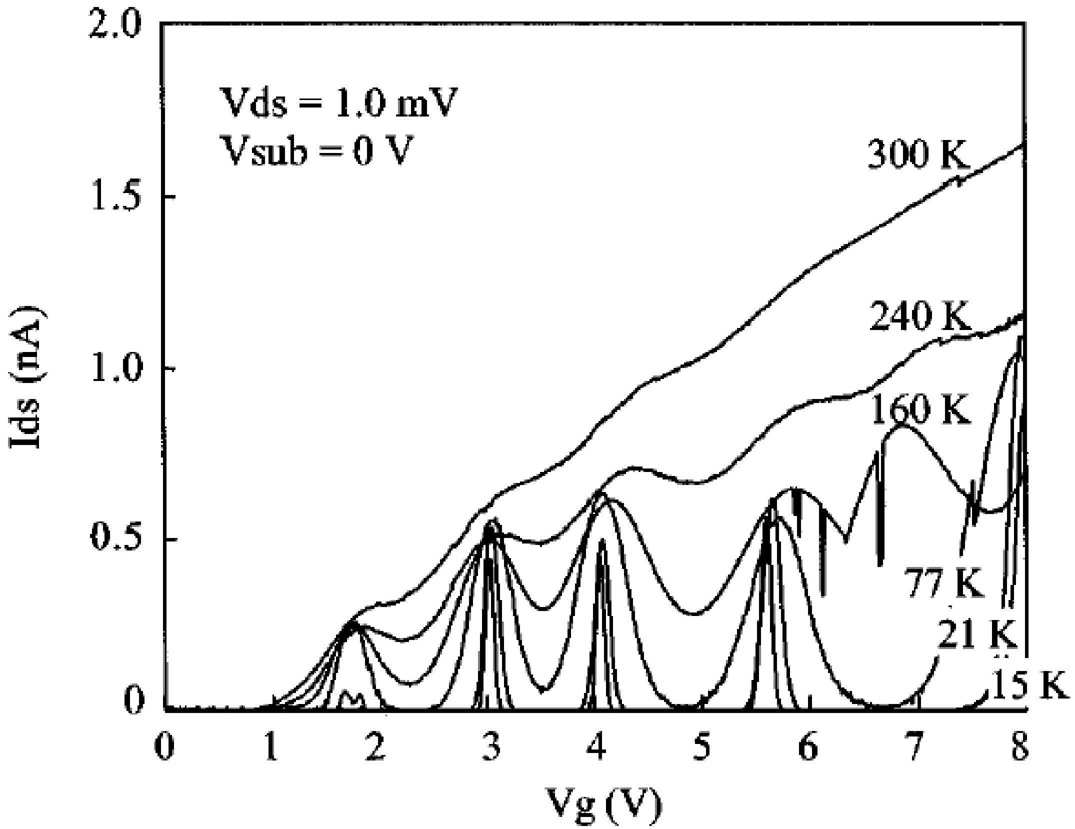



T=100mK
Vd=100 μV ac
1000
100
10
I_D (pA)
500
1000
1500
2000
2500
3000
Vg(mV)
T=100mK
Vd=100 μV ac
30
20
10
0
I_D (pA)
600
700
800
900

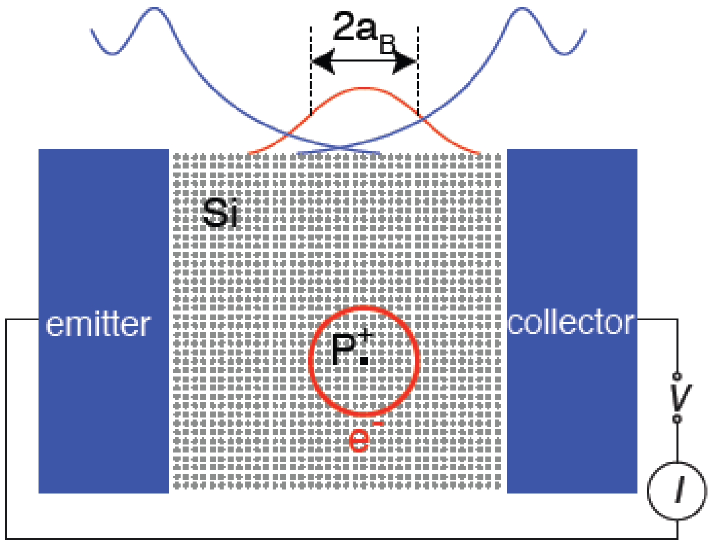
Coulomb Blockade in a SOI constriction

H. Ishikuro and T. Hiramoto
APL 71, 3691 (1997)





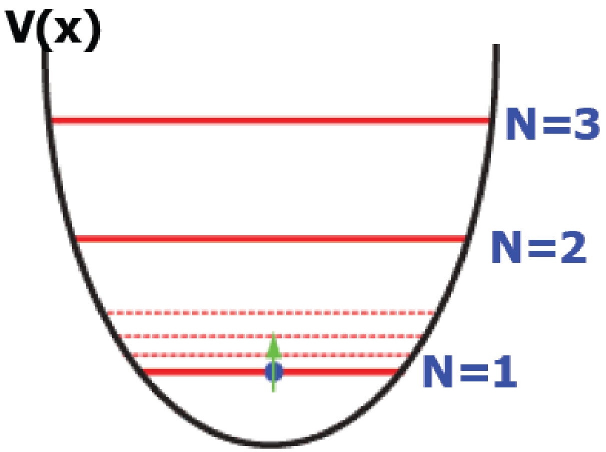
What is the physics involved in single dopant transport?



Scaling of the Bohr orbit:

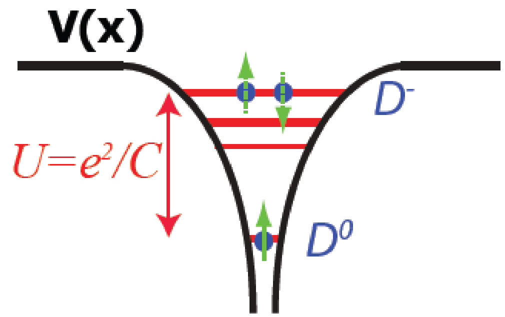
$$
\begin{aligned}
r_{\text{dopant}} &= \frac{\epsilon_{\text{r}}}{m^*} \cdot r_{\text{Hydrogen}} \\
r_{\text{Hydrogen}} &= 0.05\,\text{nm} \\
r_{\text{P:Si}} &= 2.5\,\text{nm} \\
r_{\text{P:Ge}} &= 6.4\,\text{nm}
\end{aligned}
$$

Quantum dots

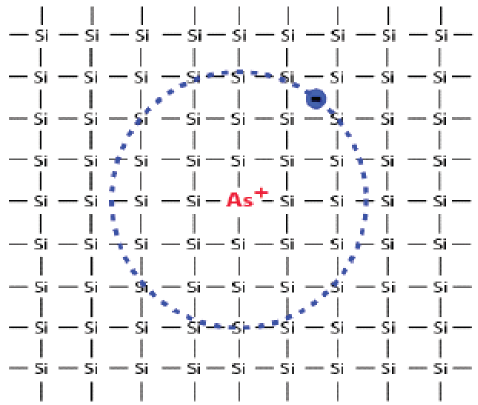


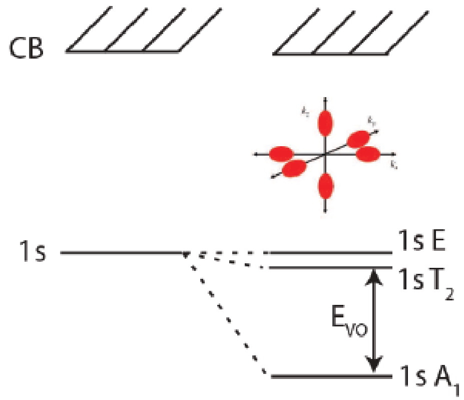
- 2D parabolic potential
- Constant charging energy
- Equidistant level spacing (excited states)

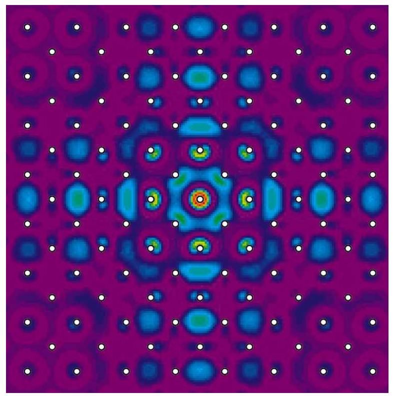
Dopant: shallow donors in Si



- 3D Coulomb potential
- Can bind up to two electrons
- Hydrogen-like level spectrum (D^0)
- Valley-orbit →

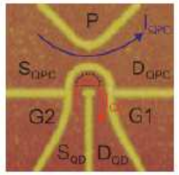


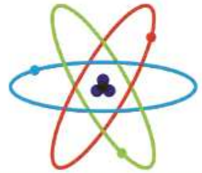




Shallow donor electronic orbital in
Silicon (Ground state, after B. Koiler et al Phys. Rev. B 70, 115207 (2004)

Relevant energy scales





Coulomb interaction:	$E_C = \frac{e^2}{8\varepsilon\varepsilon_0 r}$	1-10 meV	1-10 eV
Size quantization:	$E_Q = \frac{\hbar^2}{m^* r^2}$	0.1-10 meV	1-10 eV
Orbital magnetic energy:	$E_L = \frac{\hbar e B}{m^*}$	0-20 meV	0-1 meV
Zeeman energy:	$E_Z = g^* \mu_B B$	0-1 meV	0-1 meV
Thermal energy:	$E_{th} = k_B T$	25° C: 25 meV -273°C: 15 μeV	

One dopant, two dopants,… many dopants

One dopant, single occupation: resonant tunneling, no interaction

Two dopants: capacitive coupling, tunnel coupling, hybridization

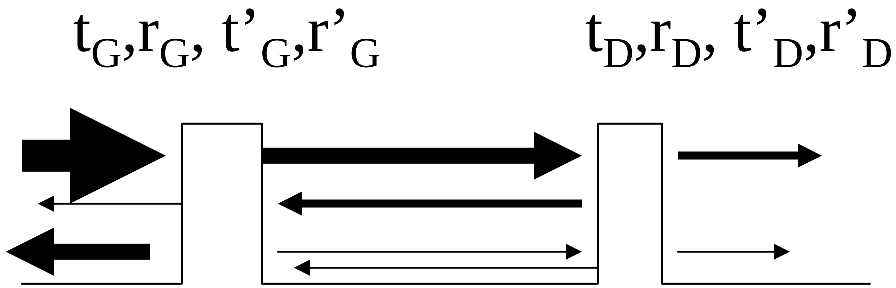
…

Many dopants: impurity band, correlations, interferences

Large mean level spacing, Resonant Tunneling,
Confinement size close to Fermi wavelength
No interaction, no Coulomb blockade

Ex: a single shallow donor
an artificial atom (very small quantum dot)

For a better understanding of the single dopant case, we consider the (single channel) Resonant tunneling



etc….

$$t_{total} = t_G t_R e^{ikL}\left(1 + r'_G r_D e^{2ikL} + (r'_G r_D e^{2ikL})^2 + \ldots\right) = \frac{t_G t_R e^{ikL}}{1 - r'_G r_D e^{2ikL}}$$

$$T = \frac{T_G T_D}{1 + R_G R_D - 2\sqrt{R_G}\sqrt{R_D}\cos(2kL + \pi)}$$

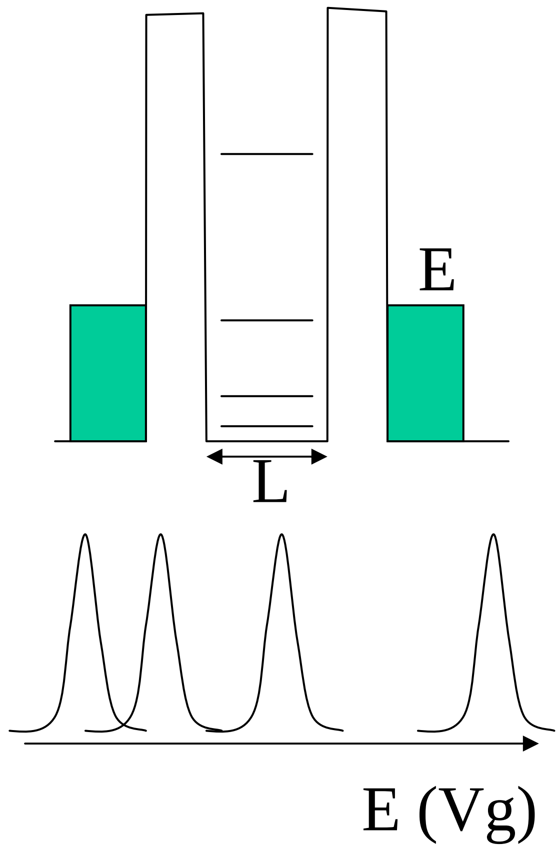
$$\pi = \phi'_G + \phi_D$$ dephasing inside (infinite) tunnel barriers

(far from any Rayleigh (elastic) resonance , the transmission is purely imaginary)

M. Buttiker IBM Journal Res. Dev. 32,63 (1988)

maximum transmission if 2kL=π, i.e. for an infinite double well cavity:

$$E_n = \left(\frac{\hbar^2 \pi^2}{2m}\right)\left(\frac{n}{L}\right)^2$$



then:

$$T_{coherent} = \frac{4T_G T_D}{(T_G + T_D)^2}$$

$T_{coherent} = 1$ if $T_G = T_D$

Far from resonance ($2kL=2n\pi$):

$$T_{coh,HR}=\frac{1}{4}T_G T_R$$

incoherent (sequential) transmission does not depend on energy:

$$T_{incoh.}=\frac{T_D T_G}{T_D+T_G}$$

$$T = \frac{T_G T_D}{1 + R_G R_D - 2\sqrt{R_G}\sqrt{R_D}\cos(2kL + \pi)}$$

$$\cong \frac{T_G T_D}{\left(\frac{T_G + T_D}{2}\right)^2 + 2\left(1 - \cos(\theta(E))\right)}$$

If T $_{D,G}$ << 1

weak transmission

$$1 - \cos(\theta(E)) \approx \frac{1}{2}\left(\theta(E) - 2n\pi\right)^2 \approx \frac{1}{2}\left(\frac{d\theta(E)}{dE}\right)^2 \left(E - Eres\right)^2$$

Near a resonance

(Taylor series development)

$$T = \frac{(\Gamma_D \Gamma_G)}{(E_F - E_N)^2 + \frac{1}{4}(\Gamma_D + \Gamma_G)^2} \qquad \Gamma_{G,D} = \frac{dE}{d\theta} T_{G,D}$$

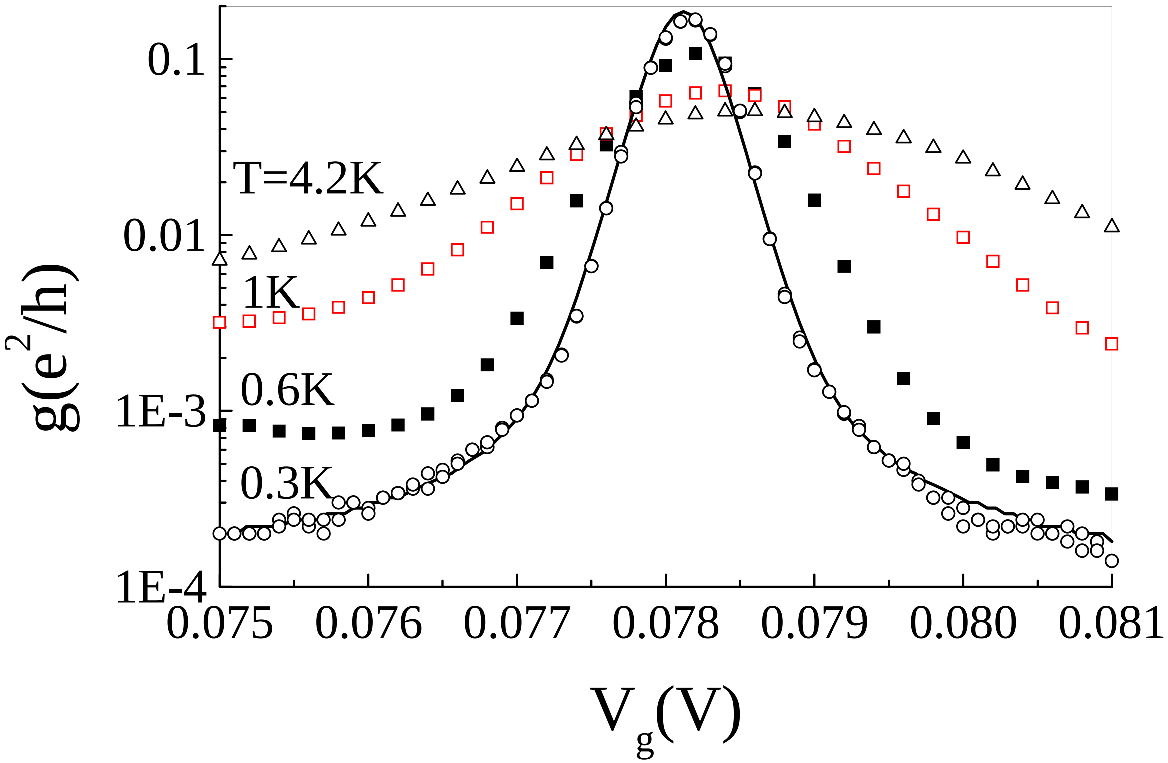
(Breit-Wigner formulae)

$$\Gamma_{G,D} = \frac{dE}{d\theta} T_{G,D} = \frac{dE}{dk}\frac{dk}{d\theta} T_{G,D} = \hbar v_F \frac{1}{2L} T_{G,D}$$

$$\Gamma = \omega (T_G + T_D) = \frac{\Delta}{\hbar}(T_G + T_D) = \frac{1}{\upsilon_{1D}}(T_G + T_D)$$

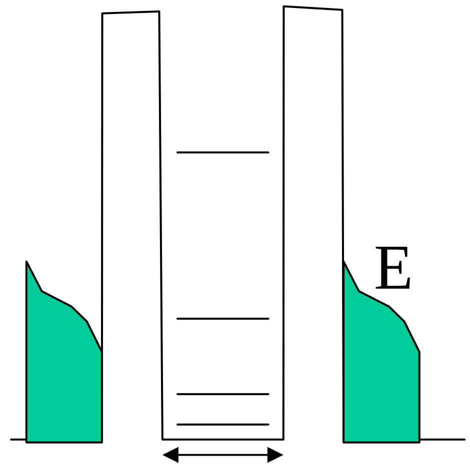
quality factor: $Q = \frac{\Delta}{\hbar \Gamma} = (T_G + T_D)^{-1}$

Thermal broadening of the resonant tunneling :



energy level separation Δ:

$\Delta > k_B T > \Gamma_{in}, \Gamma_e$



Γ_e/k_B=60mK

$$G(Vg,T) = \frac{e^2}{h}\int \frac{\Gamma_e^{\ 2}}{(e\alpha Vg - E\ )^2 + \Gamma_e^{\ 2}} \frac{df\,(E,Vg)}{dE} dE$$

$$f(E) = \frac{1}{1+exp(\frac{E-E_F}{k_B T})}$$

$$-\frac{\delta f(E)}{\delta E} = \frac{1}{4k_B T} cosh^{-2}(\frac{e\alpha Vg - E}{2k_B T})$$

Thermal broadening of the resonant tunneling (2):

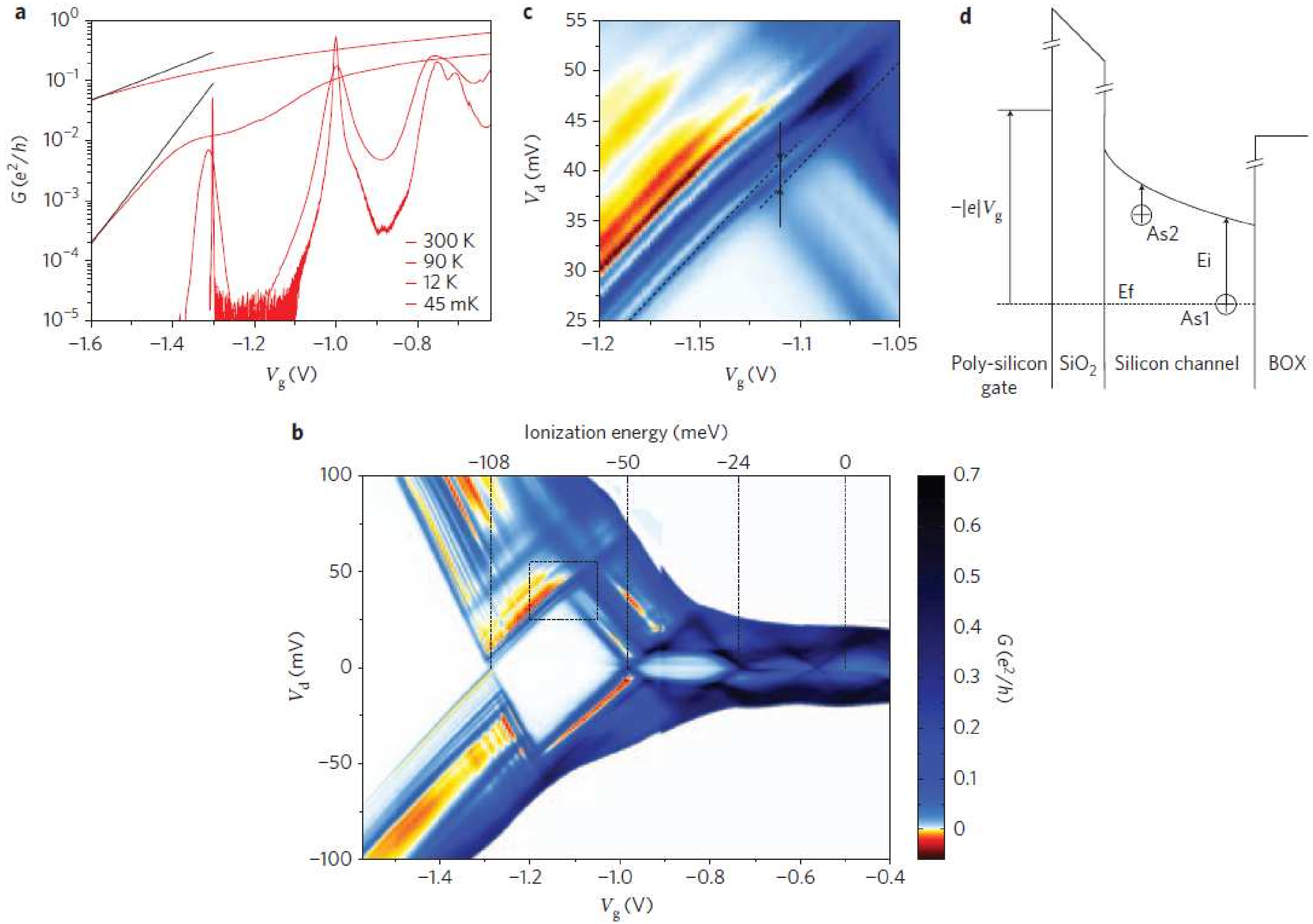
If $\Delta > k_B T >> \Gamma_{in}, \Gamma_e$

$$G(Vg,T) = \frac{e^2}{h} \int \frac{\Gamma_e^{\ 2}}{(e\alpha Vg - E\ )^2 + \Gamma_e^{\ 2}} \frac{df(E,Vg)}{dE} dE$$

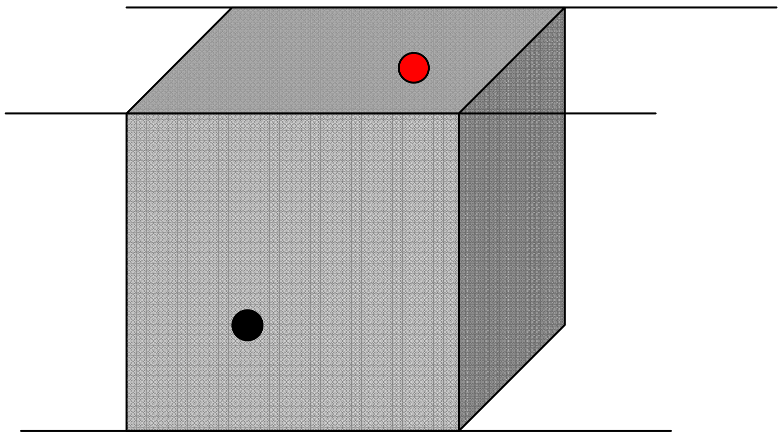
➔

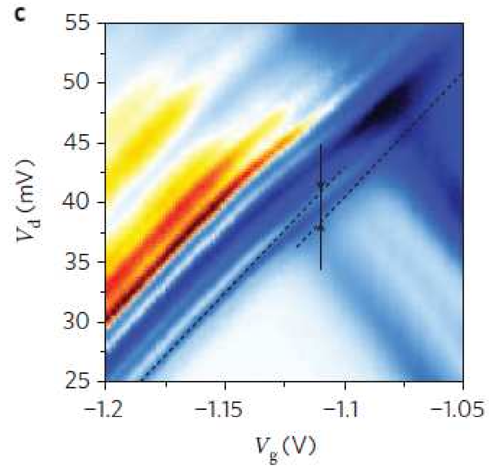
$$G(Vg,T) = \frac{e^2}{h}\left(\frac{1}{4kT}\right) A\cosh^{-2}\left(\frac{e\alpha Vg - Eres}{2kT}\right)$$

Coupled dopants: electron Coulomb repulsion



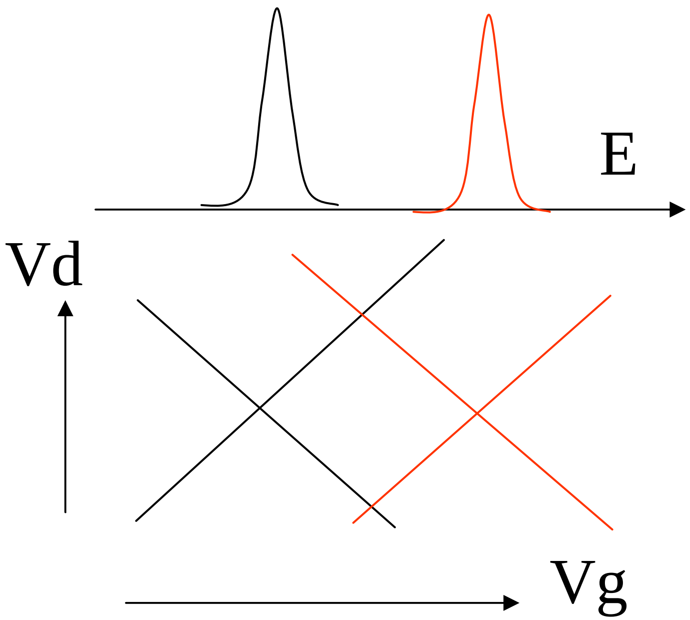
M. Pierre *et al.*, Nat. Nanotechnol. **373**, 10.1038, (2009)

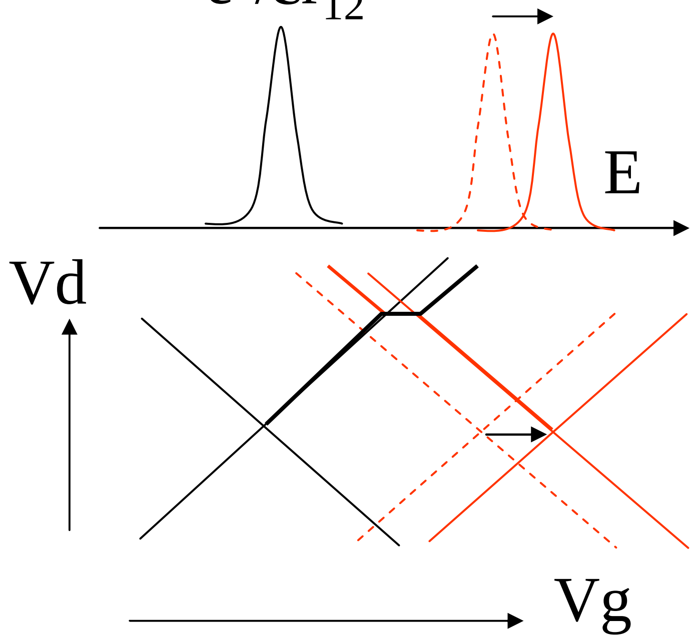


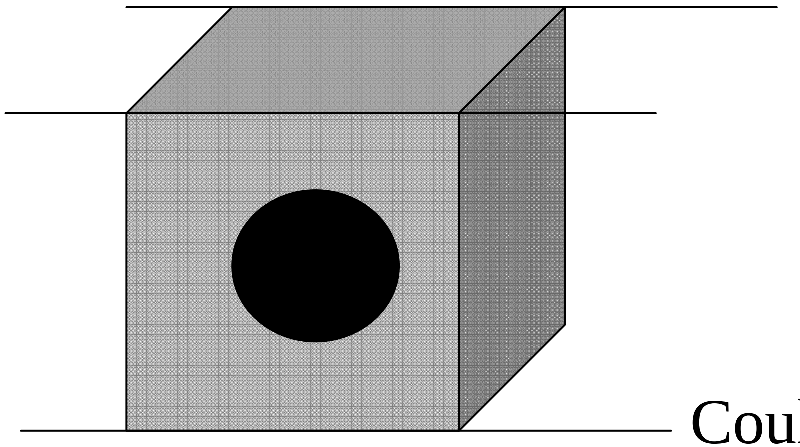


Coulomb repulsion:

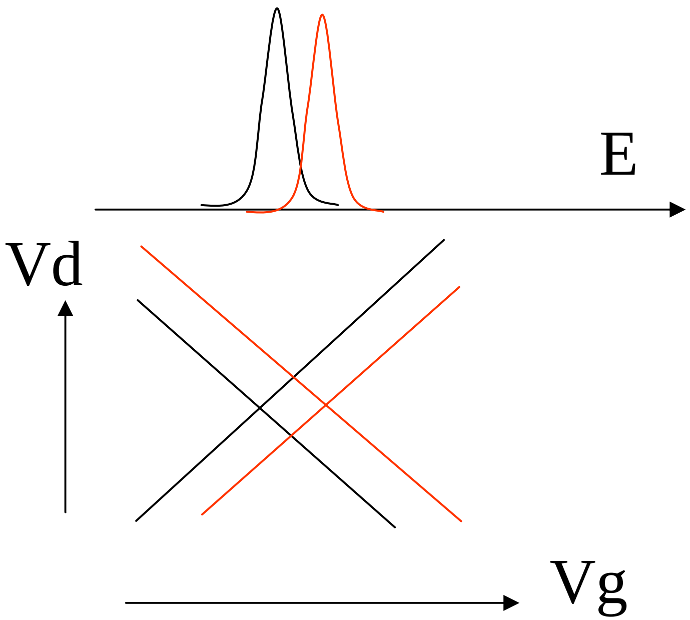
$e^2/\varepsilon r_{12}$

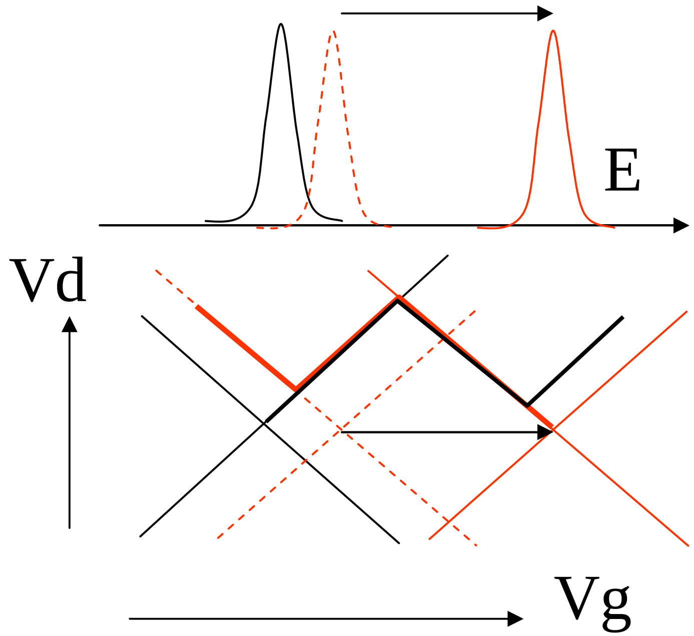




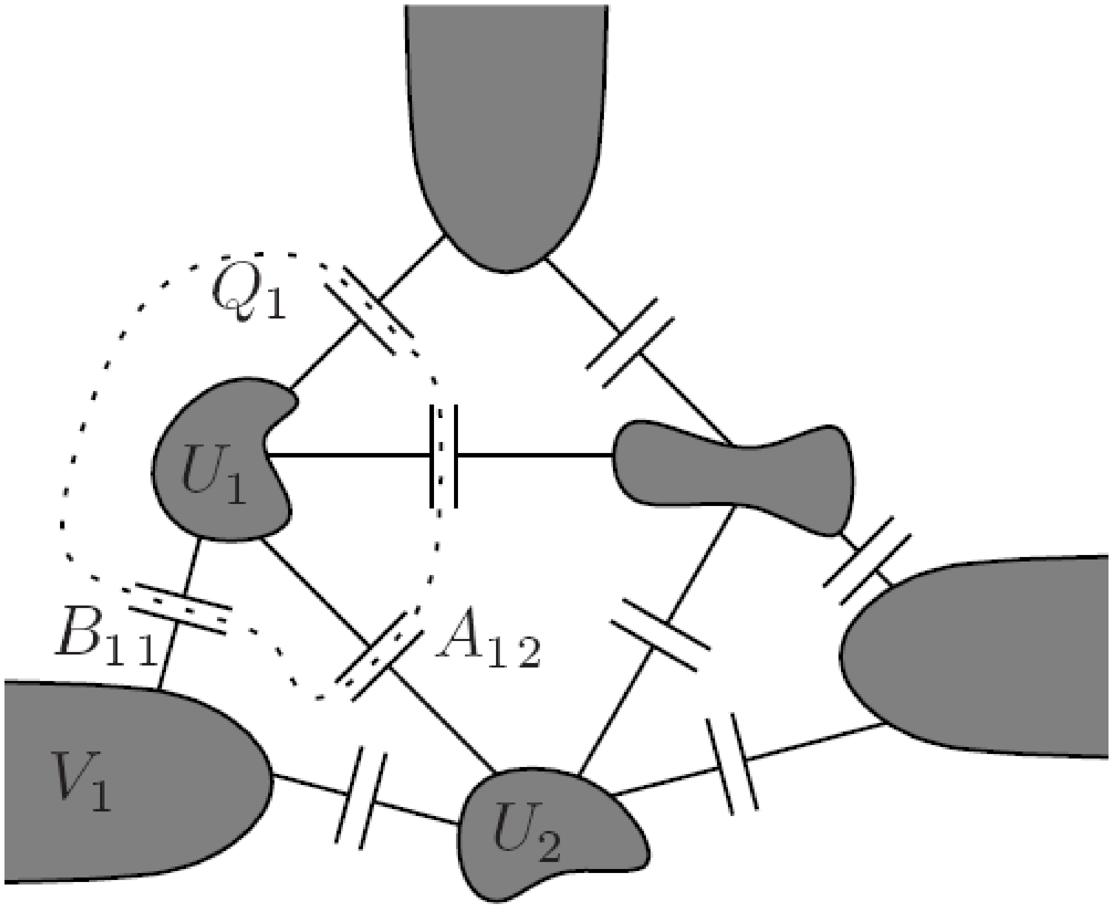


Coulomb repulsion:
e^2/C with $C \sim \varepsilon R$

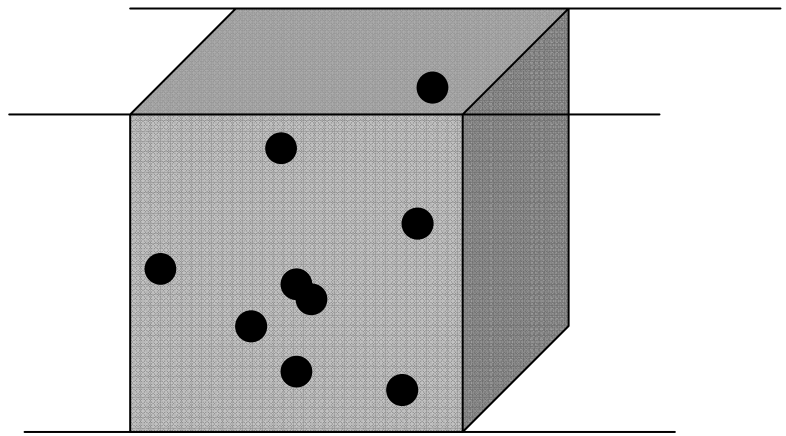




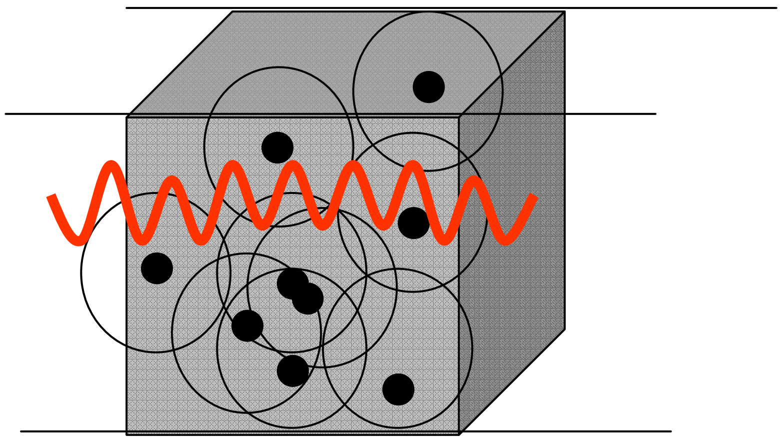
Coulomb blockade= classical effect which is due to the finite capacitances (well defined when the size are much larger than the Fermi wavelength)



Many dopants: impurity band, localization



First case: **electrons are delocalized**
and treated within the Fermi liquid theory
(interaction➔ quasi particule)



Interferences and diffraction of electronic waves

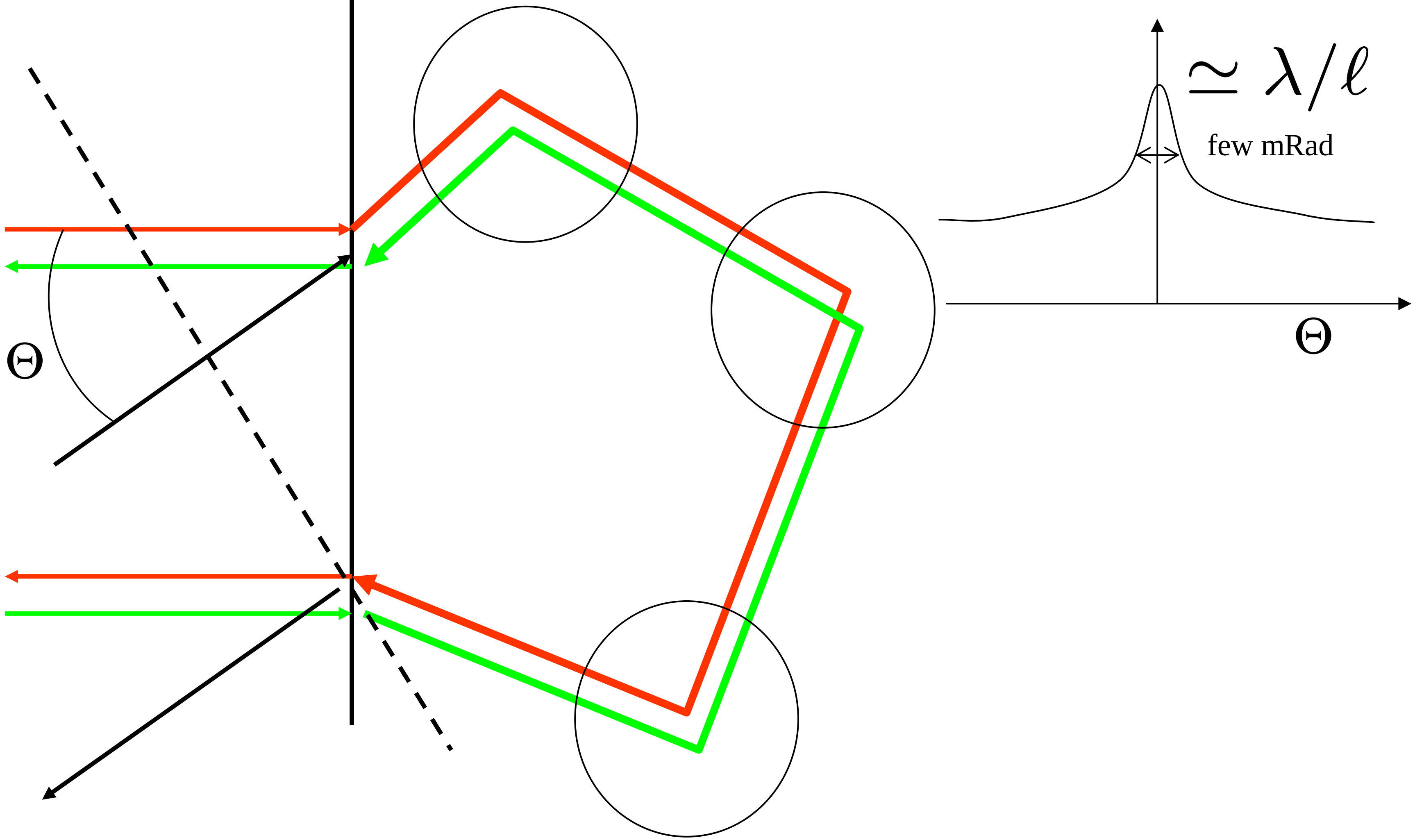
Calculate t_{ij} transmission amplitude (complex) between i and j modes

$$\left|\psi\right\rangle = Ae^{i(px-\omega t)}$$

$$\left|A_1 e^{i\phi_1} + A_2 e^{i\phi_2}\right|^2 = A_1^{\,2} + A_2^{\,2} + 2A_1 A_2 e^{i(\phi_1-\phi_2)}$$

back scattering cone for light

(Wolf, Maret PRL 55, 2696, 1985)



Boltzmann formulae neglects coherence among

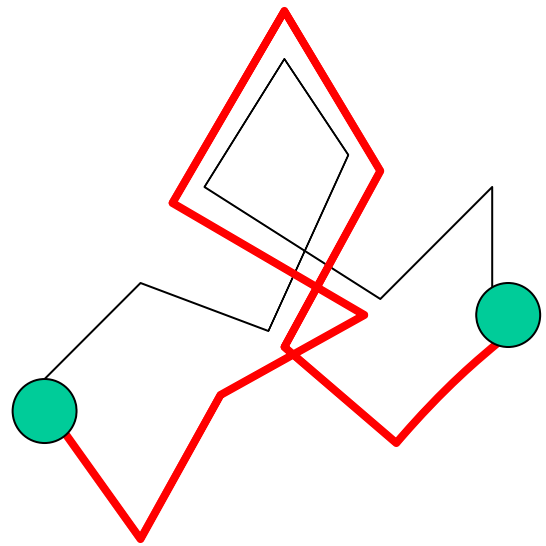
multi- diffusion.

$$\phi = \frac{Action}{\hbar} = \frac{1}{\hbar}\int V dt + \frac{e}{c} A dr + \frac{1}{\hbar}\int p dr$$

Matthiessen rule:

$$\frac{1}{\tau} = \frac{1}{\tau_{el}} + \frac{1}{\tau_{in}}$$

elastic and inelastic scattering
on an equal footing



« Feynman paths»

interference effects are small at room temperature
for silicon devices larger than 10nm:

At T=300K, inelastic Acoustic Phonon scattering dominates diffusion *(Long, Phys. Rev. 120, 2024 (1960))*

(with a certain proportion of surface roughness scattering).

$\tau\phi \approx \tau in \approx \tau e \approx 0.1ps$ (i;e. $\mu \approx 10^4 cm^2/s/V$,)

($V_F = 10^5 m/s$ gives (balist.) Lin=10nm)

<u>Large interference effect at low temperature:</u>

$\tau\phi$ versus T(K):

•at T =1K varies like 1/T (2D Altshuler)

Data for silicon (Heslinga,Klapwijk SSC84, 739 (1992)):

$\tau\phi \approx 1ns >> \tau e \approx 0.1ps$

Quantum localization by elastic multi-diffusion

(important for films and wires)

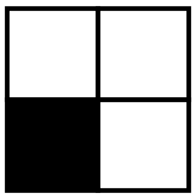
disorder parameter: $k_F\ell$

$$\sigma = e^2 D\nu \qquad \text{(Einstein)}$$

$$\Longrightarrow \quad G_{2D} = \frac{e^2}{h} k_F \ell \qquad \text{(Boltzmann)}$$

$$G_{2D} = \frac{e^2}{h} k_F \ell - \alpha \frac{e^2}{h} ln(L/\ell)$$

(interference correction, L<Lin)



$\alpha = 2/\pi$ (2D, without magnetic flux and spin-orbit)

weak localization

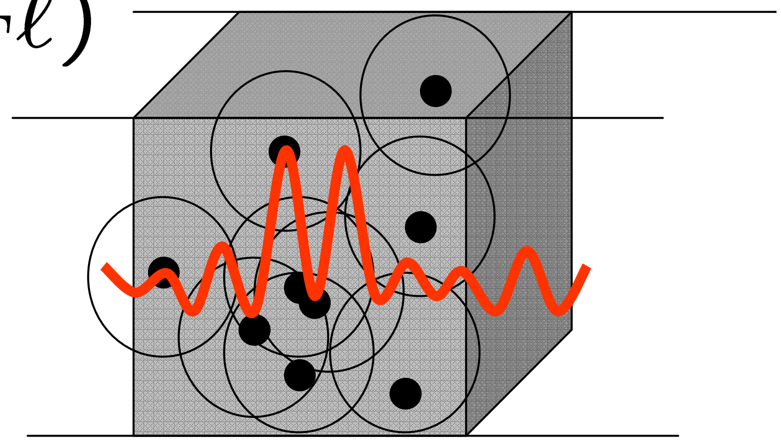
As : $k_F \ell \simeq 1$

or as: $L = \ell exp(k_F \ell)$

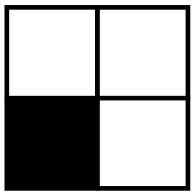
strong localization

$$G_{2D} \simeq \frac{e^2}{h} exp(-L/\xi)$$

$$\xi = \ell exp(k_F \ell)$$



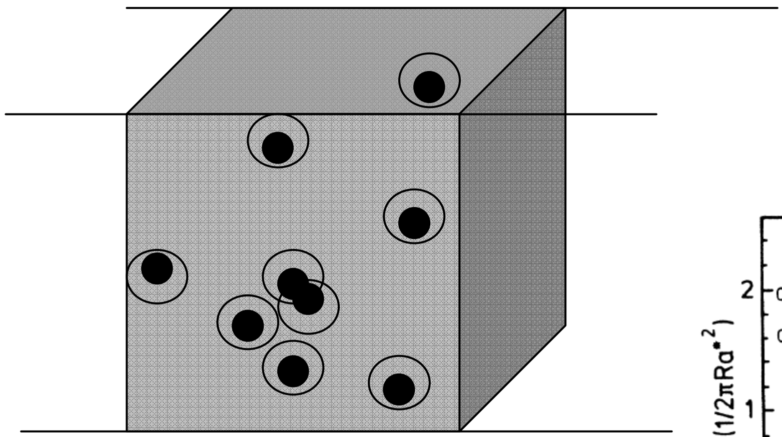
conductance depends exponentially on the length



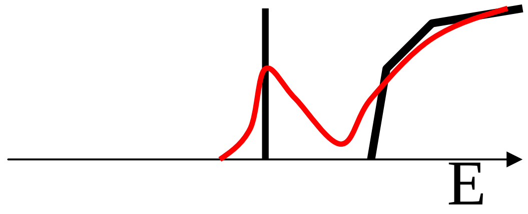
strong localisation

Second case: **electrons are localized** (lightly doped SC)
One should take care explicitely on Coulomb repulsion

Mott 1968
Efros Shklovskii « lightly doped semiconductors »1984



Impurity band



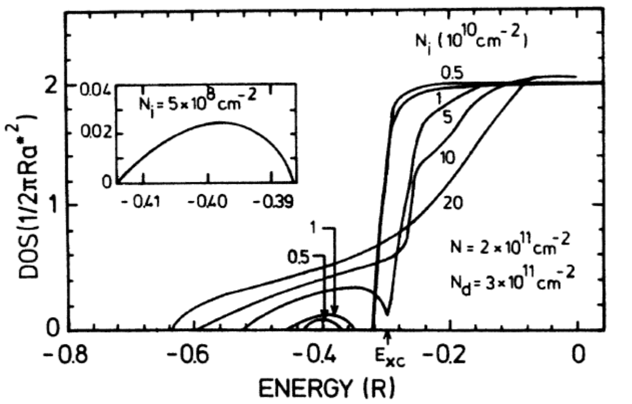
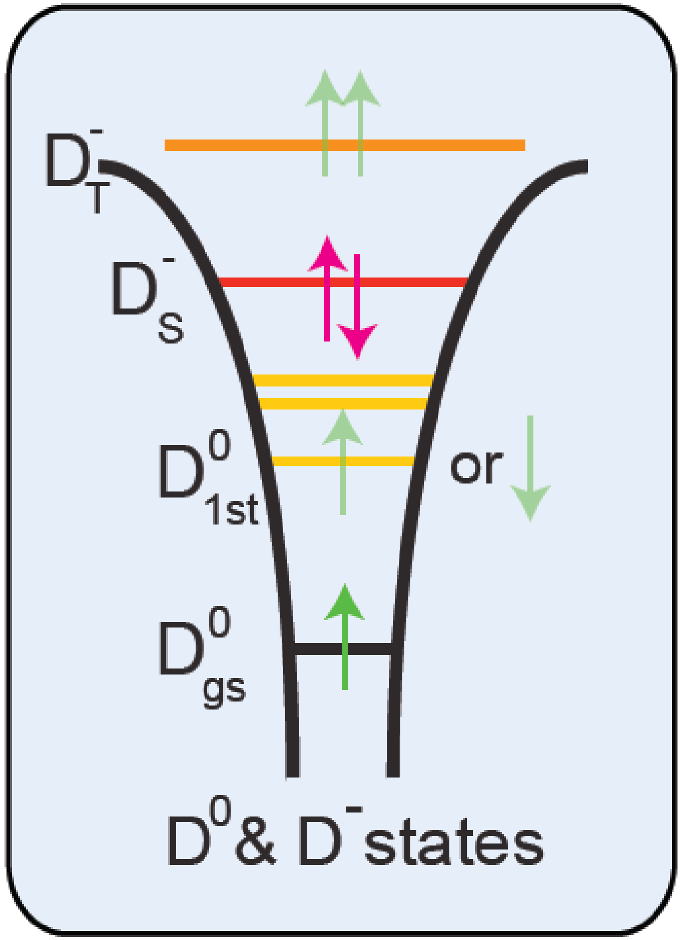


FIG. 1. Density of states (DOS) versus energy (E) for different impurity densities (N_i) according to the Klauder approximation and the random-phase approximation.

Ex: Gold et al PRB37,4589 (1988)

A « simple » problem for Coulomb repulsion

Double occupation of donors,
Coulomb blockade



eigen states

General case for a small numbers of electrons

(« artificial atom»)

N=0 E=0

$e\phi_{ext} = E_1$

N=1 $E_1 - e\phi_{ext}$

$e\phi_{ext} = E_1 + U_{11}$

N=2 $2E_1 + U_{11} - 2e\phi_{ext}$ $E_1 + E_2 + U_{12} - 2e\phi_{ext}$

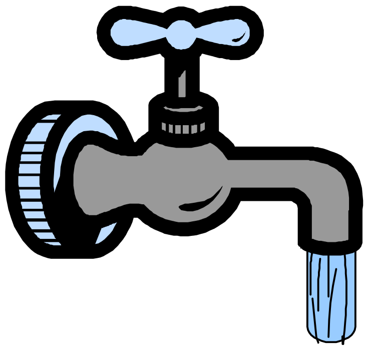
$e\phi_{ext} = E_2 + 2U_{12}$

N=3 $2E_1 + E_2 + U_{11} + 2U_{12} - 3e\phi_{ext}$ etc…

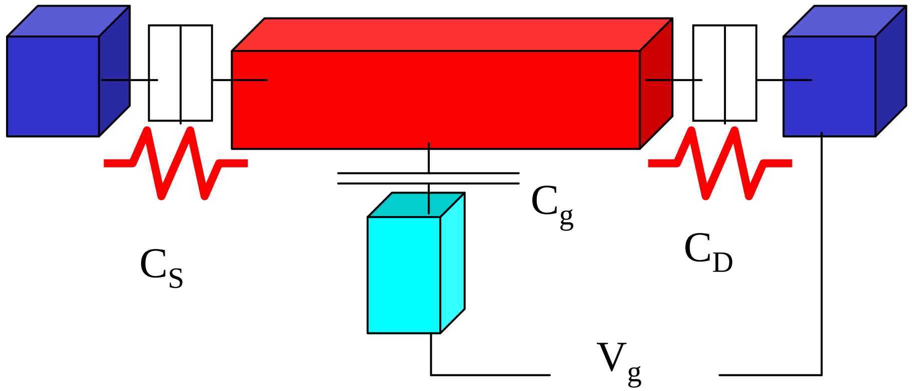
$$U_{ij} \approx \iint \psi_i^*(r_1)\psi_j^*(r_2)V(r_1 - r_2)\psi_i(r_1)\psi_j(r_2)dr_1dr_2 - \acute{e}change$$

Generally $U_{ii} > U_{ij}$, spin polarization (Hund's rules)

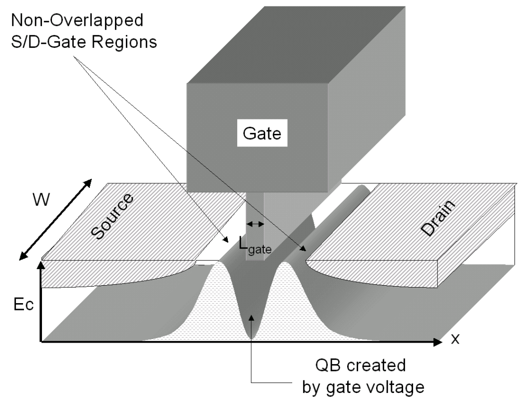
- Conventional devices (MOSFET)
  Continuous flow of charge carriers
  – Modelled by hydrodynamic equations



- Single Electron Devices
  – Operation based on the charge quantization
  – One-by-one flow of electron
  – Localization of electron wave function



$$E(N)=(Ne)^2/2C-Ne\phi_{ext}(V_g)$$

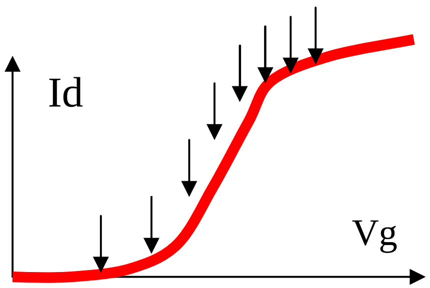


Depending on the access resistance: (T=0K)

FET ($R<h/e^2$) or SET ($R>h/e^2$)

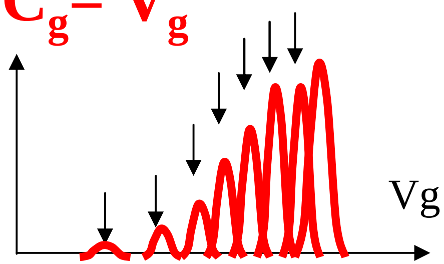
« The more electrons, the more current »

« e-periodic current: E(N)=E(N+1)»



$(N+1/2)e/C_g = V_g$

$kT<Ec=e^2/C$



Classical Coulomb blockade,
Fermi wavelength <<size

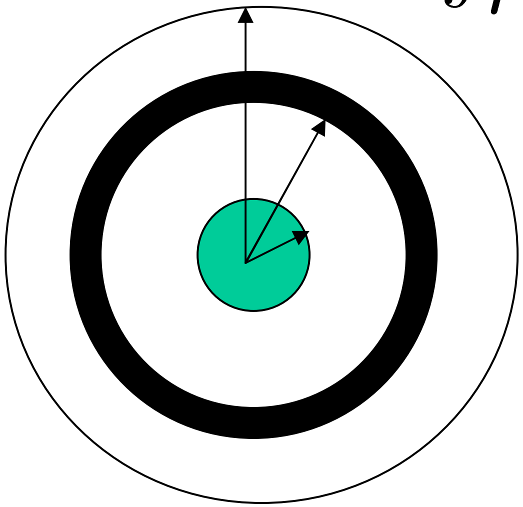
	Gate	S/D barriers	Advantages	Drawbacks
metal		Metal oxide Al2O3[7], NbOx[8]		Offset charges Medium size
		Thin films resistors[18]	Minimizes co-tunneling	Offset charges Medium size
	Thin metallic film[17]		gain	
Metallic clusters[19]	No gate	Metal oxide	Small size	No control gate
Semiconducting clusters	Lateral[14], front or back gate	oxide	Small size	Poor control
Vertical dot[20]	Lateral gate	Epitaxially grown	Few electron regime, disorder free	
Lateral multigates cavities	Lateral, front or back gate	Split gate[21], lateral gate[13], top gate [9,22] Schottky gate or local oxidation (AFM)	versatile	Medium or large size, crosstalk
Lateral single gate dot	Front or back gate	Oxide (PADOX)[10,11], etched constriction[1,2,3,5,6,12,15,16,25], doping modulation[4]	Small size One control gate	Poor control of the D/S barriers
Dynamic Quantum dots[24]		Moving electrostatic potential (surface acoustic waves..)		

Partial list in a very large literature:

1 H. Ishikuro et al. APL71, 3691 (1997)
2 L. Guo et al. APL70, 850 (1997)
3 Sakamoto et al. APL72, 795 (1998)
4 Peters et al. J of Appl. Phys. 84, 5052 (1998)
5 Tilke et al. APL75, 3704 (1999)
6 L . Zhuang et al. APL72, 1205 (1998)
7 Nakamura APL76 (2000)
8 Shirakashi APL72 (1998)
9 A. Fujiwara et al. APL 88, 053121 (2006).
10 H. Namatsu et al,
J. Vac. Sci. Technol. B21, 2869 (2003).
11 S. Horiguchi et al. Jpn. J. Appl. Phys.
Part 2, 40 , L29 (2001).
12. J. Gorman et al. PRL95, 090502 (2005).
13 P. A. Chain, H. Ahmed and D. A. Williams,
J. Appl. Phys. 92, 346 (2002).
14 Y. T. Tan et al. J. Appl. Phys. 94,633 (2003)
15 E. Leobandung et al. APL67, 2338 (1995).
16. M. Saitoh et al. APL84,3172 (2004).
17 Yu. A. Pashkin et al. APL74, 132 (1999).
18 V. A. Krupenin et al., J. Appl.. Phys., 90, 2411 (2001
19 D.C. Ralph et al. PRL74, 3241 (1995).
20 S. Tarucha et al. PRL77,3613 (1996)
21 J. A. Folk et al. PRL76, 1699 (1996).
22 D. H. Kim et al. APL79, 3812 (2001)
23 C. Single et al. APL78, 1421 (2001).
24 J.A.H. Stotz et al. Nat. Materials 4, 585 (2005)
25 R. Augke et al. APL76, 2065 (2000)

electron-electron interaction on a disk:

$$\int_{r_0}^{R} \frac{2\pi r dr}{a^2}\frac{e^2}{\epsilon r} = \frac{2\pi e^2}{\epsilon}\left(\frac{R}{a^2} - \frac{r_0}{a^2}\right)$$

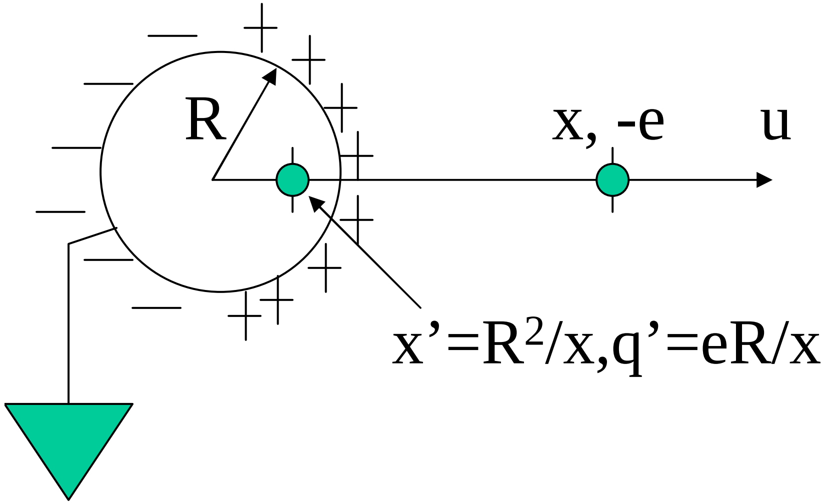


Electrostatic energy for $N=(R/a)^2$ electrons, i.e.:

$$E_1 = (Ne)^2\frac{2\pi}{\epsilon R} = \frac{Q^2}{2C} \qquad E_2 = N\frac{2\pi e^2}{\epsilon a}$$

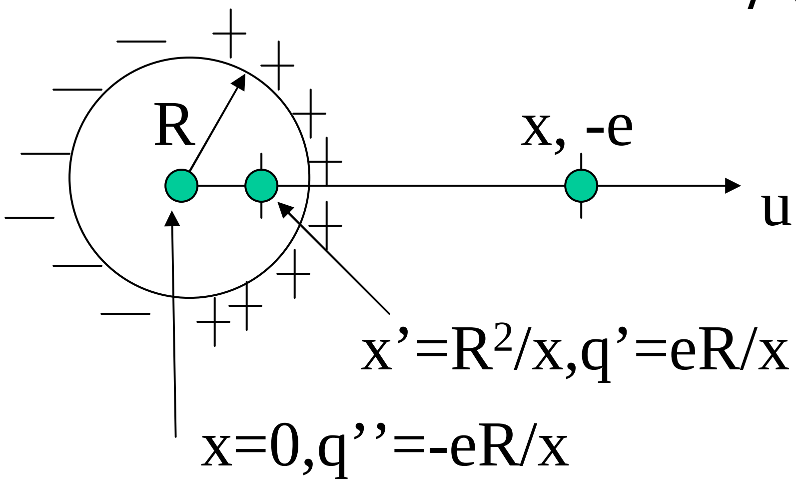
Charging energy + correlation energy (r_0=a)

Charging energy for a metallic sphere: $E_c = e^2/2C$



$$\varphi_{im} = \frac{q'}{x-x'} \approx \frac{e}{2(x-R)}\Big|_{x\to R}$$

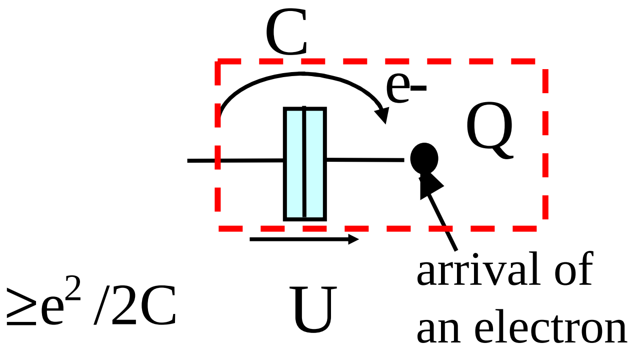
$$\epsilon_0 = 1$$



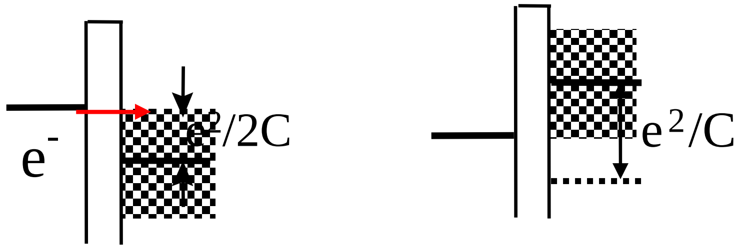
$$\varphi(u,x) = \frac{-e}{|x-u|} + \frac{eR/x}{|u-R^2/x|} - \frac{eR/x}{u}$$

For u=R, the third term varies from 0 to e/R=e/C. Work needed to charge the sphere:

$$W = \int_{+\infty}^{R} dq'' \frac{q''}{R} = \frac{1}{2}\frac{e^2}{C}$$



tunneling junction



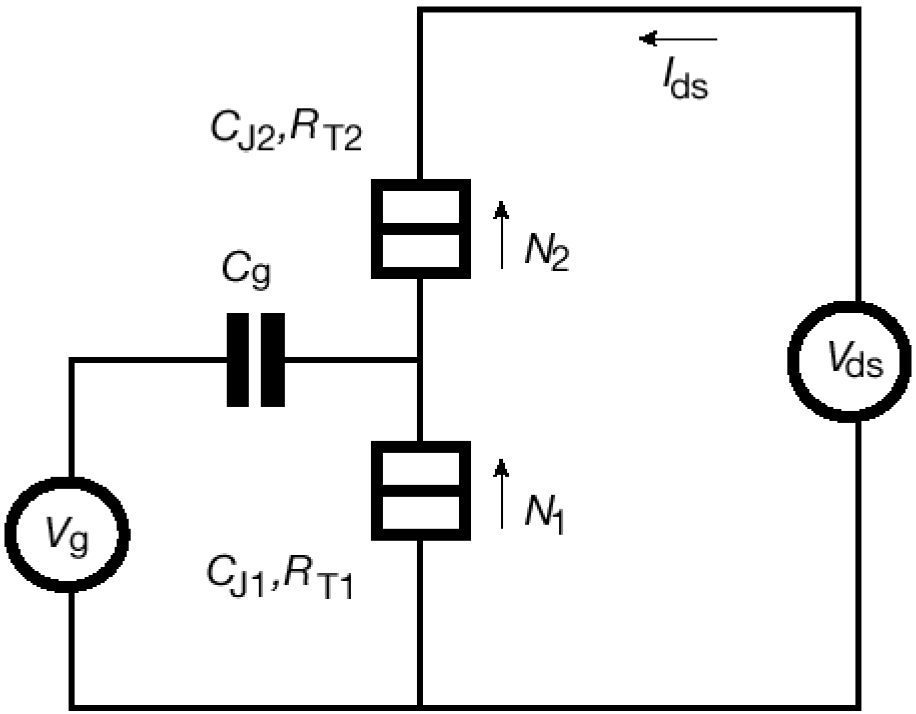
before tunneling after tunneling

$$\boxed{\Delta F = e\left(U - \frac{e}{2C}\right) \geq 0} \quad (\equiv \Delta W)$$

$\Rightarrow$ **critical junction voltage for the arrival of an electron:** $\boxed{\mathbf{U > e/2C}}$

The Coulomb repulsion opens a gap for the arrival or departure of electrons

K. Likharev, FED Journal
Vol.6 Suppl.1, pp.5-14, 1995



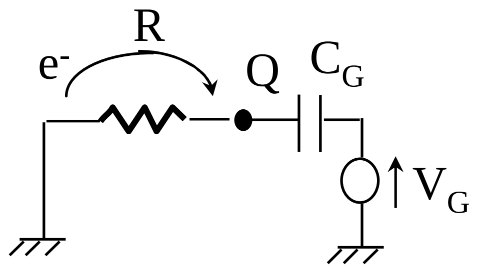
single-electron transistor

$$E_C = e^2/C_{\sum}$$

$$E_{\rm el} \quad = \quad E_{\rm C}[N_2 - N_1 - (C_{\rm g} V_{\rm g}/e) - (C_2 V_{\rm ds}/e) + q_0]^2 - eN_2 V_{\rm ds}$$

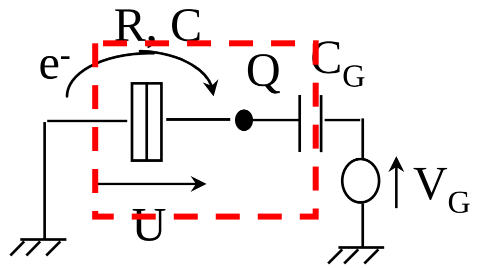
NB: sequential ($\Delta N_2 \neq \Delta N_1$) prevented, correlated events ($\Delta N_2 = \Delta N_1$) favorable

- Capacitor charged through a resistor



- Capacitor charged through a tunnel junction

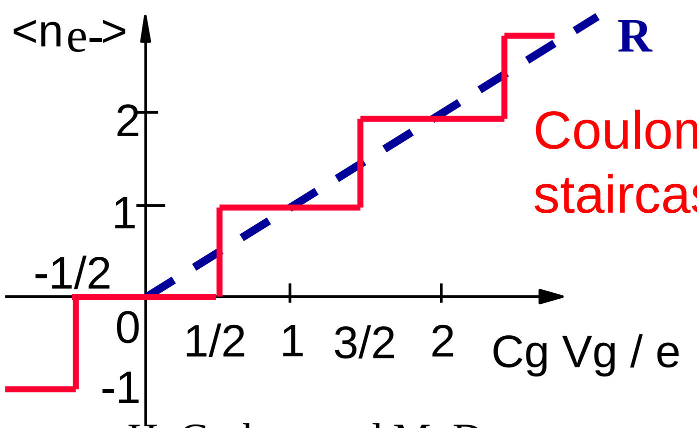
$$kT \ll E_C = \frac{e^2}{2C_\Sigma}$$

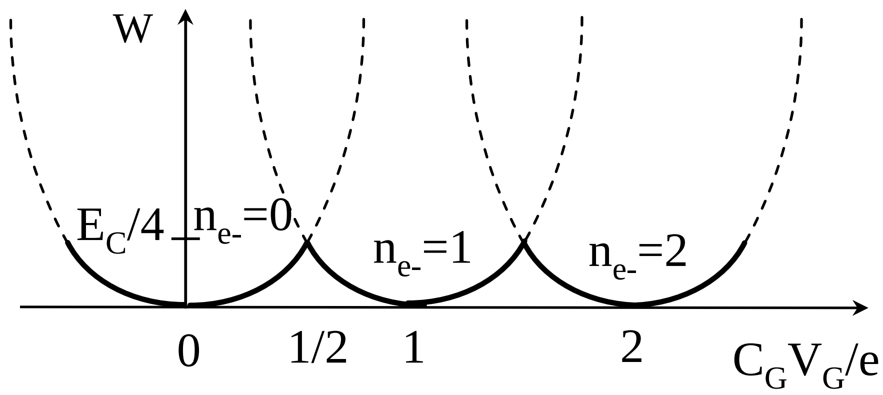


island

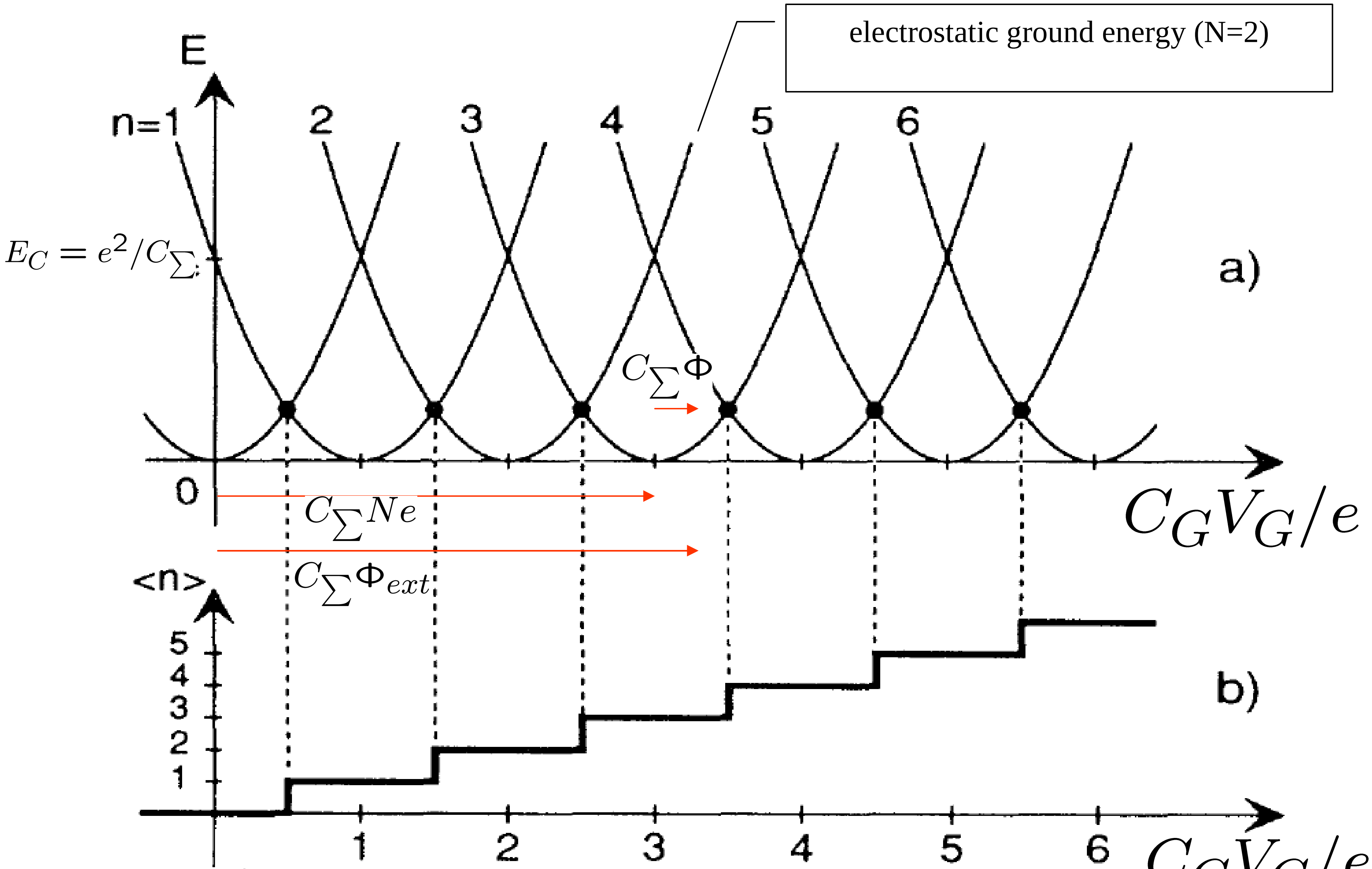
Q=- n e-

Electron Box





H. Grabert and M. Devoret
NATO ASI Series B, Vol.294, Plenum Press, New-York, physics edition, 1992



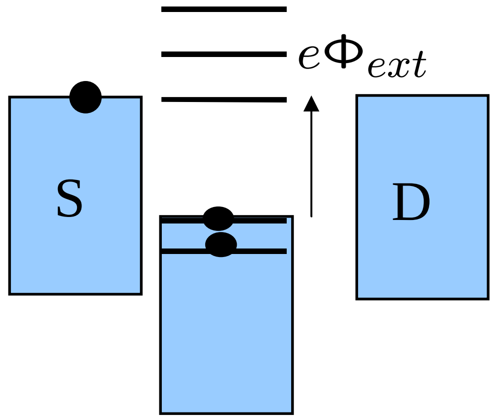
$$E_N = \frac{e^2}{2C_\Sigma}(N - C_G V_G/e)^2 = N^2 e^2/2C_\Sigma - Ne\Phi_{ext} + C_\Sigma \Phi_{ext}^2/2$$

$$\Phi_{ext} = \frac{C_G V_G}{C_\Sigma} \qquad \Phi(N) = -eN/C_\Sigma + \Phi_{ext}$$

electrochemical potential= energy to add one electron:

μ(N)=E(N)-E(N-1)

$$\mu(N) = E(N) - E(N-1) = (N-1/2)\frac{e^2}{C_\Sigma} = \phi_{ext}$$

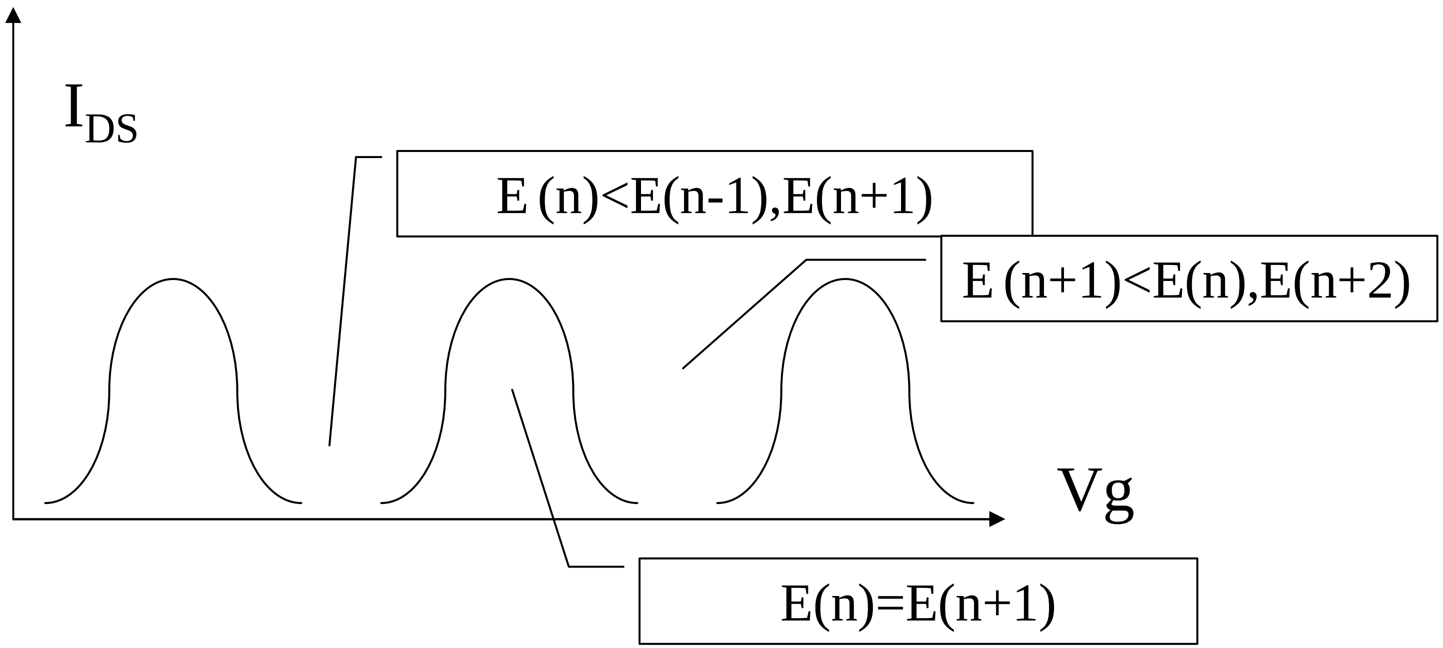


$$E_{elec}(N) = E_{elec}(N+1)$$

$$N^2e^2/2C_{\sum} - Ne\Phi_{ext} + C_{\sum}\Phi_{ext}^2/2 = (N+1)^2e^2/2C_{\sum} - (N+1)e\Phi_{ext} + C_{\sum}\Phi_{ext}^2/2$$

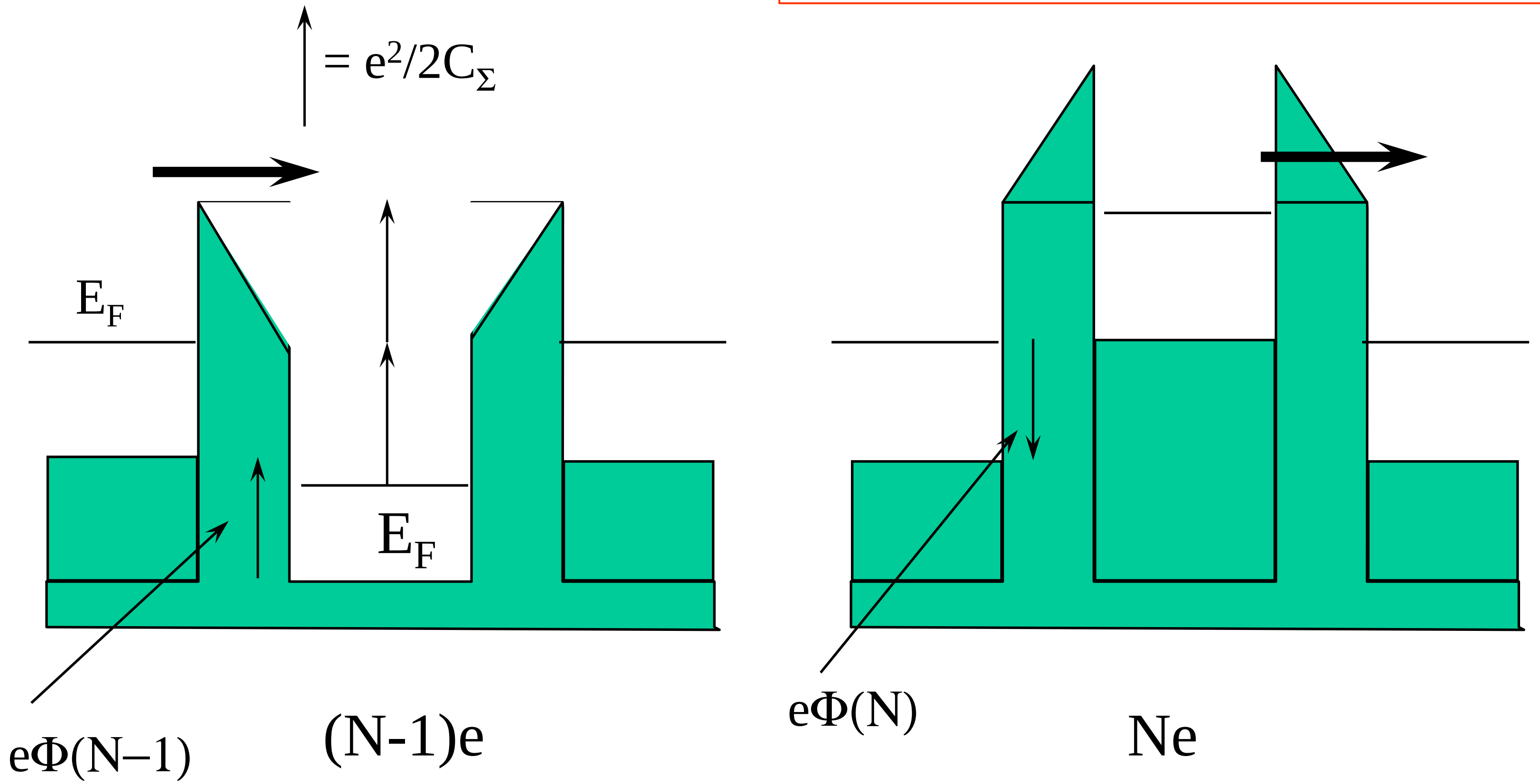
$$\Phi_{ext} = \frac{C_G V_G}{C_{\sum}}$$

$$e(N+1/2)/C_G = V_G$$



$$\boxed{\frac{e^2}{2C_\Sigma} + E_F = E_F + e\phi\left(N-1\right)}$$

$$\boxed{-\frac{e^2}{2C_\Sigma} + E_F = E_F + e\phi\left(N\right)}$$

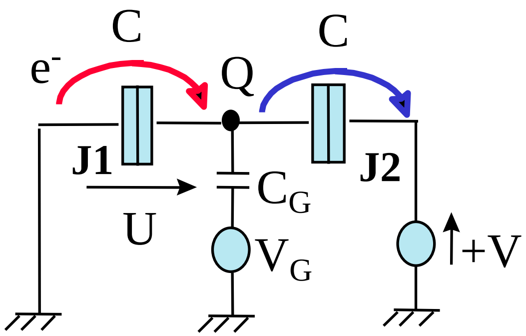


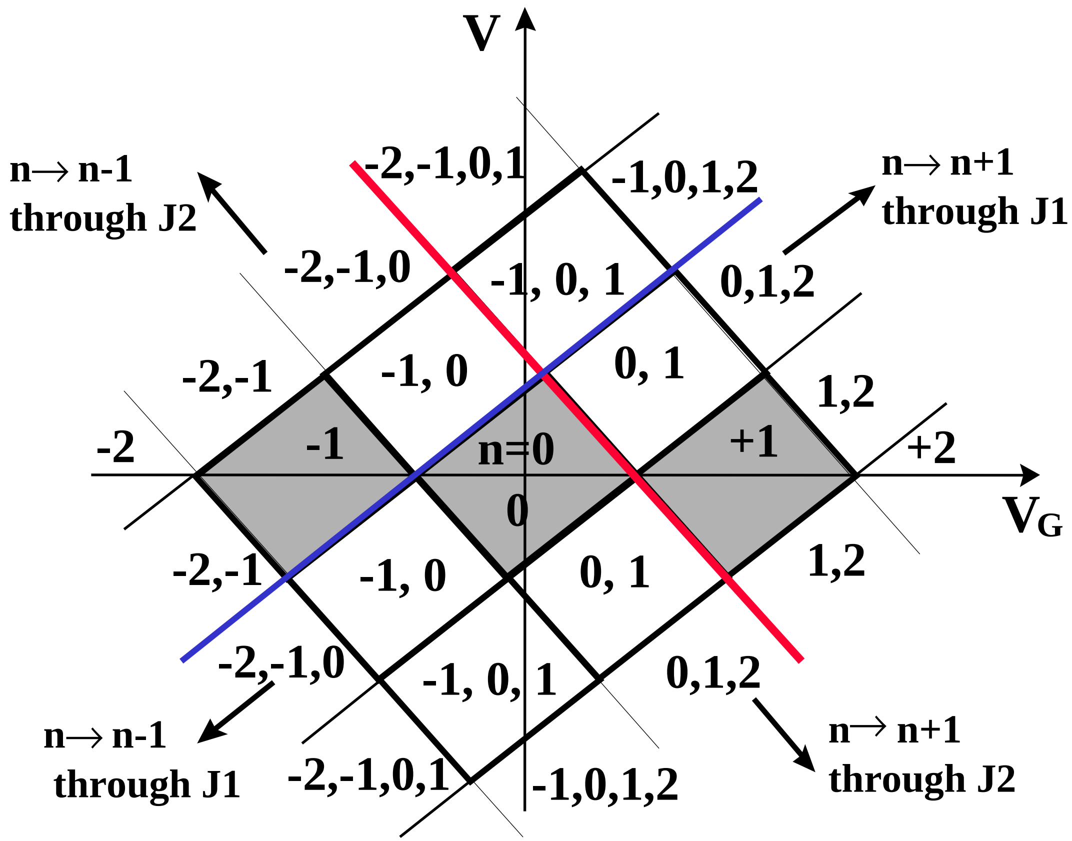
$$\Phi(N) = -eN/C_{\sum} + \Phi_{ext}$$

$$\Phi_{ext} = \frac{C_G V_G}{C_{\sum}}$$

(e>0)

Charge states of SETs





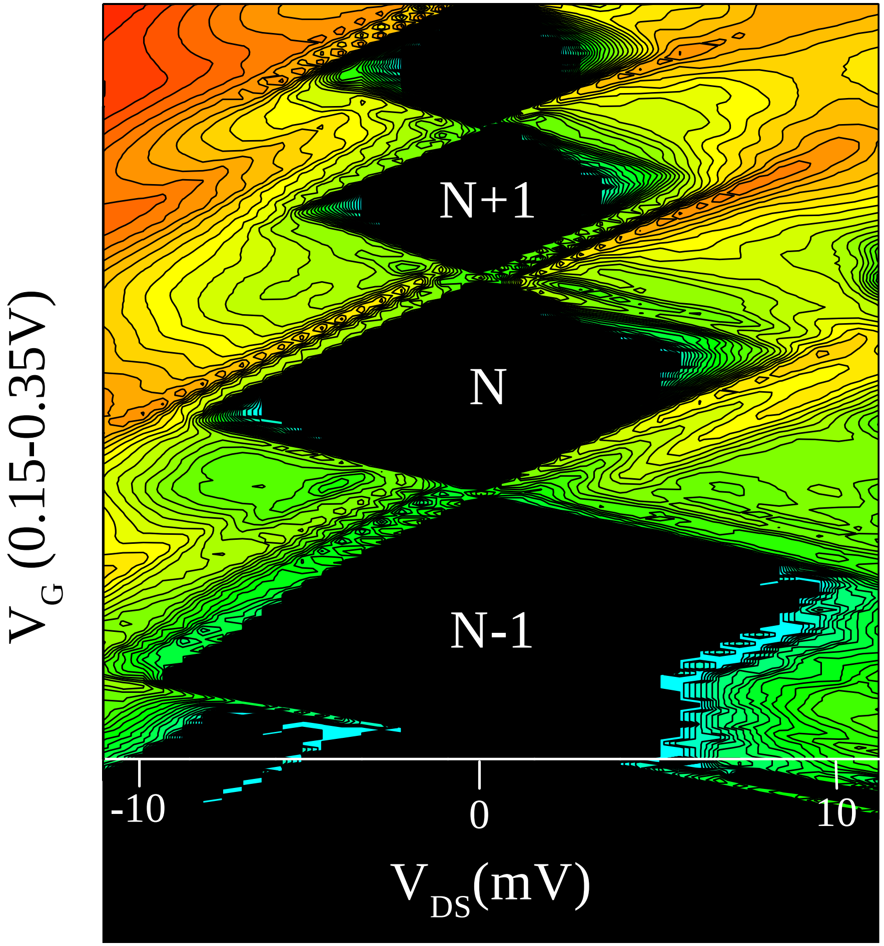
SET off → CB

SET on
(sequential arrival and departure of electrons)

n→ n+1 through J1

$$\boxed{\Delta W = \frac{e}{C_\Sigma}\left(-\frac{e}{2} - ne + CV + C_G V_G\right) \geq 0}$$

Stability diagram:



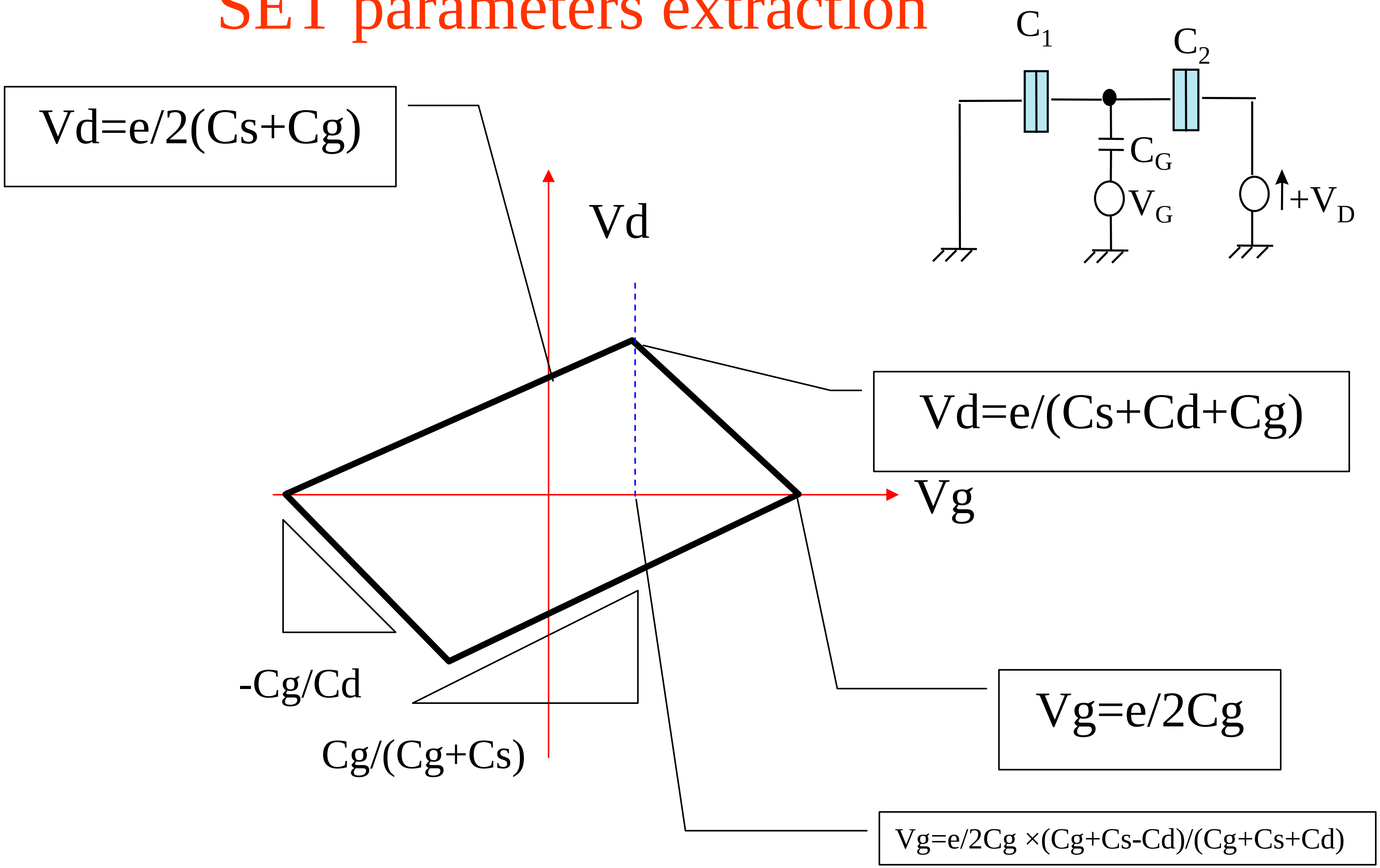
Si-MOSFET

gate length
= 100nm
W=400nm
T=0.1 K

$C \cong 80\text{aF}$

$E_C \cong 2\text{meV}$

SET parameters extraction



Schema for Cg=Cs=Cd

-CBO period → C_G

-Voltage gain $G_{SET}=C_G/C_2$

-$V_{th} = e/2(C_1+C_G)$

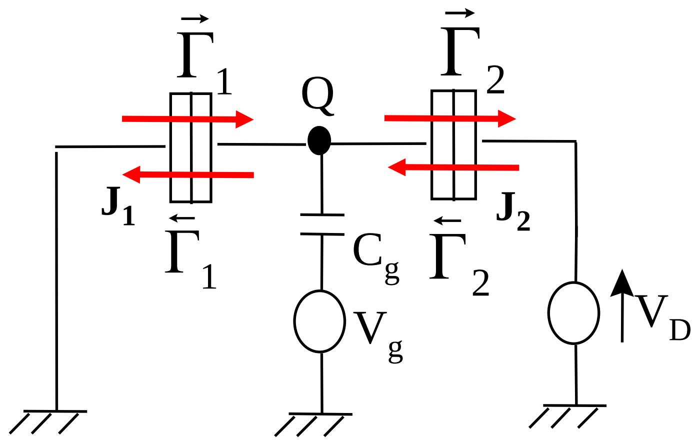
Calculation of the currents:

Hypothesis: stationary case, markovian process (no history),

No cotunneling (only single electron tranfer, inelastic process: "simultaneous" tunneling of e- through two junctions,elastic process: same electron tunnels through both junctions)

electrostatic problem treated (outside the electron tranfer, constant external potential)

Step 1: count all the possible single electron transfers



The **state** of the device is characterized by the number n of electrons on the island. Let p_n the probability to find the device in the state n.

Q=-ne

P_n may change:

-by leaving state n to n-1 or n+1

-or by coming into state n from n-1 or n+1

$$\Rightarrow \boxed{\dot{p}_n = \overbrace{\Gamma_{n,n+1}\, p_{n+1} + \Gamma_{n,n-1}\, p_{n-1}} - \overbrace{(\Gamma_{n+1,n} + \Gamma_{n-1,n})}\, p_n} \qquad (1)$$

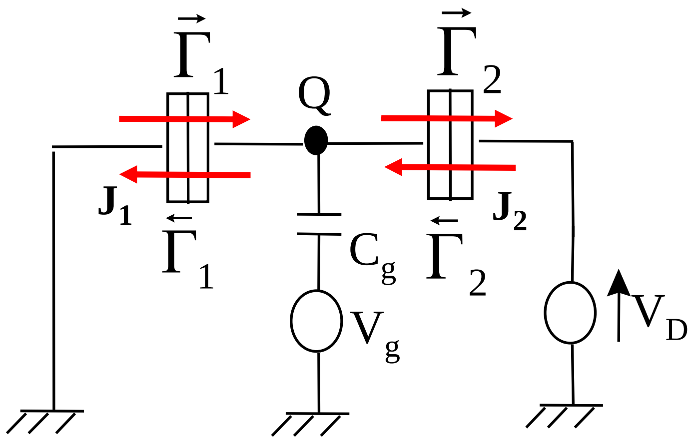
Where $\Gamma_{k,l}$ is the rate for a transition from state l to state k

$$\boxed{\Gamma_{n+1,n} = \vec{\Gamma}_1(n) + \overleftarrow{\Gamma}_2(n)} \qquad \boxed{\Gamma_{n-1,n} = \overleftarrow{\Gamma}_1(n) + \vec{\Gamma}_2(n)} \qquad (2)$$

Where $\overleftrightarrows{\Gamma}_i$ are the electron tunneling rates

Step 2: Derivate the current (for the stationary case)

$$\dot{p}_n = 0$$



Current conservation

$$\boxed{I(\mathrm{V}) = \mathrm{e}\left(\sum_{n=-\infty}^{+\infty} p_n\left(\vec{\Gamma}_1(n) - \overleftarrow{\Gamma}_1(n)\right)\right) = \mathrm{e}\left(\sum_{n=-\infty}^{+\infty} p_n\left(\vec{\Gamma}_2(n) - \overleftarrow{\Gamma}_2(n)\right)\right)}$$

I=e Γ

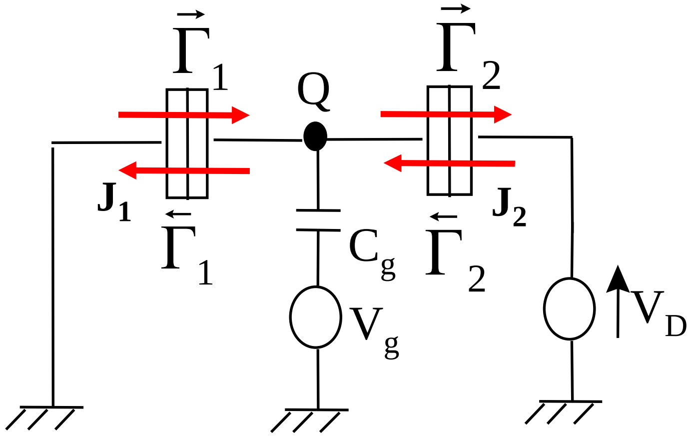
0.16 attoAmpère= 1électron/sec

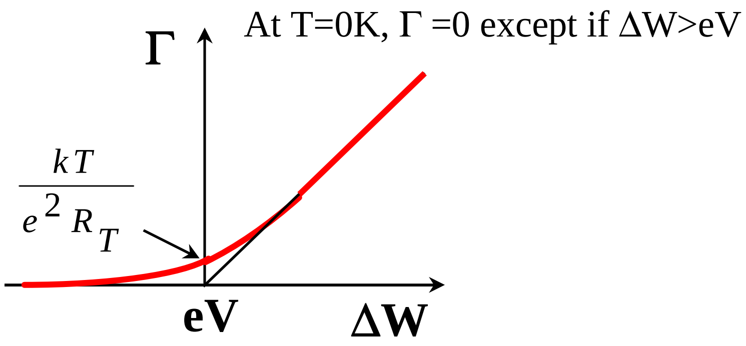
Step 3: calculate the tunneling rates (function of electrostatic energy before and after each electron transfer)

the rate of tunneling events is given by the "orthodox" theory, for the weak tunneling regime ($R_T >> R_Q$):

$$\boxed{\Gamma = \frac{\Delta W - eV}{e^2 R_T \left(1 - \exp\left(\frac{-(\Delta W - eV)}{kT}\right)\right)}}$$

- ΔW: drop of electrostatic energy
- R_T: tunneling resistance
- eV: work to be done by the voltage source





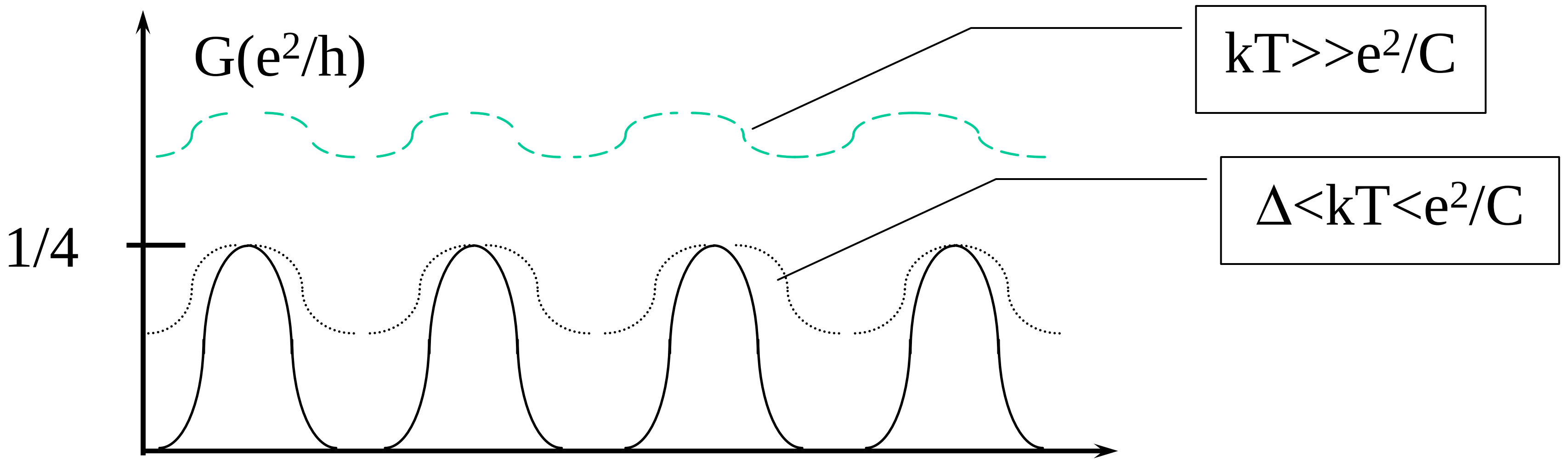
I. Kulik and R. Shekhter 1975, D. Averin and K. Likharev 1985

Results for the orthodox model in the linear regime (V~0)

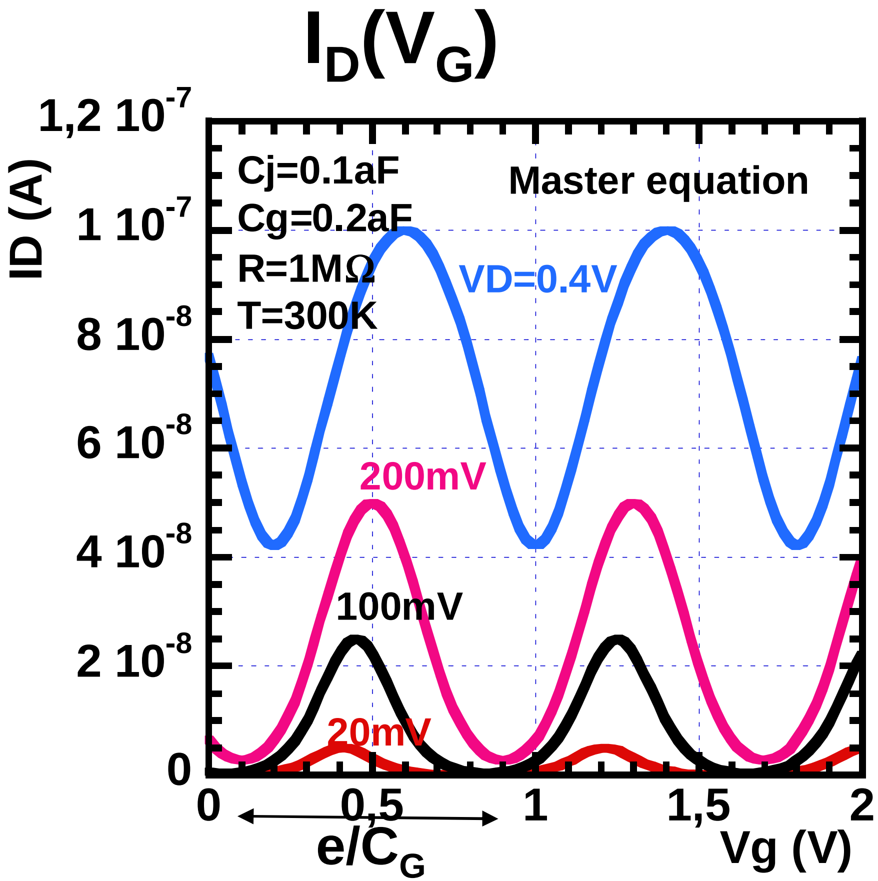
$$G_{\max} = \frac{e^2\rho}{2}\frac{\Gamma^l\Gamma^r}{\Gamma^l+\Gamma^r} \quad \text{if } \Delta E \ll kT \ll e^2/C,$$

$$G = e^2\rho\frac{\Gamma^l\Gamma^r}{\Gamma^l+\Gamma^r} \cong G_\infty \quad \text{if } \Delta E, e^2/C \ll kT \ll \bar{\mu}, E_F$$

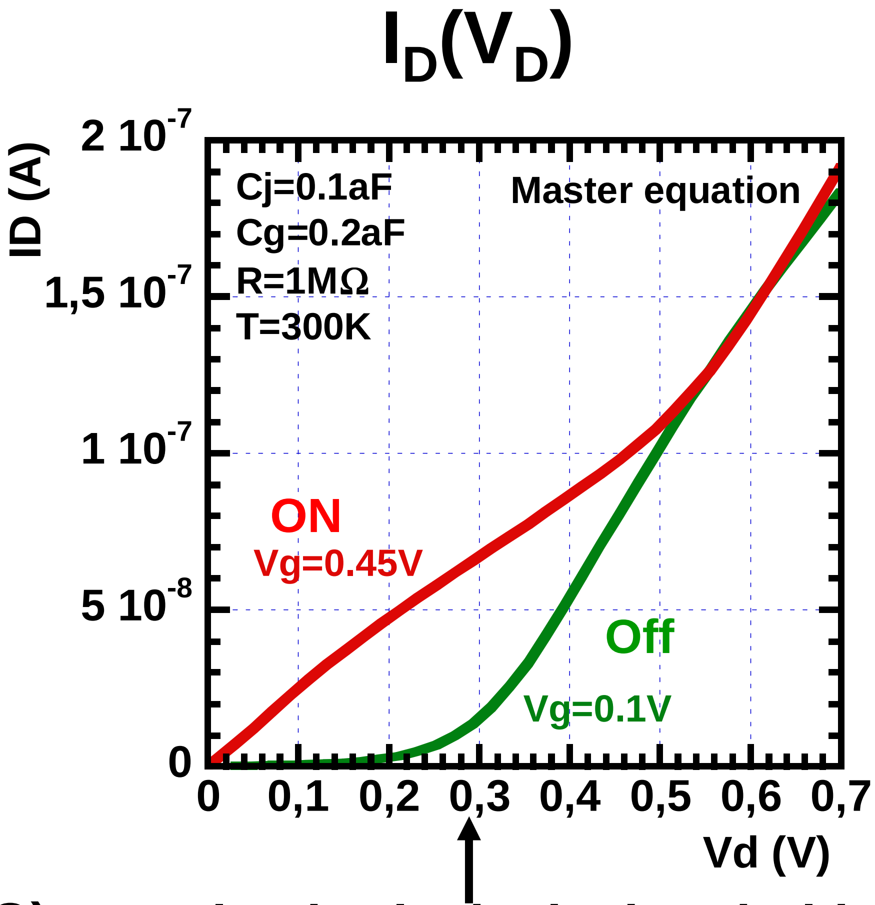
(means 2 barriers in series)



Exemple for a SET working at room temperature



Coulomb Blockade Oscillations (CBO)



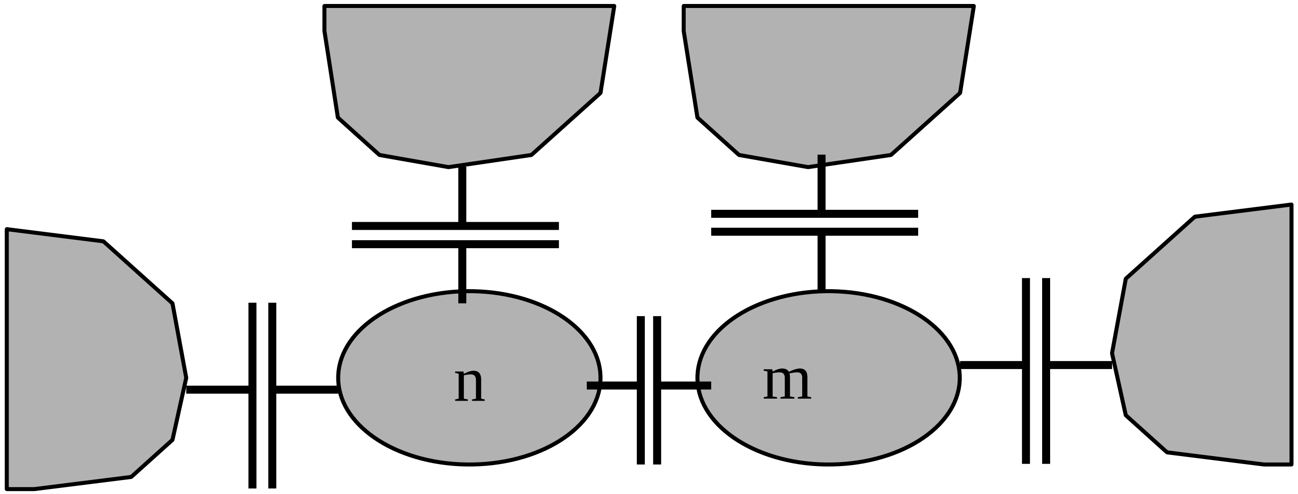
Coulomb Blockade threshold

"on" → <n_e-> half-integer

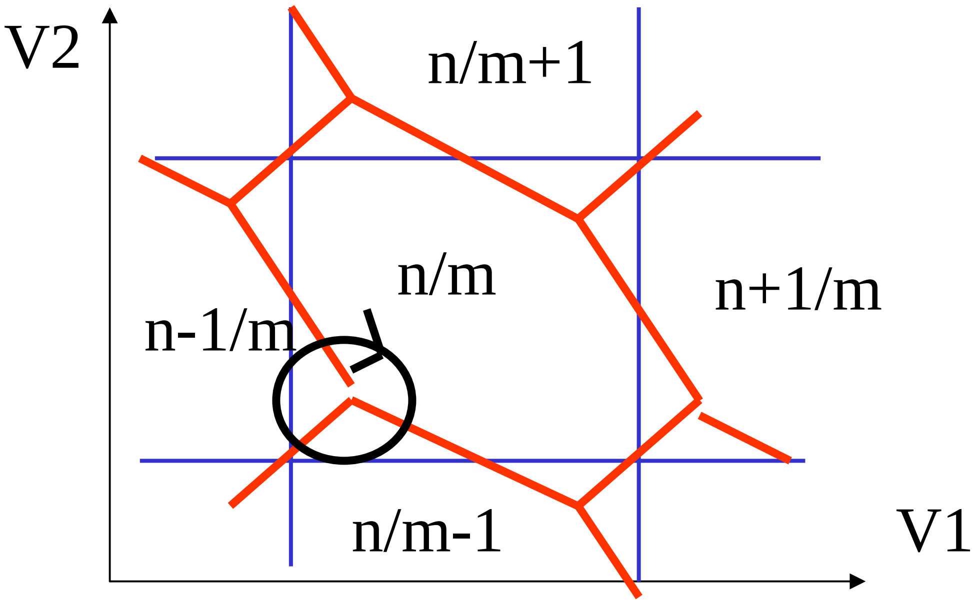
"off" → <n_e-> integer

Low V_D operation

Double dots & their applications as pumps

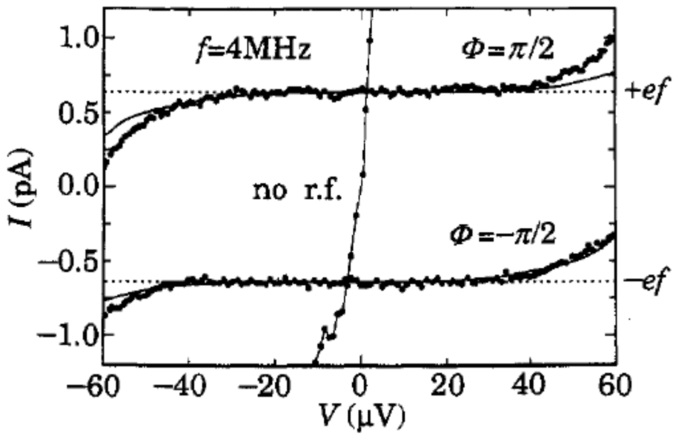


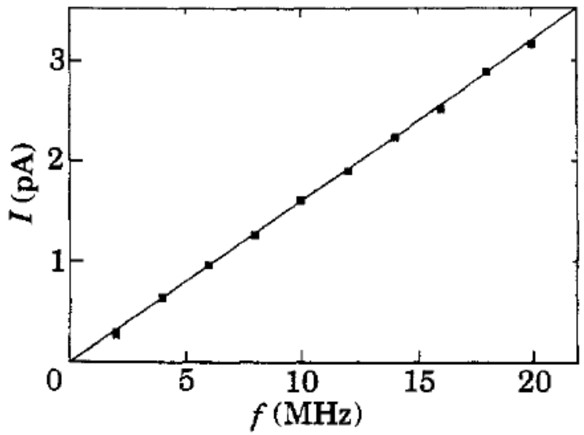
Doubled quantum dot= electron pump

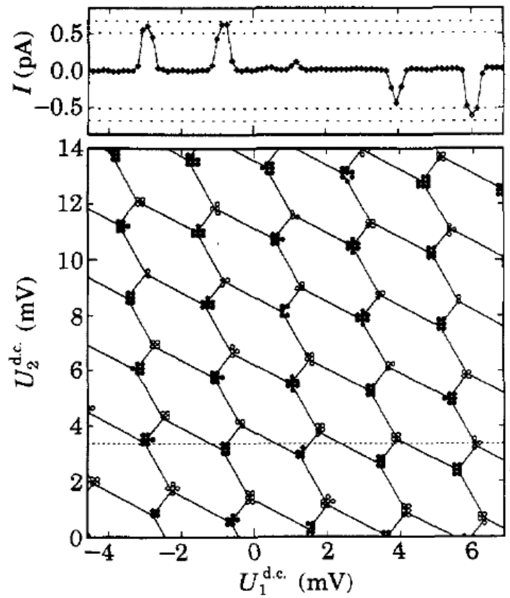


Van der wiel et al.
Rev. Modern Phys.75,1 ('03)

Pothier et al. Europhys. Lett; 17 249 (1992)

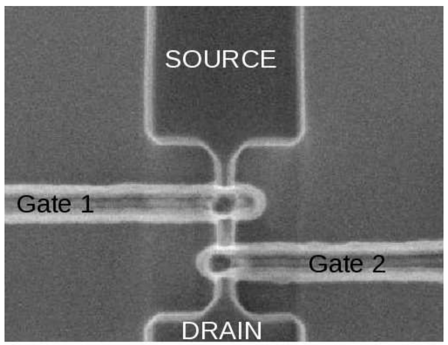


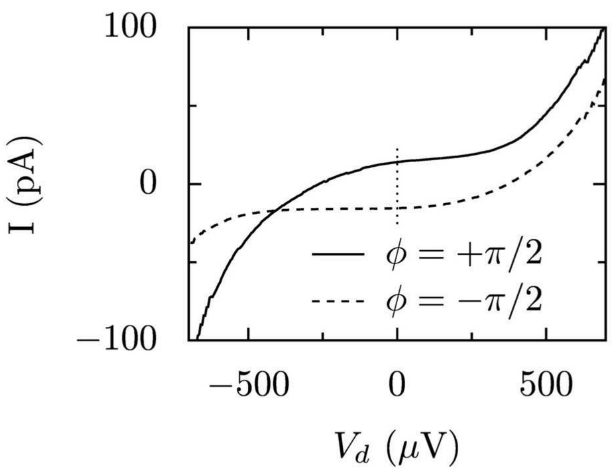


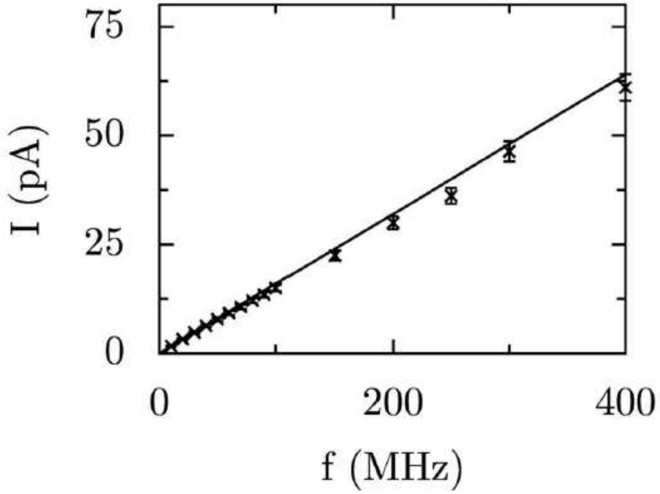


See also: A. Fujiwara et al., “Nanoampere charge pump by single-electron ratchet using silicon nanowire MOSFET ”, *Appl. Phys. Lett.*, vol. 92, 042102, 2008.

M. Pierre et al. CPEM2010



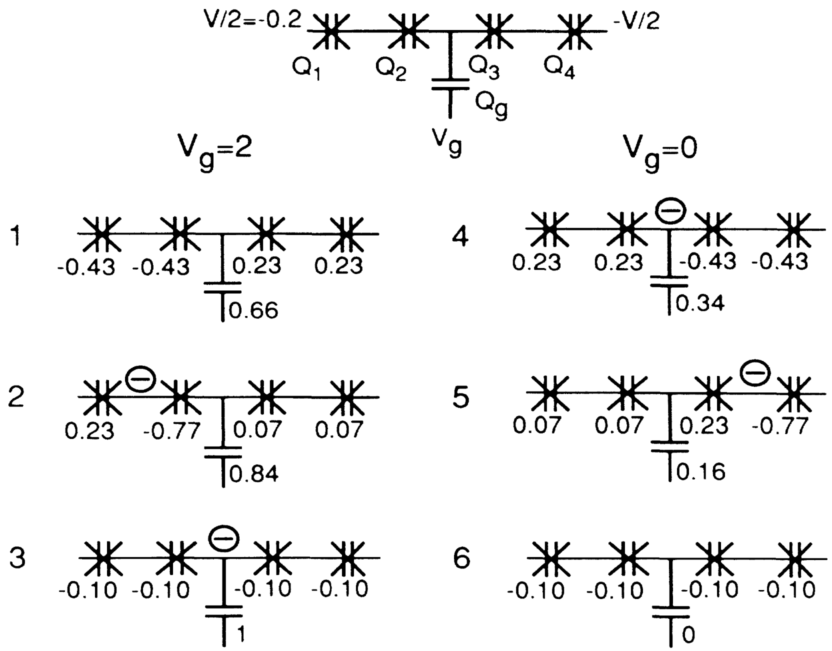




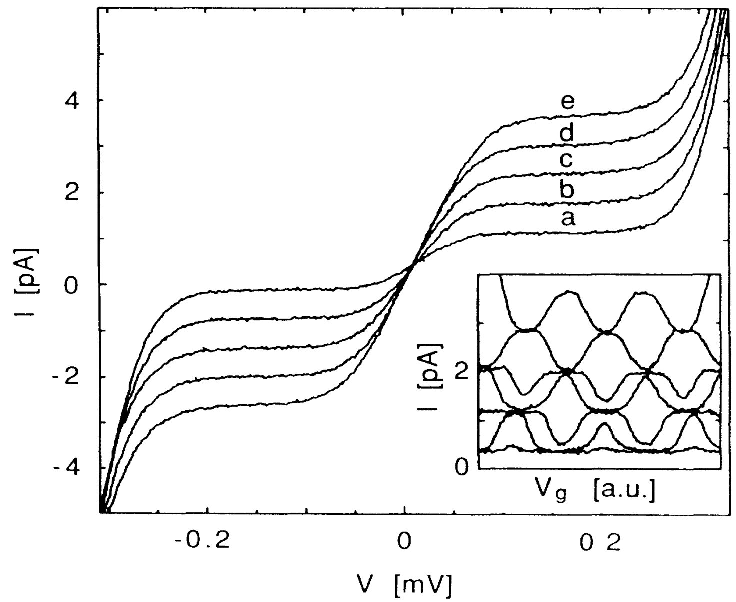
« electron turnstile », (non stochastic) current given by the frequency :

$$I = ef$$

f=4,8,12,16,20 Mhz

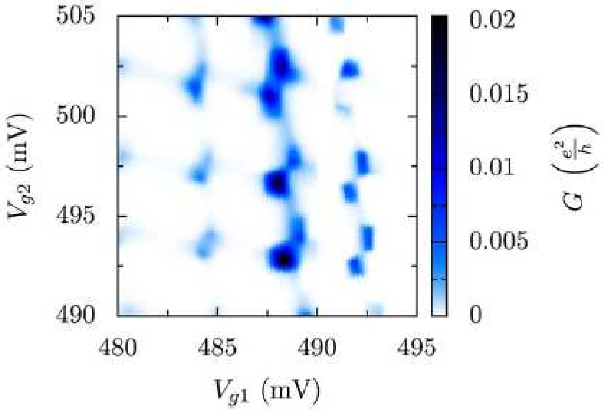


Pothier et al. 1991

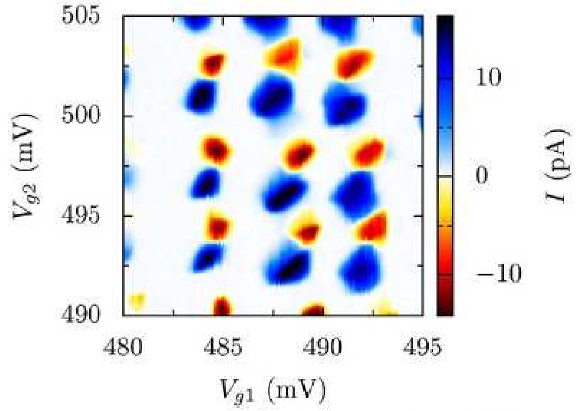


electron pump (see later on)

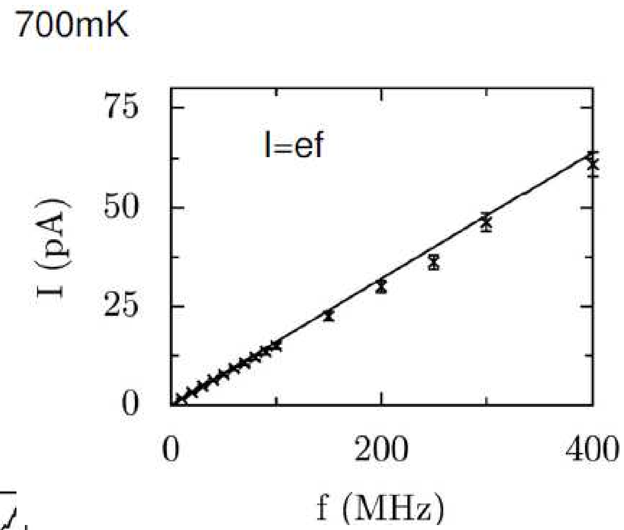
M. Pierre et al. CPEM2010

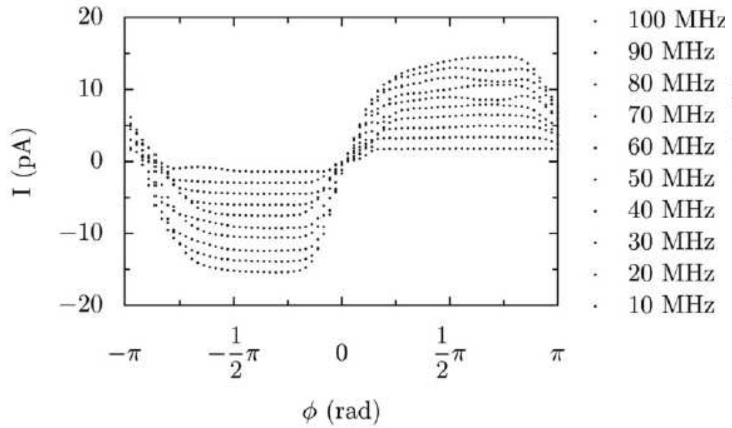


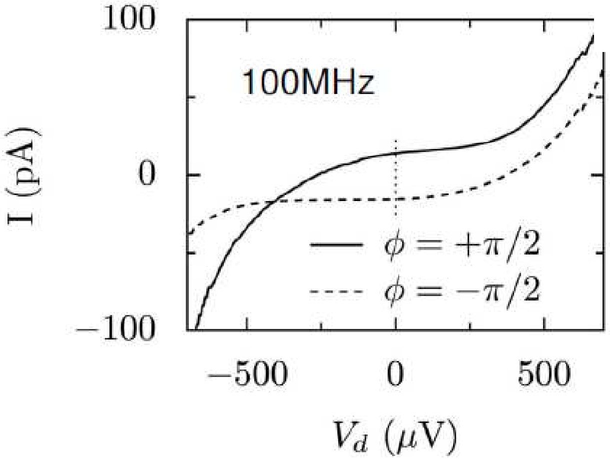
Low bias conductance
(lock-in with Vac=50μV)



Pumped current at 100MHz

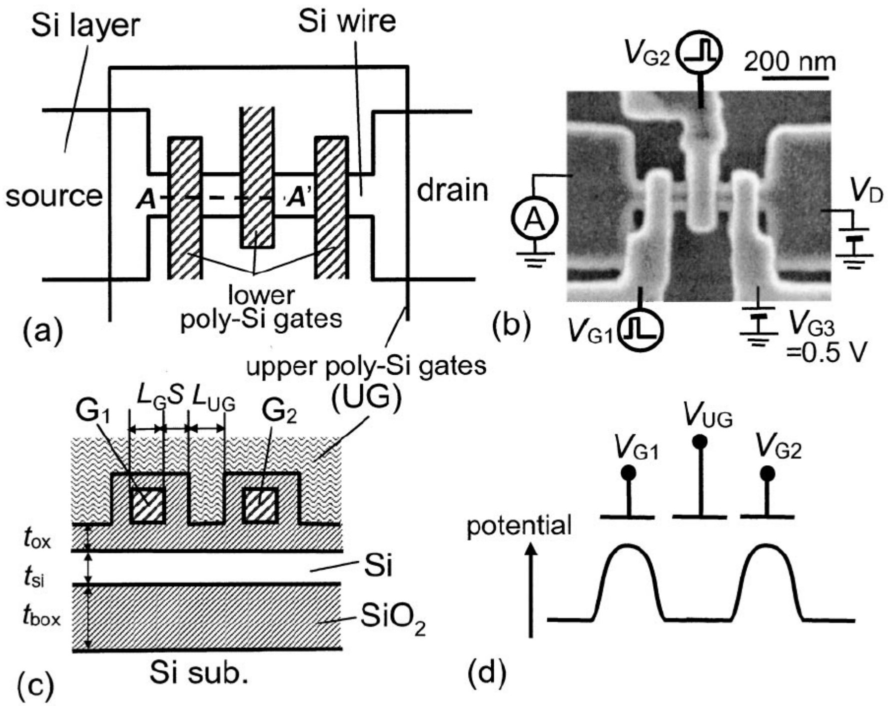


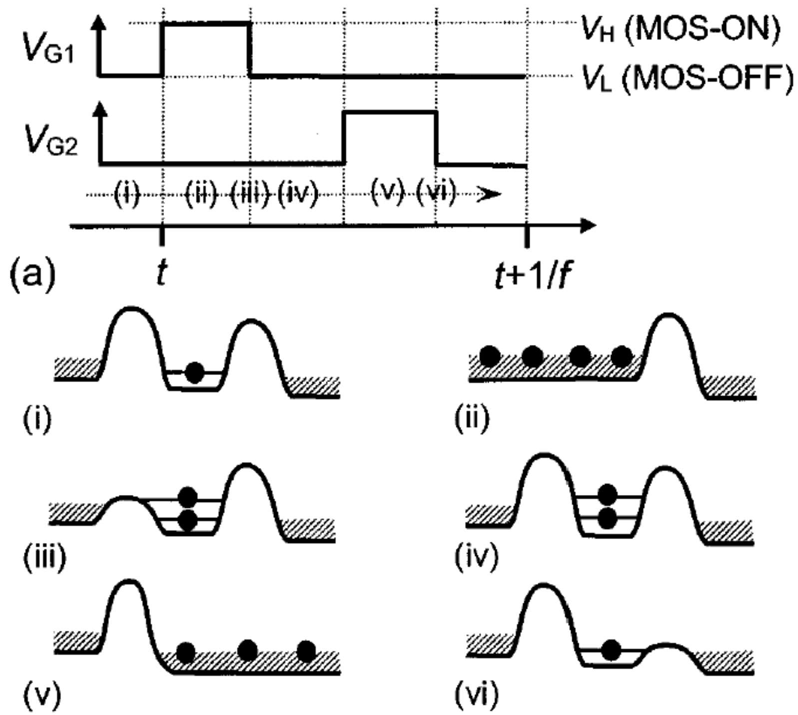




CB vs CCD electron pump

Fujiwara et al. APL2004

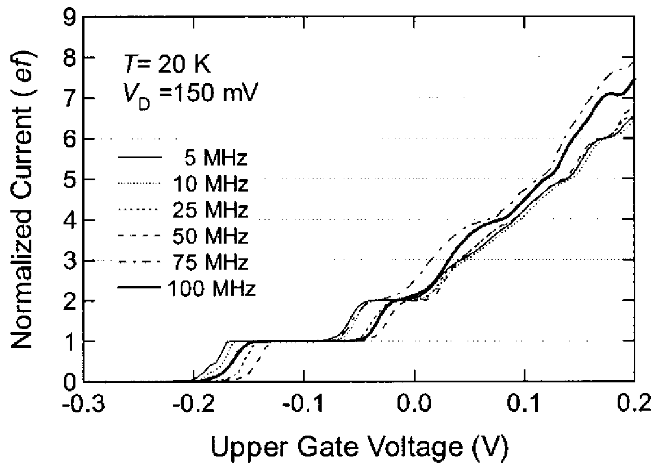




$$I = Nef$$

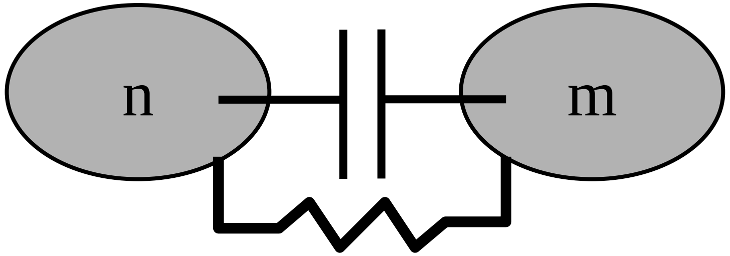
f= pump frequency

(not eV_d/h) ;N depends on potentials



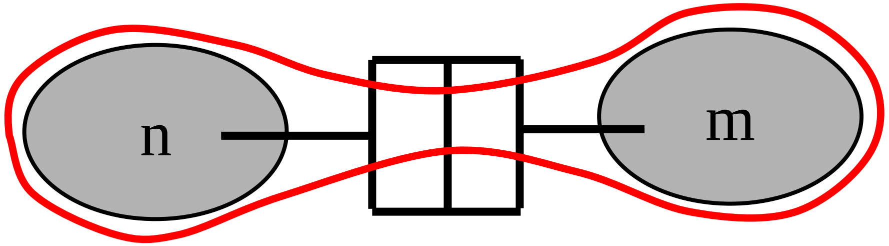
Double dots for quantum bits

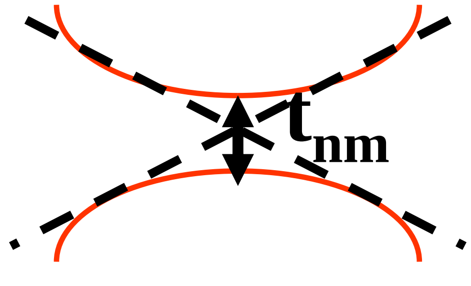
fréquence des pompes à électrons:
RC^{-1} (processus stochastiques)
faible en général (voir transparent 49)



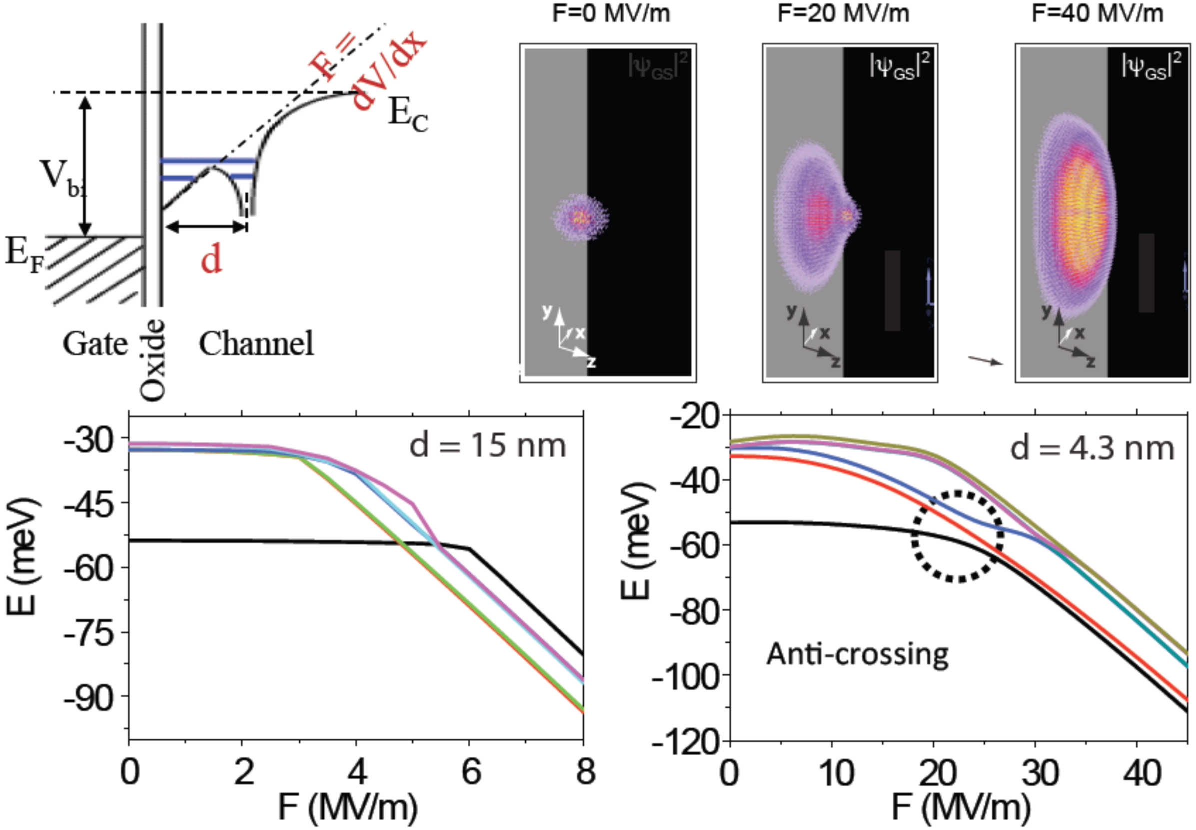
$\neq$

oscillations de Bloch (entre deux états quantiquement cohérents)





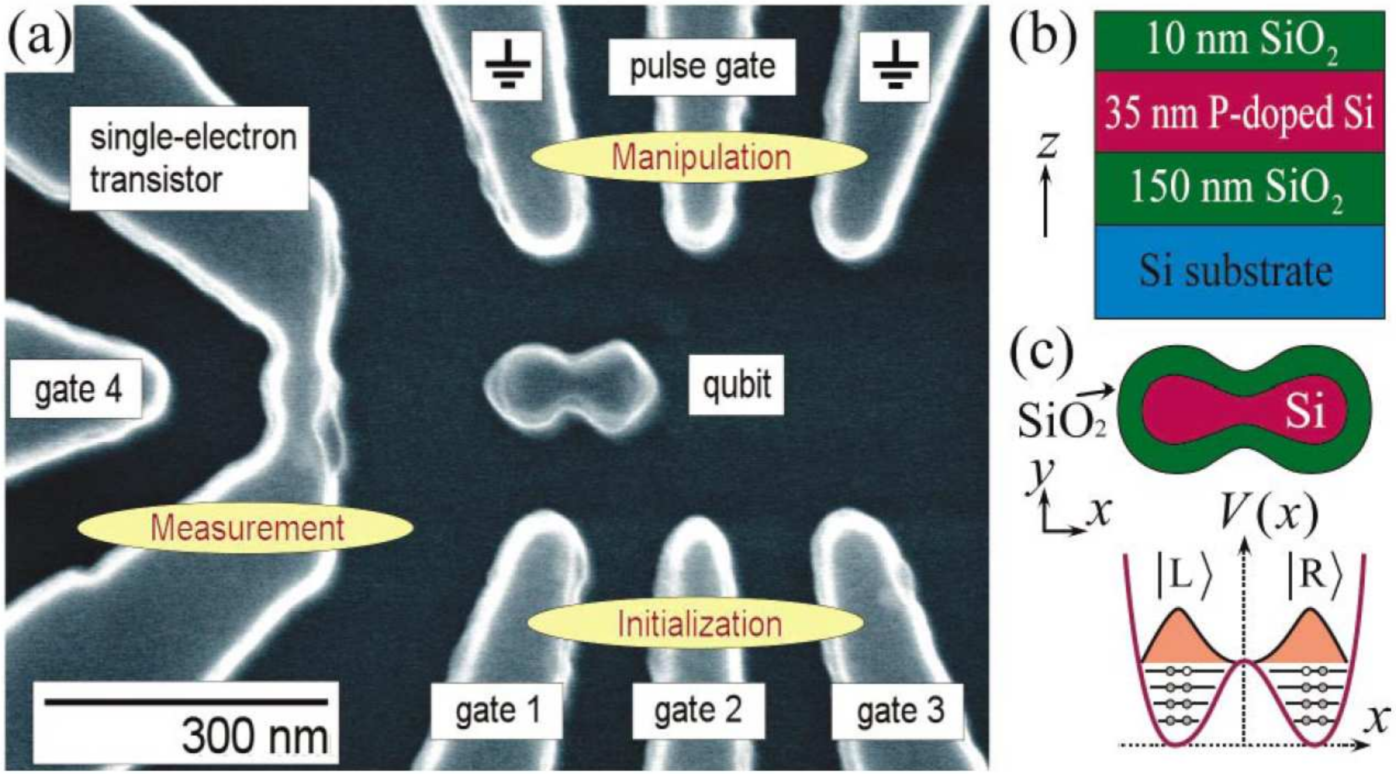
Hybridization with well state leads to a molecular system



[Lansbergen & Rahman, Nature Physics 4, 656, 2008]

Charge Qubit

J. Gorman,et al, PRL**95,** 090502 (2005)



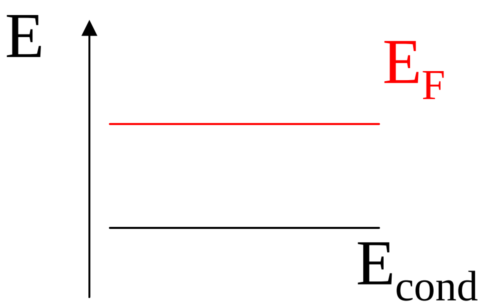
NB: Double dots are also important for spin qubits
(via spin/charge conversion for detection and for coupled spin Qubits)

Ballistic transport, Landauer formula, diffusive transport, transfer matrices, …

Nota:

degenerate case mainly considered
(n_S> few 10^{12}cm^{-2} at T= 300K):

$$k_B T \leq E_F - E_{Cond}$$



quantum interferences revealed at low temperature

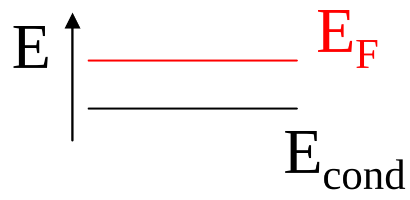
Transport properties are determined at the Fermi level

$$J = env_{drift}$$

« All the electrons participate »

$$v_{drift} = \frac{eE\tau}{m}$$

$$n = \nu(E_F - E_{cond})$$



Warning: degenerate case!

$$J = en\frac{v_{drift}}{v_{Fermi}}v_{Fermi}$$

$$\frac{nv_{drift}}{v_{Fermi}} \ll n$$

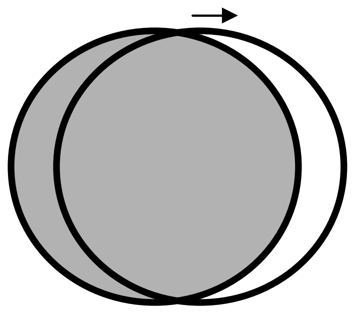
(typ 10^5m/s for 10^{12}cm^{-2}) v_{Fermi}

« only a small part of electrons at the Fermi level participate »

$$k_{drift} = eE\tau/\hbar$$

$$\nu = \frac{\delta n}{\delta\mu} = \frac{mg_Vg_Sk_F}{\hbar^2\pi^2} = \frac{g_Vg_Sn}{2E_F}$$

3D



$$\nu = \frac{g_Vg_Sm}{\pi\hbar^2}$$

2D

$$\sigma = ne\mu \qquad \mu = e\tau/m$$

small electric field (J proportional to E, constant mobility, without saturation)

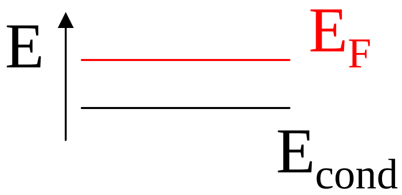
conductivity proportional to n and to the mobility

but equivalently

$$\sigma = e^2 D\nu \qquad D = \frac{1}{d} v_F^2 \tau$$

(Einstein)

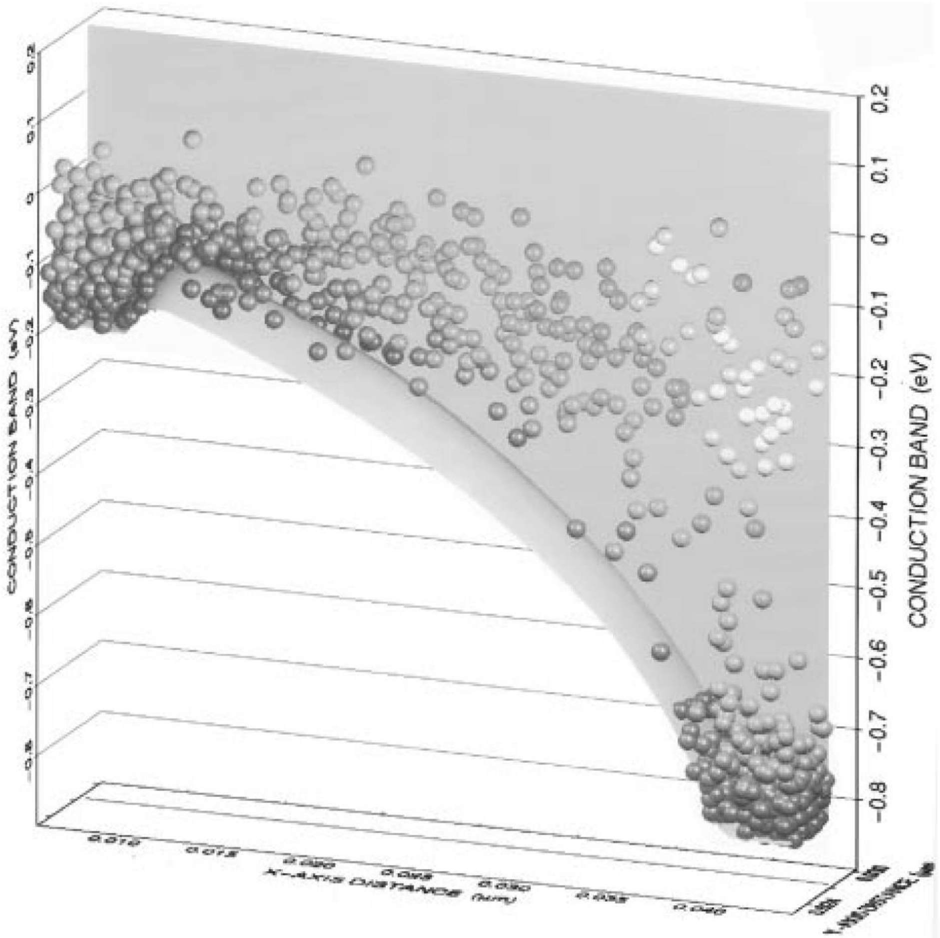
conductivity proportional to D and not to n.



$$\left.\begin{aligned} eD/\mu &= E_F - E_{cond} \\ n &= \nu(E_F - E_{cond}) \end{aligned}\right\} \text{degenerate}$$

non-degenerate:
$$\left\{\begin{aligned} &eD/\mu = k_B T \\ &(k_B T \geq E_F - E_{Cond}) \end{aligned}\right.$$

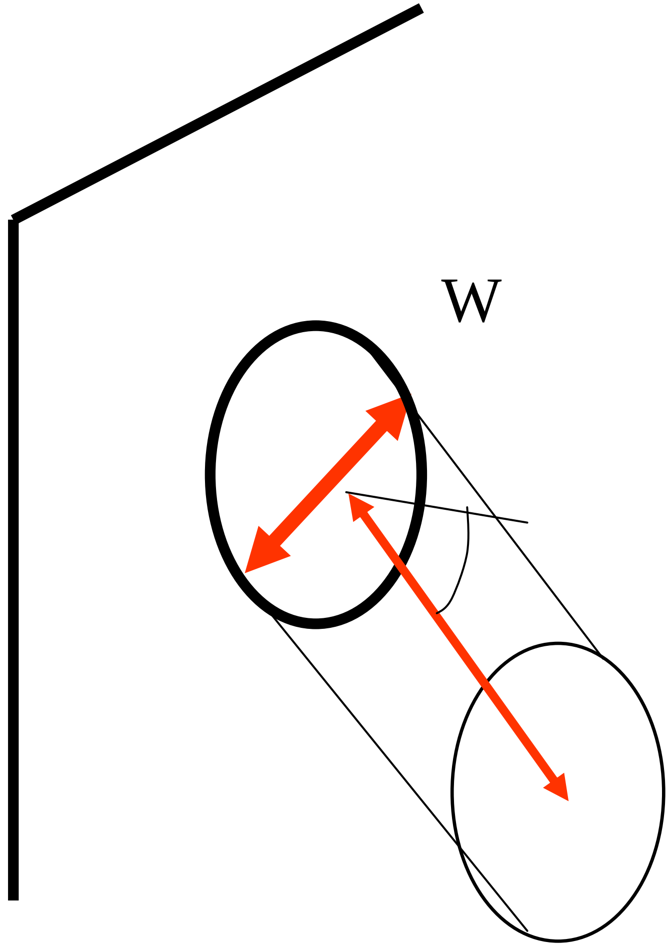
replace ν par $n \times k_B T$ (end of the nota)



Monte Carlo simulation of electron energy versus position down the channel of an
n-channel double-gate MOSFET. The points represent electrons and the line indicates the conduction
band edge. The height of the points above the band edge indicates their kinetic energy. Taur et al. IEDM1997

	D (degenerate)	ν	$\sigma_{Boltzmann} = e^2 D\nu$
1D	$v_F^2\tau$	$\frac{g_V g_S m}{2\pi\hbar^2 k_F}$	$\frac{e^2}{h}\ell \times g_V g_S$
2D	$\frac{1}{2}v_F^2\tau$	$\frac{g_V g_S m}{\pi\hbar^2}$	$\frac{e^2}{h}(k_F\ell) \times \frac{g_V g_S}{2}$
3D	$\frac{1}{3}v_F^2\tau$	$\frac{g_V g_S m k_F}{\hbar^2\pi^2}$	$\frac{e^2}{h}k_F(k_F\ell) \times \frac{2g_V g_S}{3\pi}$

Sharvin resistance of a point-contact



$$I = \frac{eV\nu}{\tau} e \int W cos(\theta) d\theta \ell$$

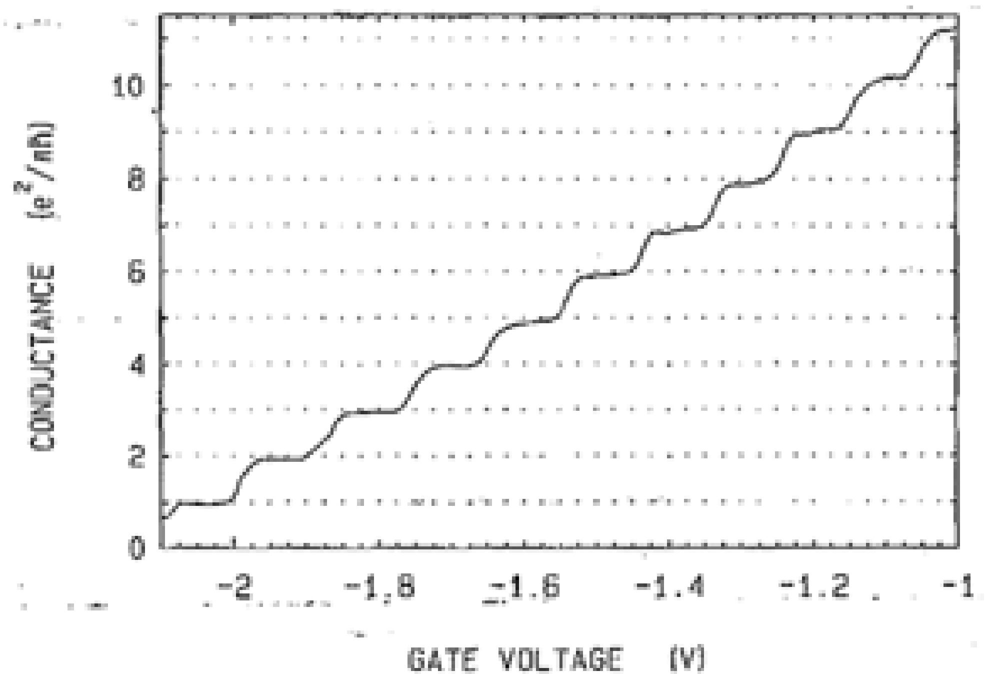
$$G = \frac{e^2}{h} W k_F^2 = \frac{W}{\ell} \sigma_{3D}$$

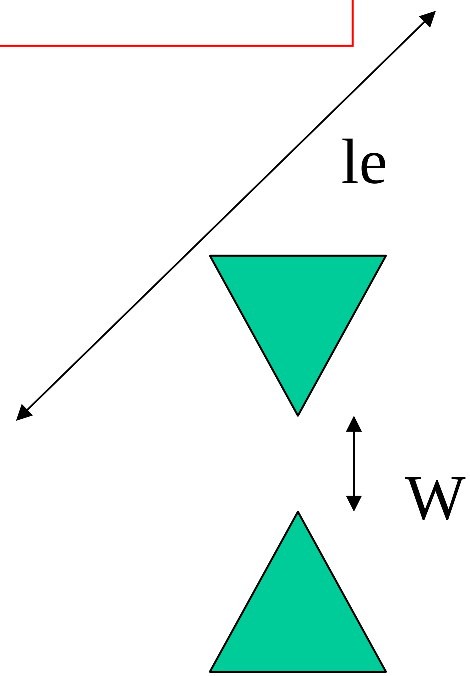
$$\sigma_{3D} = e^2 D\nu = \frac{e^2}{h} k_F (k_F \ell)$$

ballistic quantum point-contact (experiments):

The conductance -as function of width- is quantized

van Wees et al. , Wharam et al. (1988)



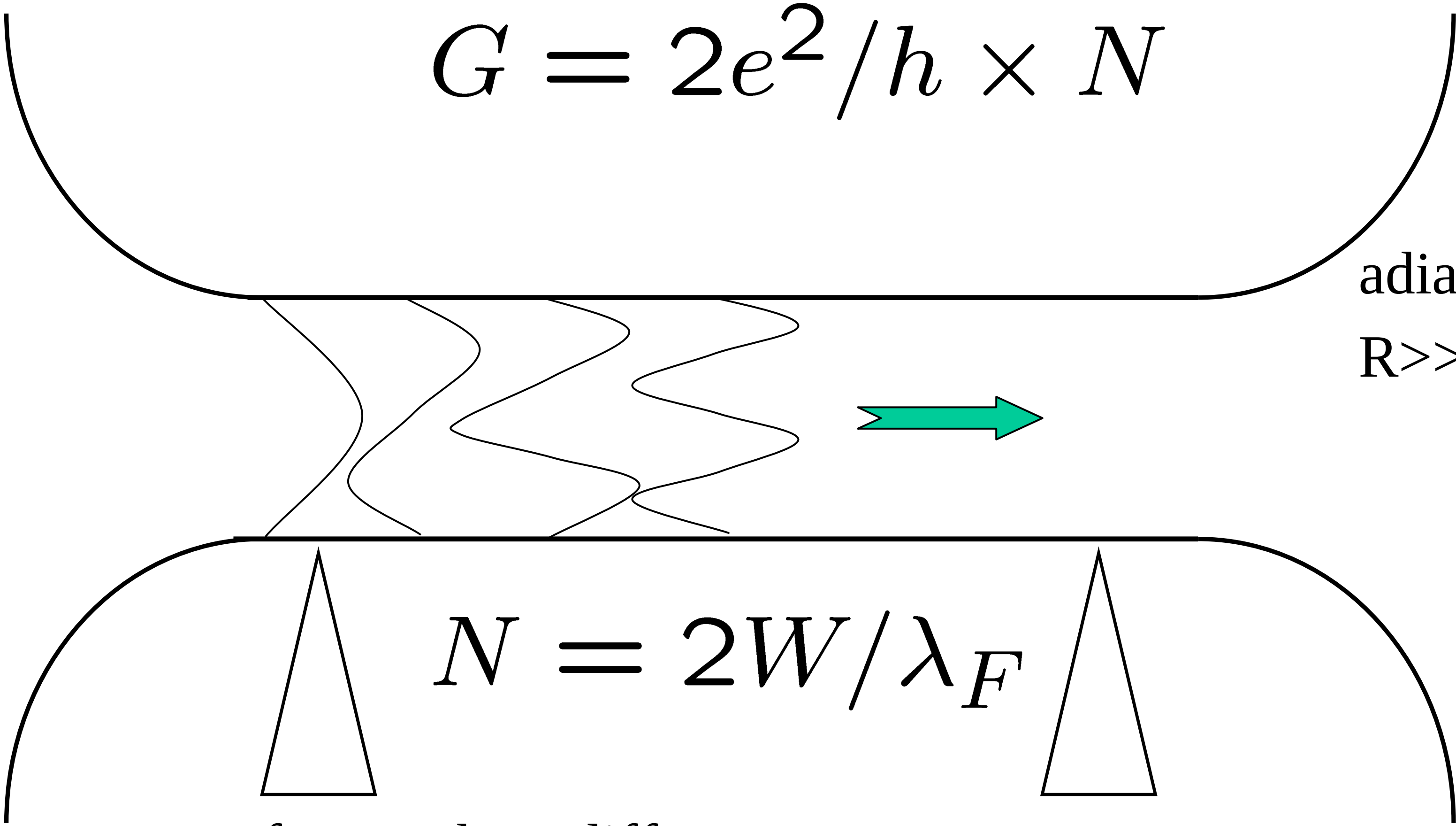


at low temperature

see Buttiker (PRB 41, 7906, '90) for a realistic simulation of the energy-dependent transmission

Landauer formulae (two-probes, ballistic)

$$G = 2e^2/h \times N$$



adiabatic contact:
R>> λ_F

$$N = 2W/\lambda_F$$

four probes=different case

$$I = e \times eV/h \times 2N$$

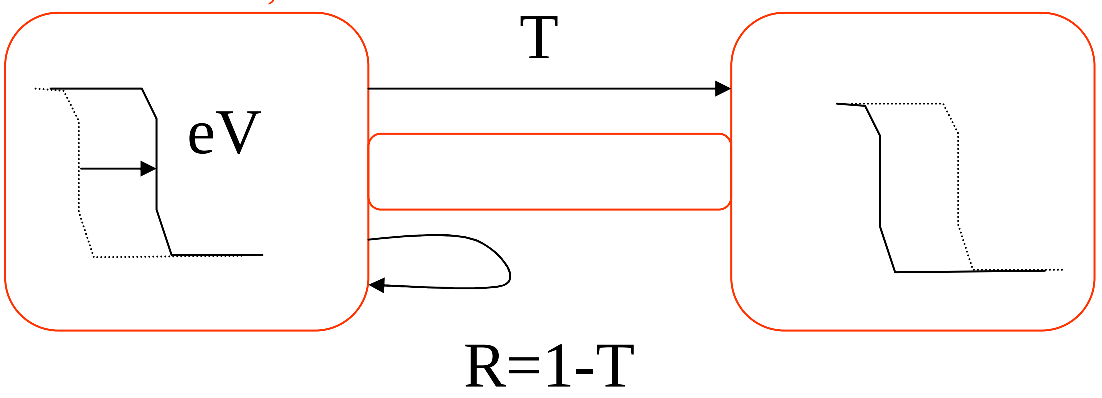
Conductance = Transmission

electrons emitted at frequency: eV/h

$$I = e \times eV/h \times T$$

I =[e/h][energy]

Perfect Réservoirs = no resistance, black bodies



Classically: either T=1 or T=0, stochastic emission

Quantum mechanics: 0<T<1 partition noise

$$I = e \times n \times v_F = e \times \nu eV \times v_F$$

$$I = e \times \frac{1}{hv_F} eV \times v_F$$

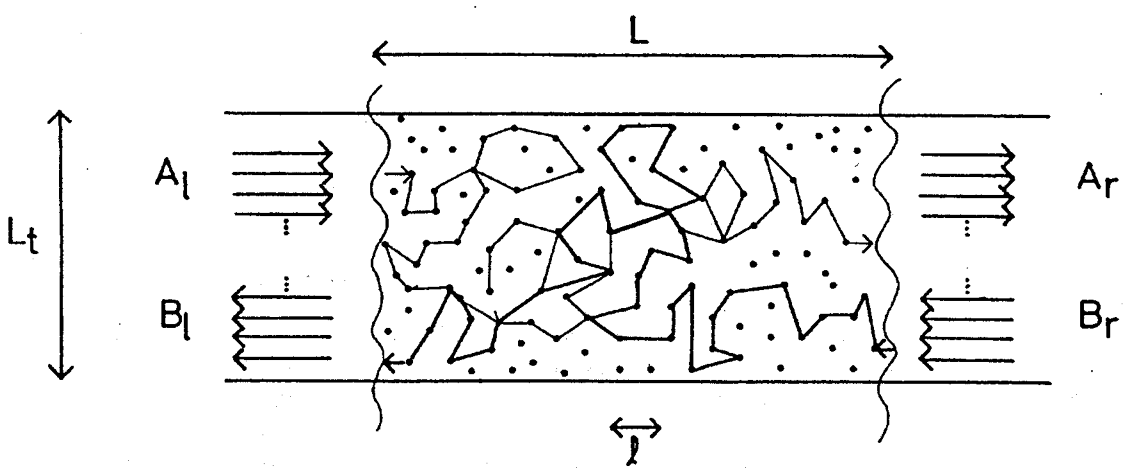
$$\nu_{1D} = \frac{g_V g_S m}{2\pi\hbar^2 k_F} = \frac{g_V g_S}{hv_F}$$

I does not depend on the Fermi velocity, but on the transmission < 1

$$I = e \times eV/h \times T$$

about 40nA/meV

Landauer Formulae (two-probes)



$$\begin{pmatrix} Ar \\ Br \end{pmatrix} = t \begin{pmatrix} Al \\ Bl \end{pmatrix}$$

matrix traces properties

$$G = 2\frac{e^2}{h} tr\left(tt^+\right) = 2\frac{e^2}{h}\sum_{i,j}^{N,N}\left|t_{ij}\right|^2 = 2\frac{e^2}{h}\sum_{i}^{N} T_i$$

$$I = e \times eV/h \times T$$

$$I = \frac{e}{\pi\hbar}\int T(E)\left[f\left(E - \left(E_F + eV_B\right), T\right) - f\left(E - E_F, T\right)\right] dE$$

T(E)=T'(E)

Summing all modes with identical energy

$$I=\frac{e}{\pi\hbar}\int T(E)\left[f\left(E-\left(E_F+eV_B\right),T\right)-f\left(E-E_F,T\right)\right]dE$$

$$G=\frac{e^2}{\pi\hbar}\frac{1}{k_BT}\int T(E)f(E)\left[1-f\left(E\right)\right]dE=\frac{e^2}{\pi\hbar}\int T(E)\left[-\frac{\partial f\left(E\right)}{\partial E}\right]dE$$

$$f(E)=\frac{1}{1+exp(\frac{E-E_F}{k_BT})}$$

$$-\frac{\delta f(E)}{\delta E}=\frac{1}{4k_BT}cosh^{-2}(\frac{E-E_F}{2k_BT})$$

Landauer formulae does not tell anything about [Ti]

addition rules (see next slide)
diffusive case:

$$\langle T \rangle = \frac{\ell}{L+\ell}$$

NB: small electric field

Demo: using addition rules:

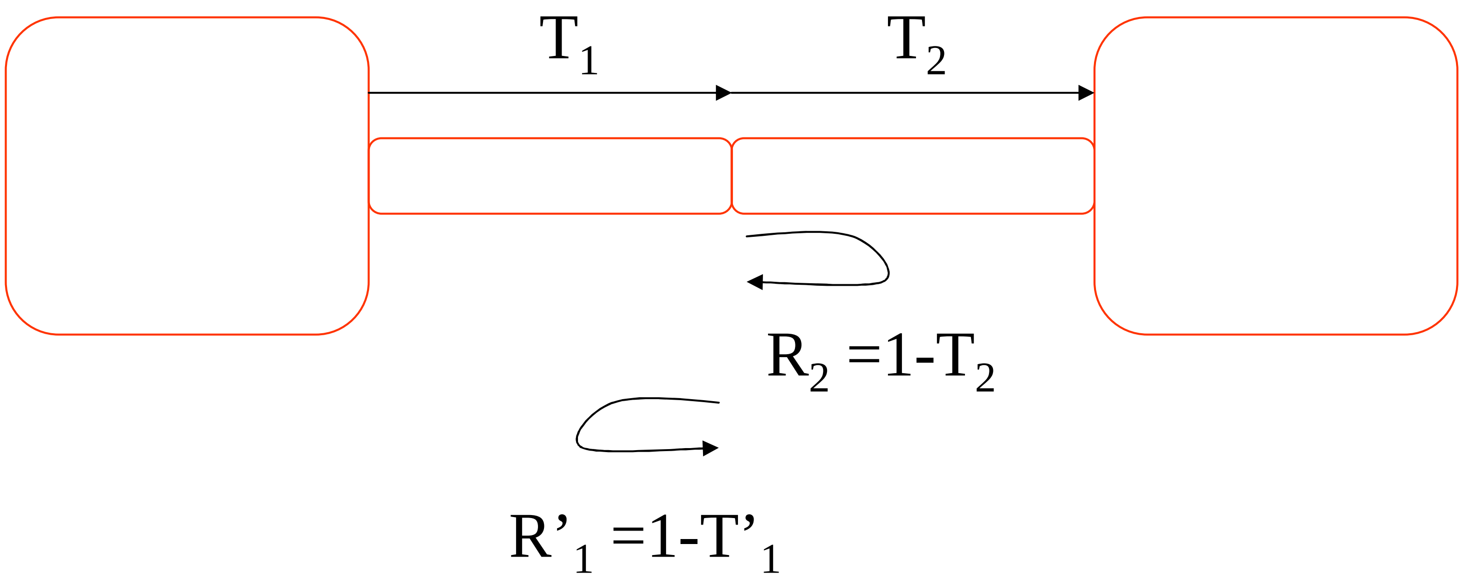
$$\frac{1-T_N}{T_N} = N\frac{1-T_1}{T_1}$$

$$T_N = \frac{T_1}{N(1-T_1)+T_1} \quad N = L \times \nu_{scatterers}$$

$$\ell = \frac{T_1}{\nu_{scatterers}(1-T_1)} \qquad 1 - T_1 \simeq 1$$

$$T_{12} = T_1 \times T_2$$

Is not finished!



$$T_{12} = T_1 T_2 \times (1 + R_2 R'_1 + R_2 R'_1 R_2 R'_1 + ...) = \frac{T_1 T_2}{1 - R'_1 R_2}$$

addition rules: $$\frac{1 - T_{12}}{T_{12}} = \frac{1 - T_1}{T_1} + \frac{1 - T_2}{T_2}$$

Landauer formulae is equivalent to Boltzmann, Einstein, and Thouless formula *for the diffusive case*:

$$\boxed{G=\frac{e^2}{h}N\frac{\ell}{L+\ell}\Leftrightarrow\sigma_{3D}=\frac{ne^2\tau}{m}\Leftrightarrow\sigma=e^2D\upsilon\Leftrightarrow G=\frac{e^2}{h}\frac{E_{Th}}{\Delta}}$$

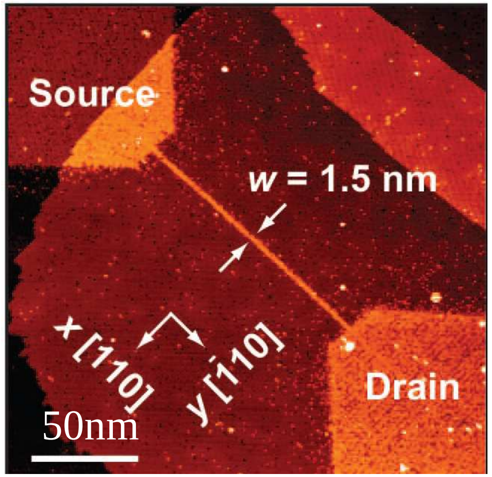
(3D case) $D=\frac{1}{3}v_F^2\tau$ degenerate case, if not : $|e|D/\mu=k_BT$

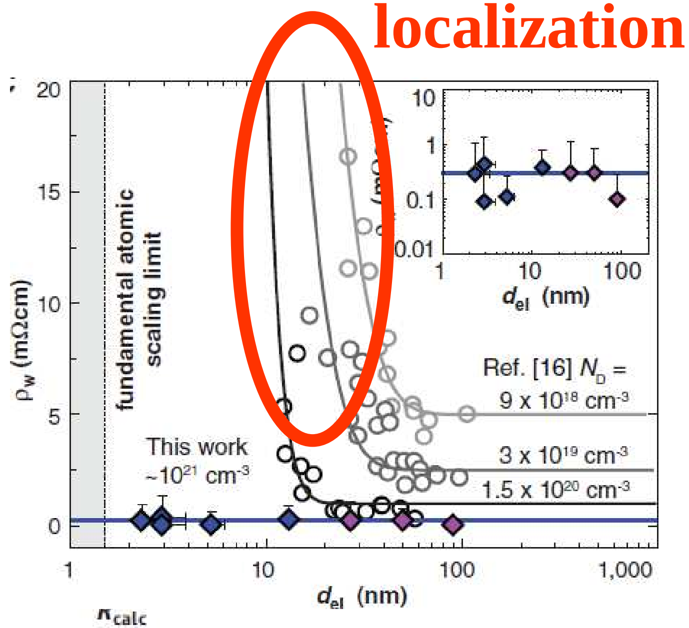
$$E_{Th}=\frac{2}{\pi}\frac{hD}{L^2}$$

Thouless energy , diffusion time across length L

Ohm's Law Survives to the Atomic Scale

B. Weber et al. Science 335 ,64, 2012





Sample	w (nm)	L (nm)	A_{el} (nm^2)	d_{el} (nm)	R_W (kΩ)	ρ_W (mΩ cm)	R_{calc} (kΩ)
W1	11.0	312	27.5	13.1	48.6	0.43	30 ± 4
W2	4.6	47	9.8	5.2	5.3	0.11	7 ± 1
W3	2.3	54	5.0	2.9	10.1	0.10	15 ± 2
W4	2.3	20	5.0	2.9	17.1	0.42	6 ± 1
W5	1.5	106	3.8	2.3	82.3	0.26	31 ± 4

T=4.2K

As long as the resistance is below h/e^2 the Boltzmann resistance is valid (if small disorder) : $G= e^2/h \times N \times \ell/L$

Comparison diffusive/tunnel:

$$G = 2\frac{e^2}{h} tr\left(tt^+\right) = 2\frac{e^2}{h}\sum_{i,j}^{N,N}\left|t_{ij}\right|^2 = 2\frac{e^2}{h}\sum_{i}^{N} T_i$$

$$< T_i > = \ell/L$$

$$\ell \ll L$$

but N=S/(λ_F/2)^2 channels, some of them lℓ/Ly transmissive (T=1) and some of them poorly transmissive (T=0)

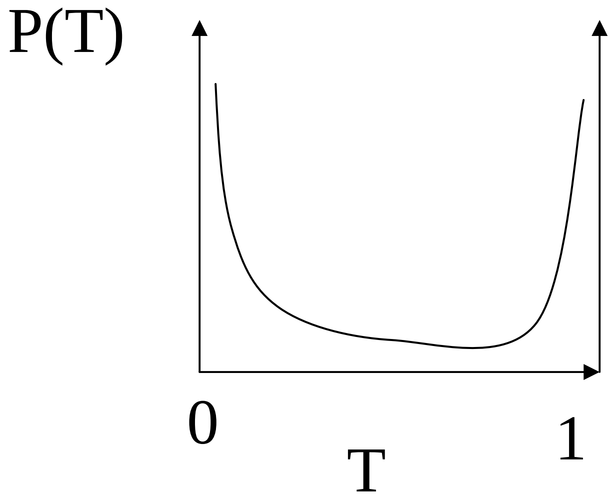
Tunnel case: G_T=e^2/h S ν^2t^2

with <Ti> =t^2 et N=S/(λ_F/2)^2 $\propto$ S.ν^2 identical channels

(Dorokhov bimodal distribution of Ti)

$$L >> \ell$$

$$P(T) = \frac{\ell}{2L}\frac{1}{T\sqrt{1-T}}, T\min = 4\exp(-\frac{2L}{\ell}) < T < 1$$

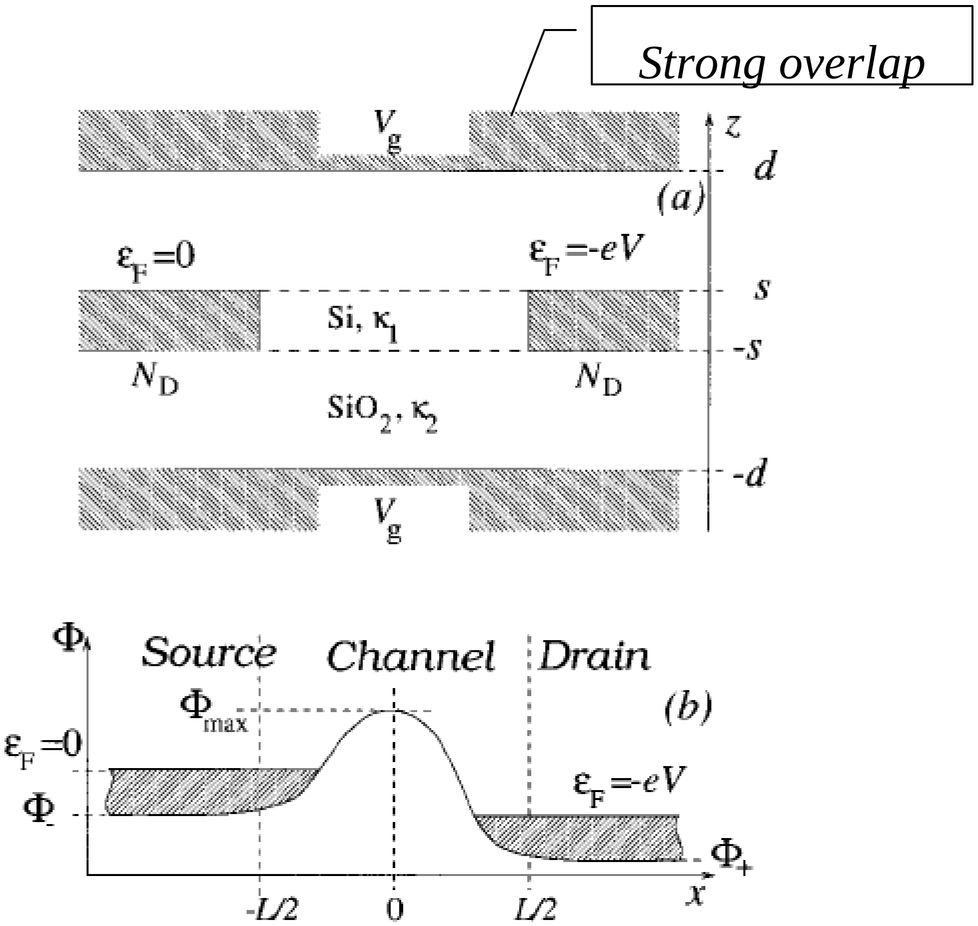


Fano factor(shot noise reduction):

$$F = \frac{\langle T(1-T)\rangle}{\langle T\rangle} = \frac{1}{3}$$

Physical limits for an ideal MOSFET

T=300K



Pikus, Likharev APL71 3661 ('97)

Hypothesis: perfect reservoirs

T=300K

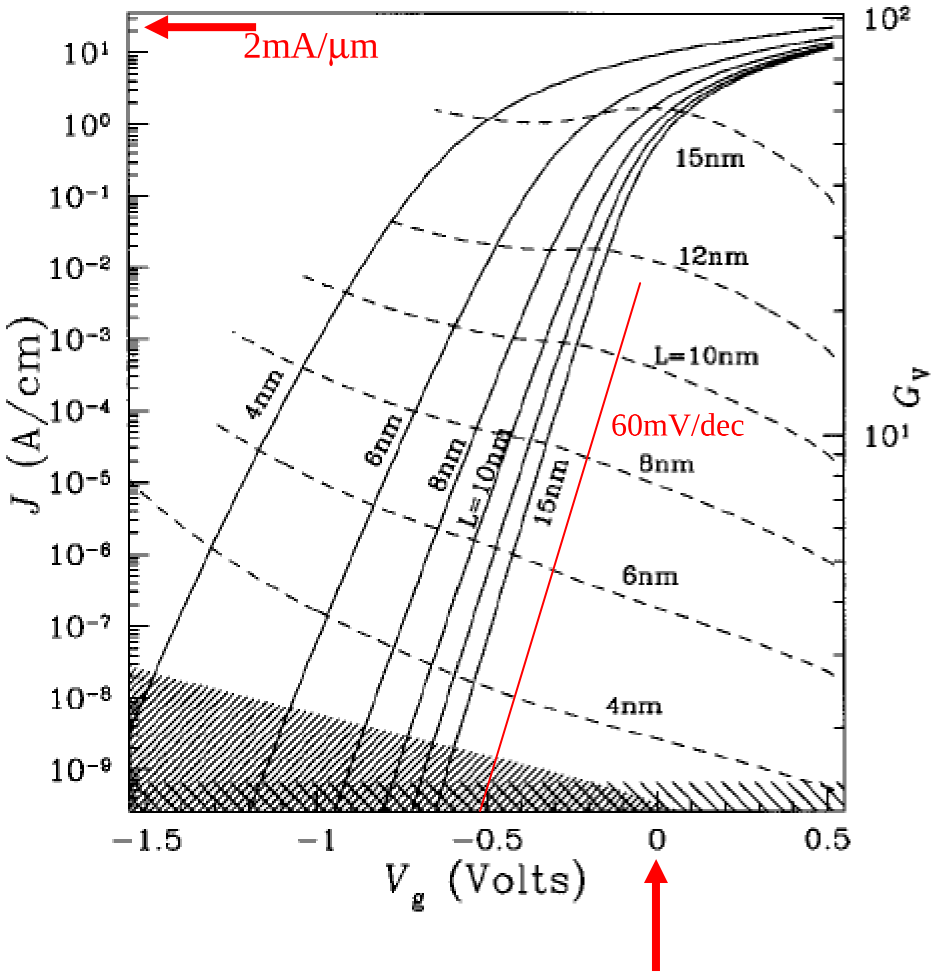


FIG. 4. Linear current density j (solid lines) and voltage gain $G_V = dV/dV_g|_{I=\text{const}}$ (dashed lines) as functions of gate voltage V_g for various channel lengths L and source-drain voltage near the onset of saturation ($V = 0.52$ V). The fine hatching shows the area of parameters where the gate leakage current exceeds the drain current. The coarse hatching shows the region where the intrinsic carriers in the channel cannot be ignored.

Pikus, Likharev

APL71 3661 ('97)

Gate oxide 2nm, channel thickness 1.5nm

screening length 2.5nm

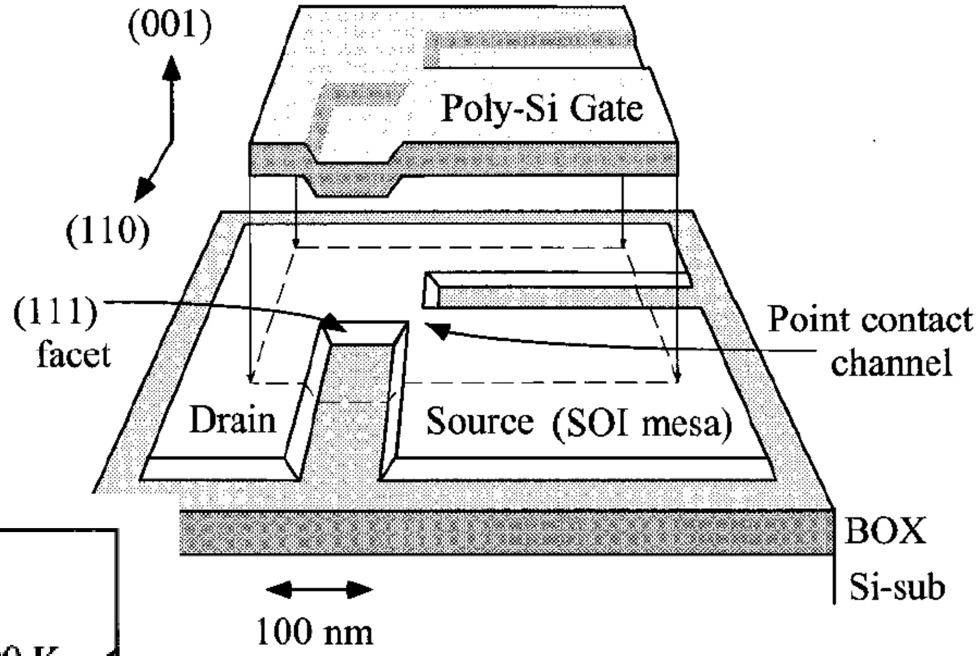
S/D doping:

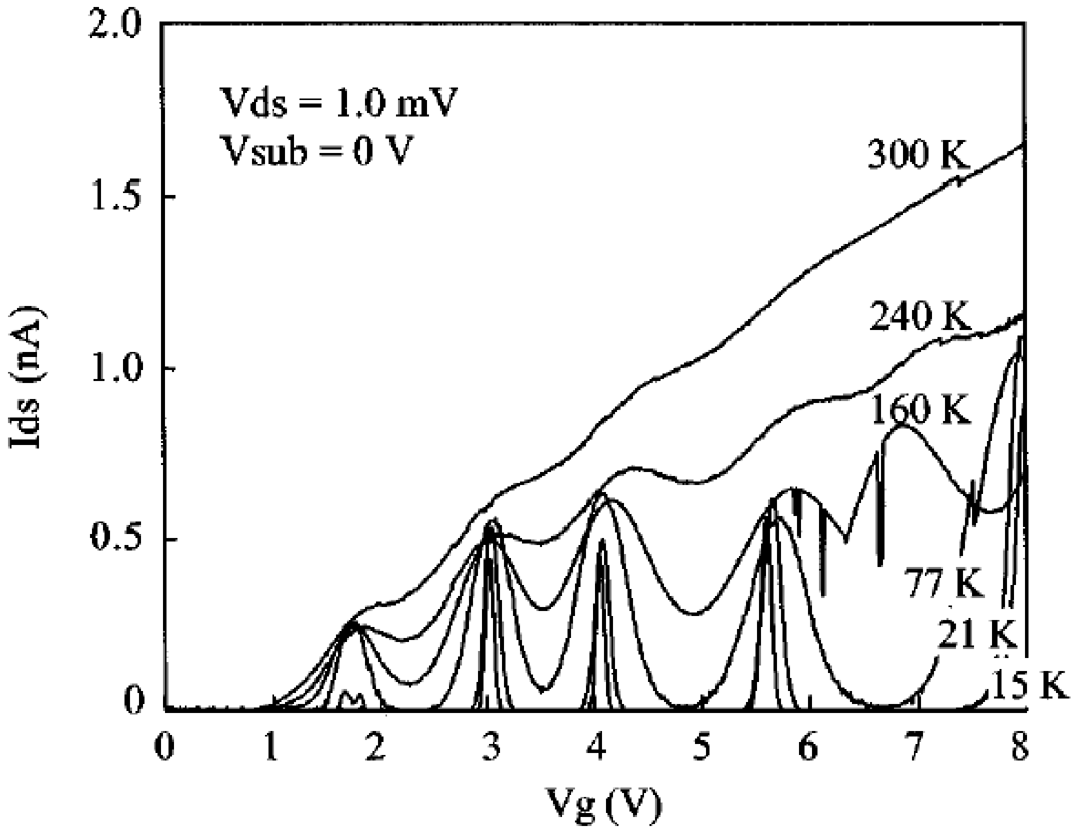
3 10^{20}cm^{-3}

Room temperature SETs

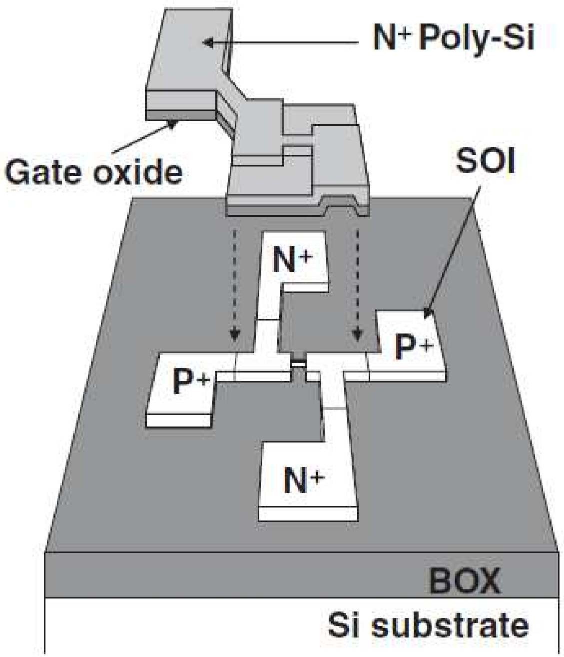
Coulomb Blockade in a SOI constriction

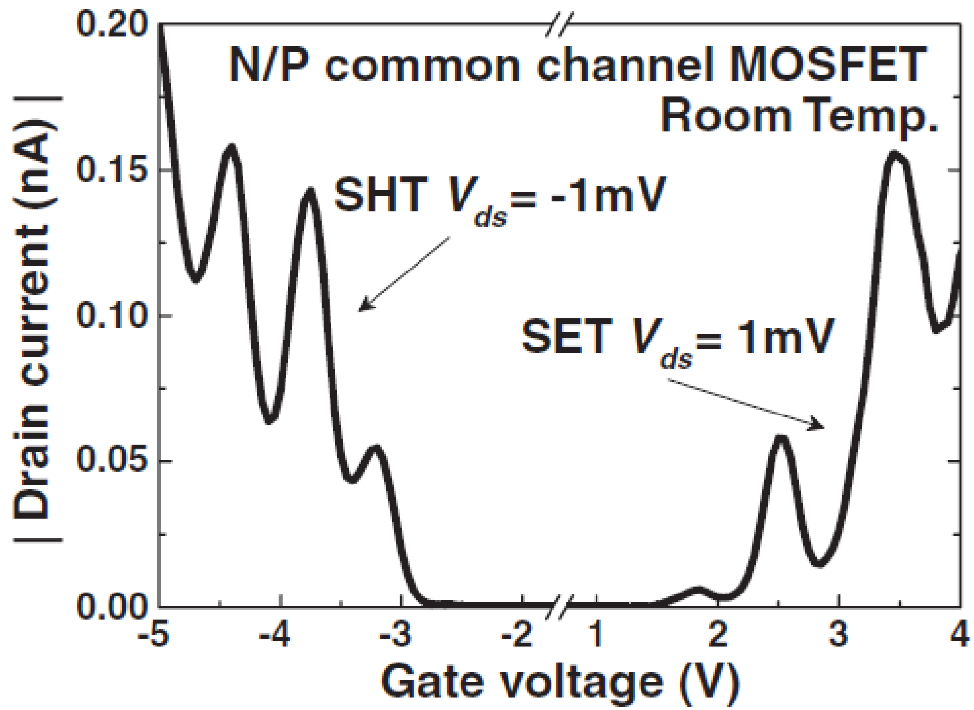
H. Ishikuro and T. Hiramoto
APL 71, 3691 (1997)





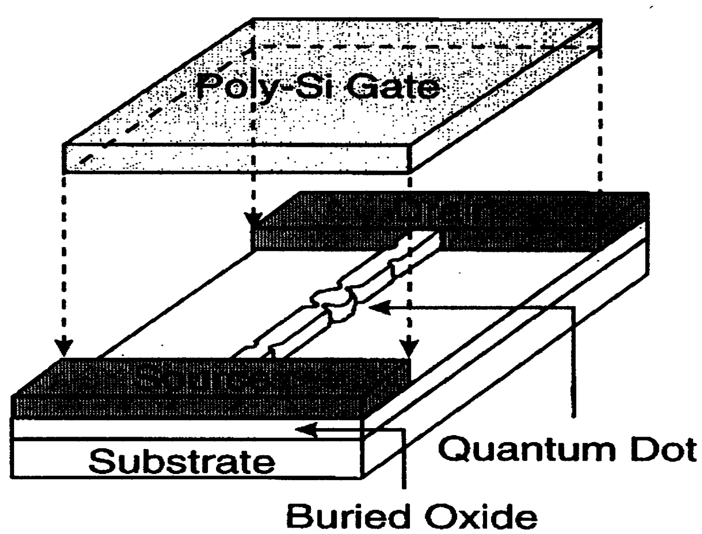
M. Kobayashi et al, Jpn. J. Appl. Phys., Vol. 47, 1813(2008)





Ex: CB in silicon devices at room temperature

•Hiramoto, Ishikuro (Tokyo) point contact MOSFETs
(Ishikuro et al. APL71, 3691 (1997).)
•Chou, Tsui (Princeton) MOSFET quantum dot
(L. Guo et al. APL70, 850 (1997).)
•Sakamoto, Baba (NEC) doped Silicon (SOI)
(Sakamoto et al. APL72, 795 (1998).)
•Peters, Dijkhuis (Utrecht) SOI MOSFET
(Peters et al. J of Appl. Phys. 84, 5052 (1998).)
•Kotthaus, Wharam (Munchen, Tubingen) SOI wires
(Tilke et al. APL75, 3704 (1999).)
•....



Δ varies like L^{-d} , E_c varies like L^{-1}

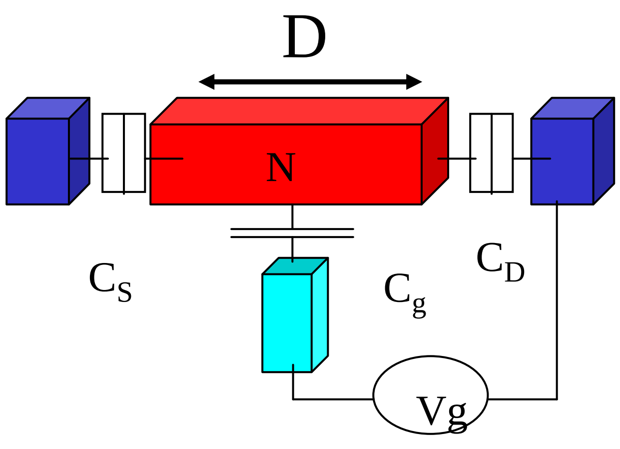
system	estimated size	Charging energy	mean level spacing
doped SiDot (baba)	20nm	1.4meV (56 aF)	0.5 meV
SOI FET wire (Tilke)	70nm	15meV	-----
SOI FET wire (Rokhinson)	10-40nm	13meV (12.3aF)	-----
SOI FET wire (Zhuang APL72, 98)	12nm	96meV	17meV
SOI point contact (Ishikuro)	6nm	58 meV (1.4aF)	30meV
Niobium island (Shirakashi APL72,98)	<10-20nm	1000 meV(0.16aF)	negligible (orthodox theory)
Aluminum island (Nakamura APL76,2000)	2-4nm	45-115meV	25-100meV
SOI FET wire (Augke)	20nm	4.9meV (32aF)	1.4meV

Large
(e^2/C) means small size

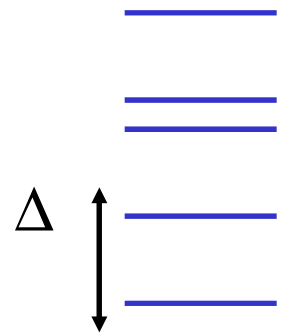
small size + semiconductors means large Δ

SET at room temperature means size of few nm.

Room temperature SET: D ≈1-2nm



kT<<Ec



•Size of the box >> λ_F : many electrons in the box

$Ec=e^2/2(C_g+C_S+C_D) >> \Delta$ mean level spacing due to confinement

•Decrease of the size:

$$Ec \text{ increases as } D^{-1}, \Delta \text{ increases as } D^{-2}$$

•Small size: large Ec ≈ Δ, Coulomb blockade + resonant tunneling:

$$D \approx \lambda_F$$

How robust are charging and quantum effects at room temperature ?

$e^2/2C$ (charge effect) and Δ (resonant tunneling) (much) larger than k_BT=25meV at T=300K

Also larger than eVd : small Vd, small current (eventually high current density)

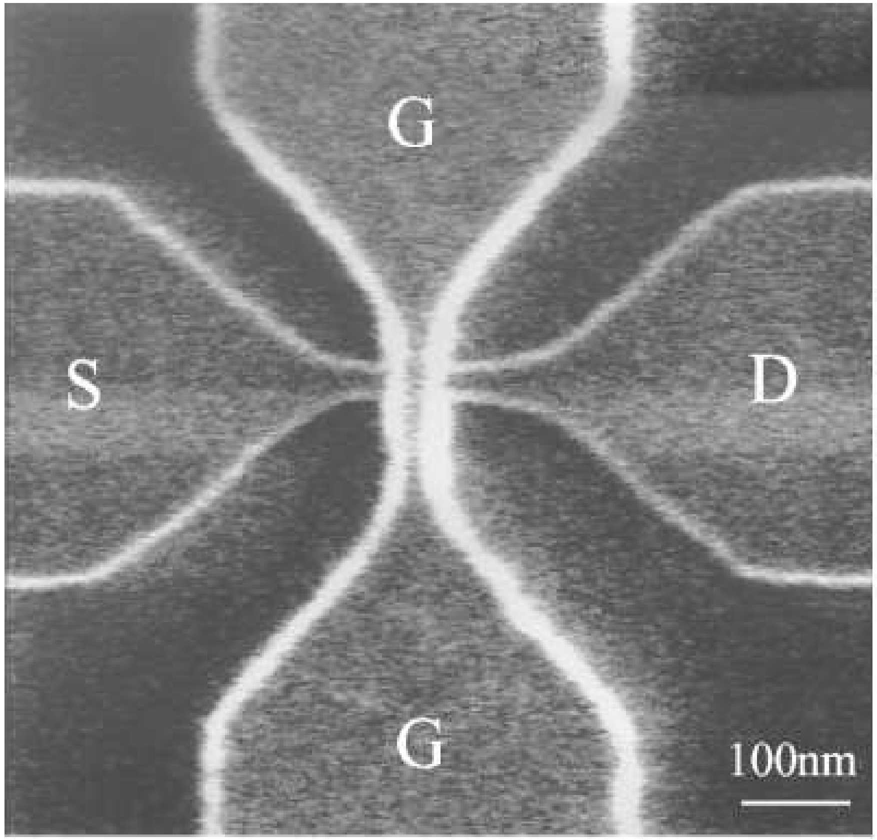
Should be verified for 2-5 nm size

ultra short MOSFET : Ballisticity

Pikus, Likharev APL71 3661 ('97)

VersuS

ultra short MOS-SET :
Coulomb Blockade due to
access resistance

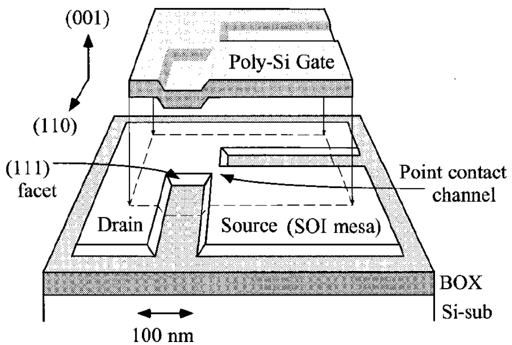


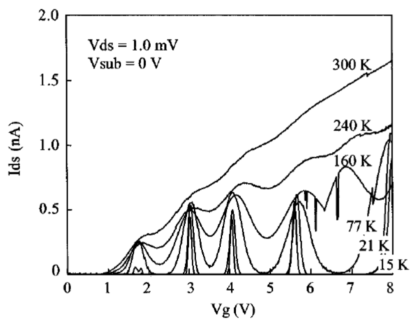
Lin(T,E): 10nm at 300K

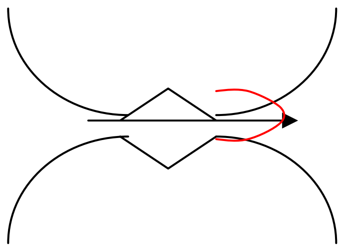
Lin comparable to ℓ at 300K

Lin :few 100 nm at low temperature Lin>> ℓ

sufficient confinement resistance at 300K means small cross-section of the wire (comparable to the Fermi wave length)



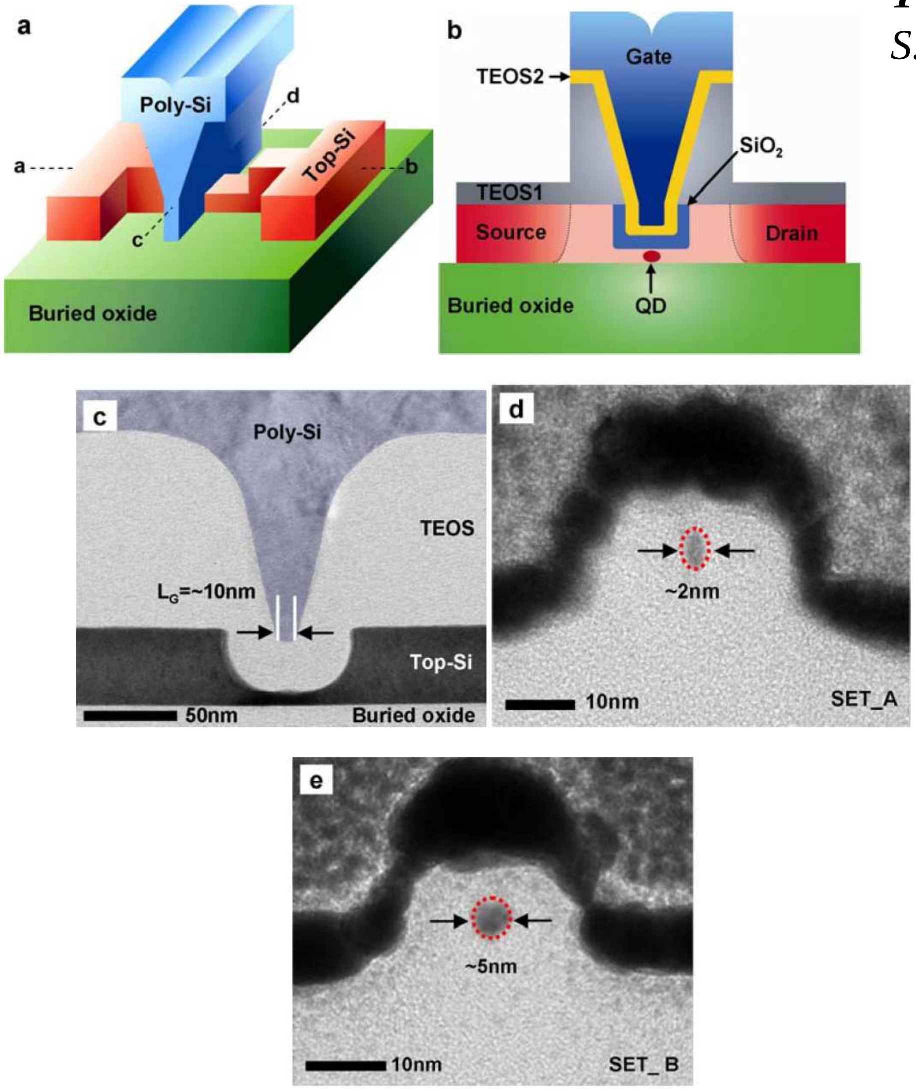


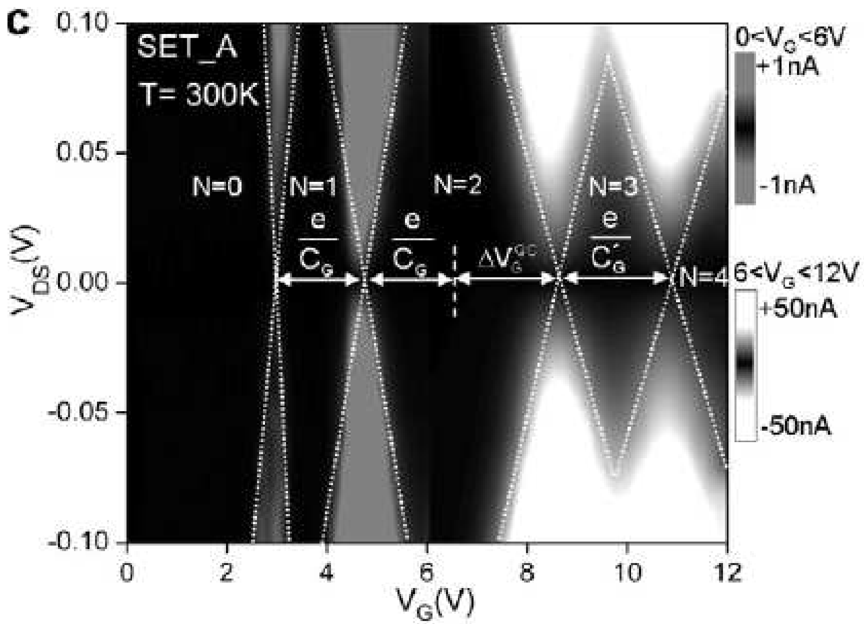


A room temperature MOS-SET

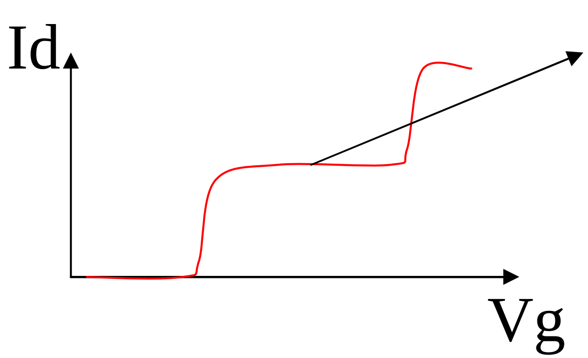
Enhanced Quantum Effects in an Ultra-Small Coulomb Blockaded Device Operating at Room-Temperature

S. J. Shin, et al. APL **97**, 103101 (*2010)*

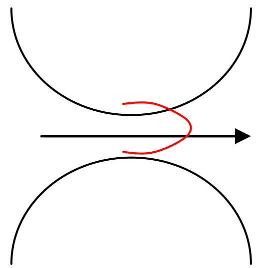


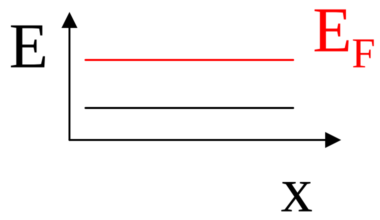


Ultimate Ballistic FET= Quantum point contact



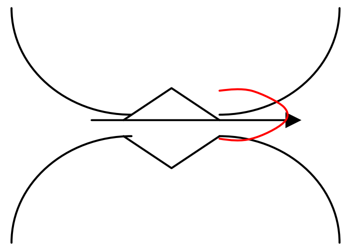
$G = e^2/h * 1 * 1 = e^2/h$

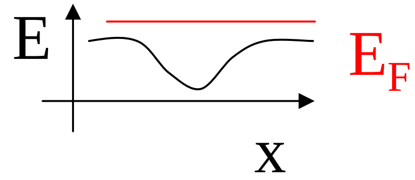


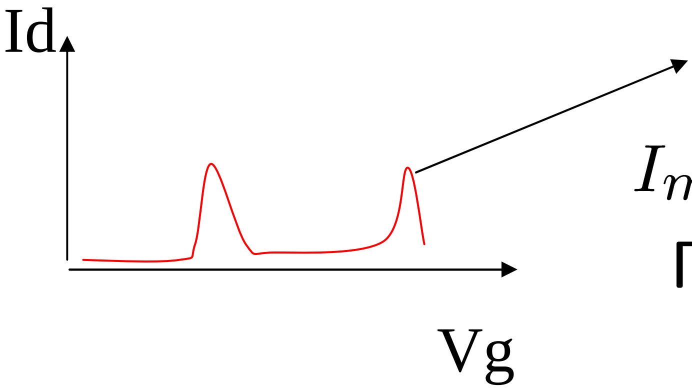


$$I_{bal} = \frac{e^2}{h} V$$

Ultimate SET = Resonant tunneling through a few-electrons state





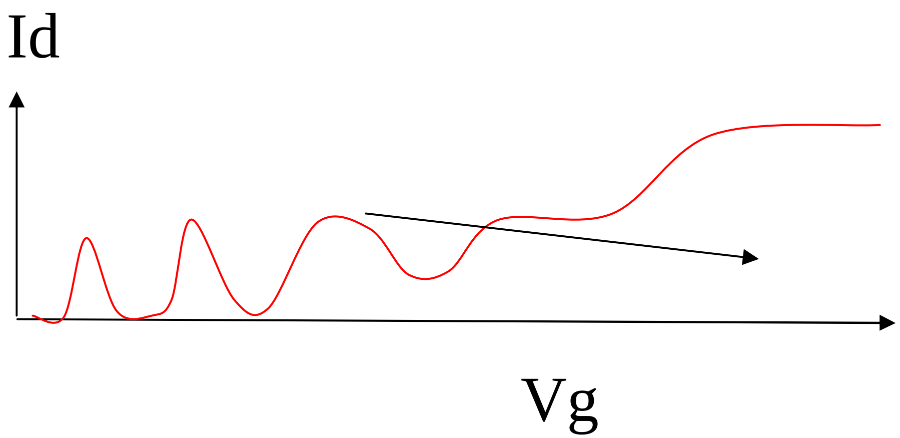


$G=e^2/h$

$$I_{max} = \frac{e^2}{h}\Gamma$$

$$\Gamma \leq eV \leq \Delta_1$$

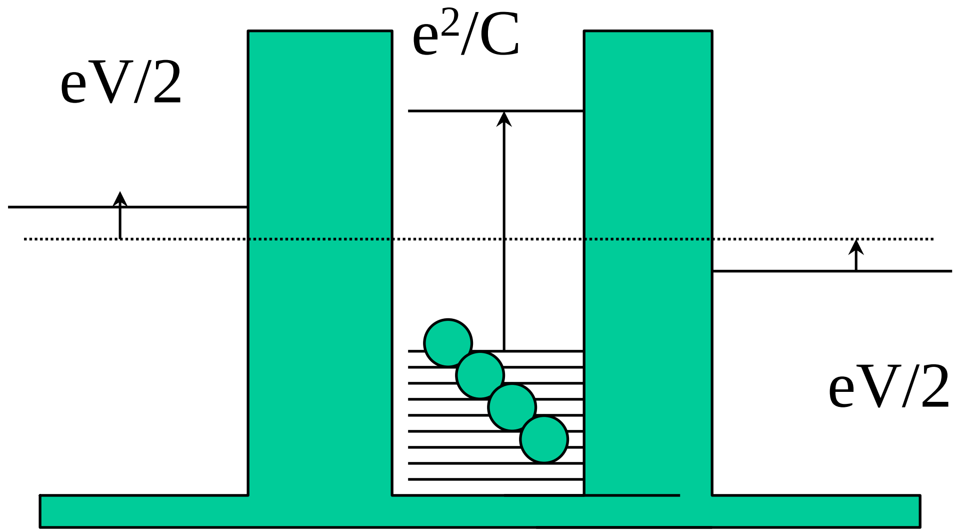
$$\Gamma \geq 10^{13} Hz$$ for T=300K

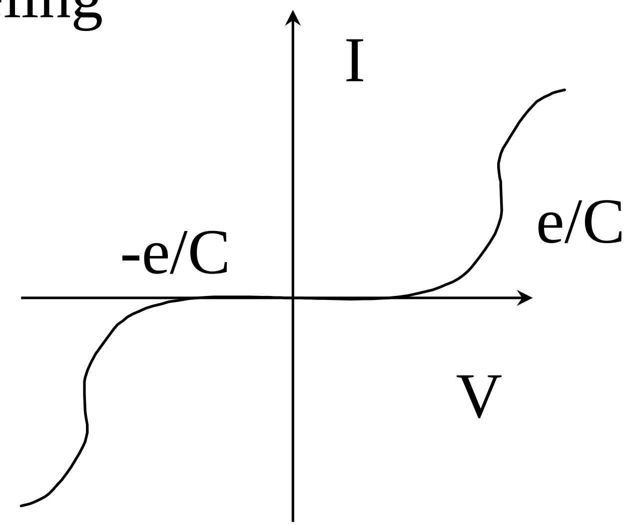


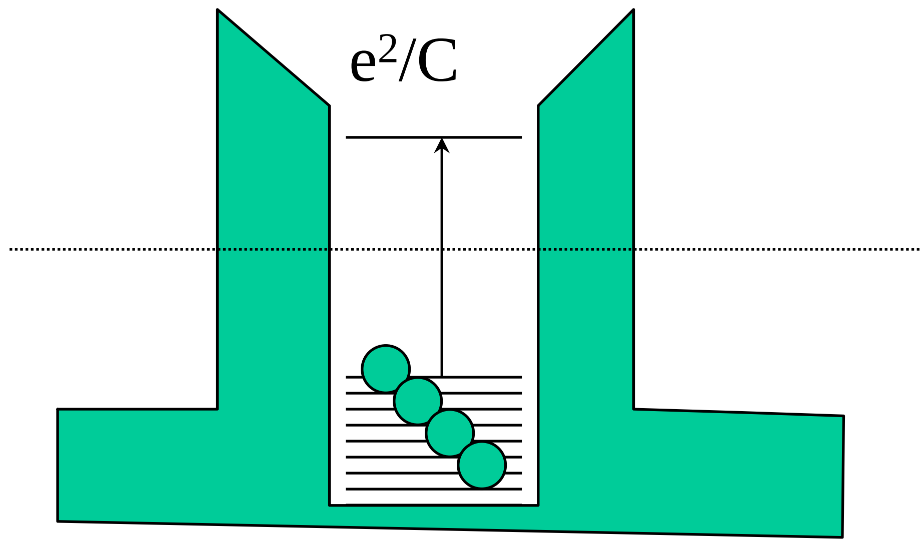
$$\Gamma \simeq \Delta_1$$

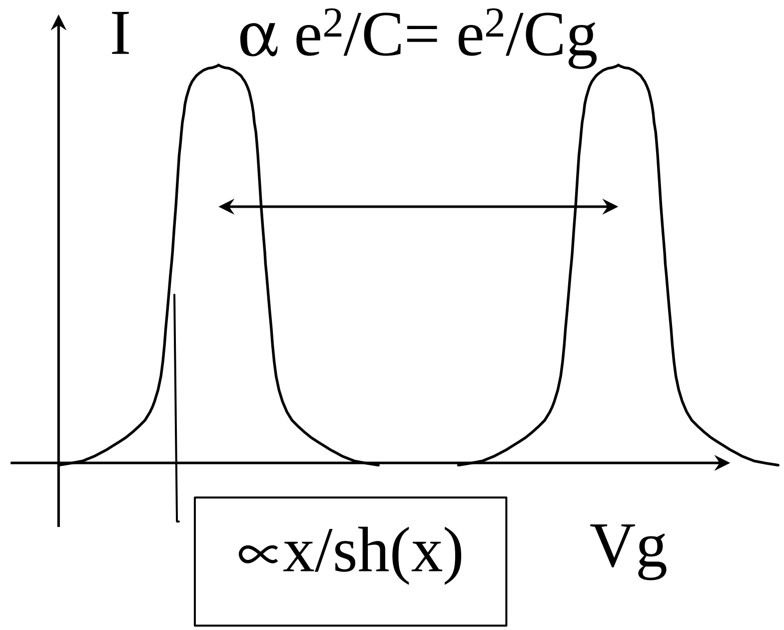
« Kondo ridge »=spin dependent cotunneling
(Goldhaber-Gordon et al. PRL81 5225 ‘98)

charging effect without resonant tunneling

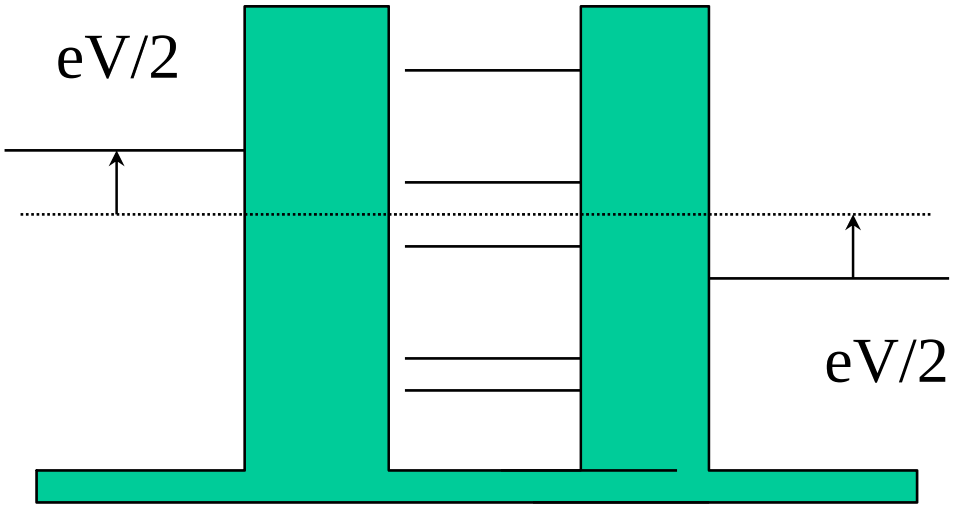


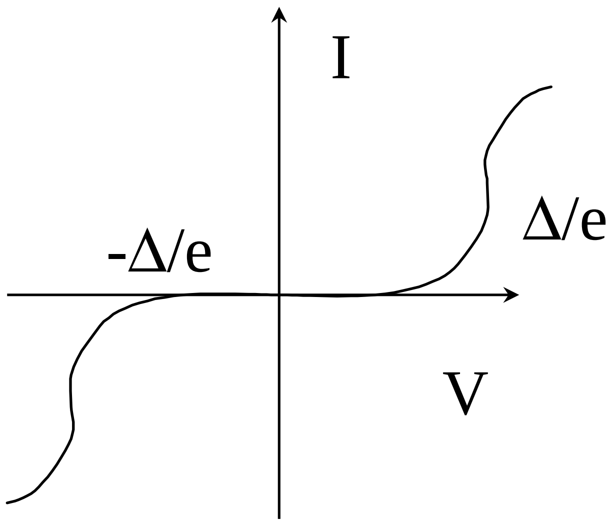


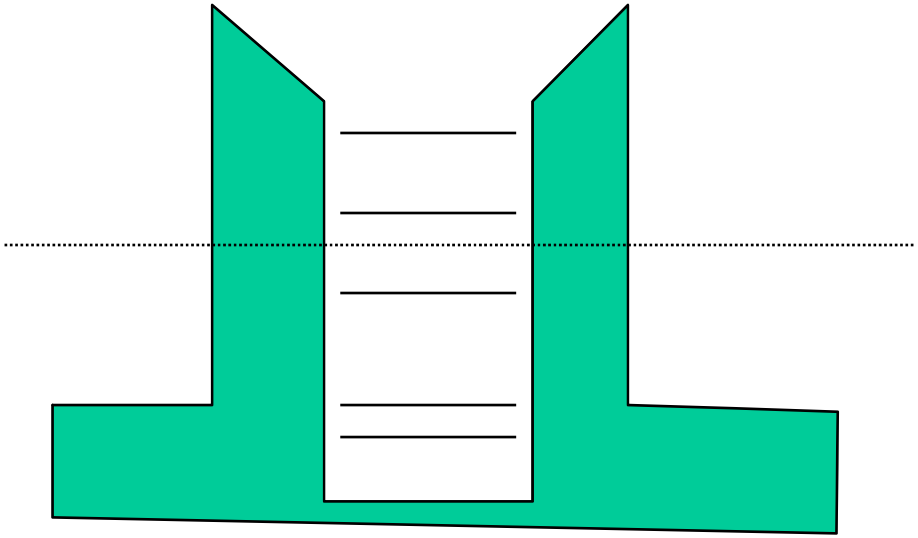


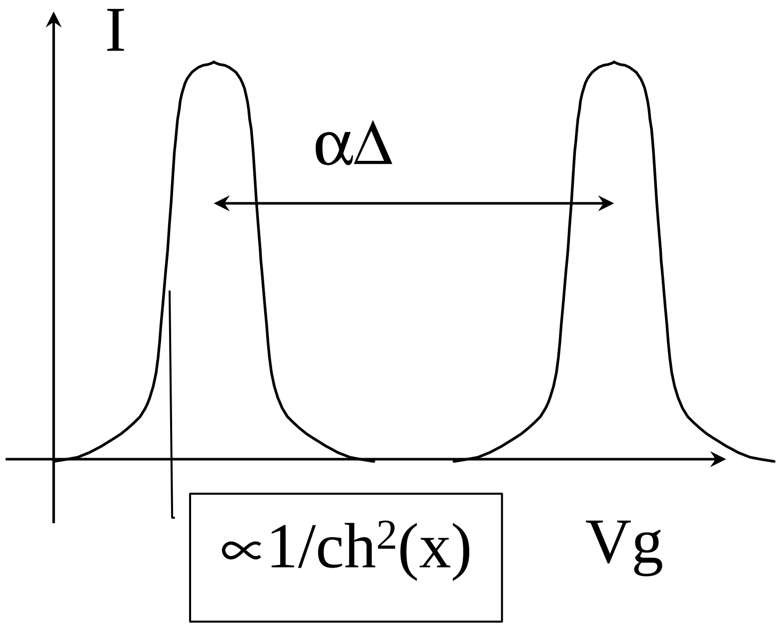


resonant tunneling without charging effect







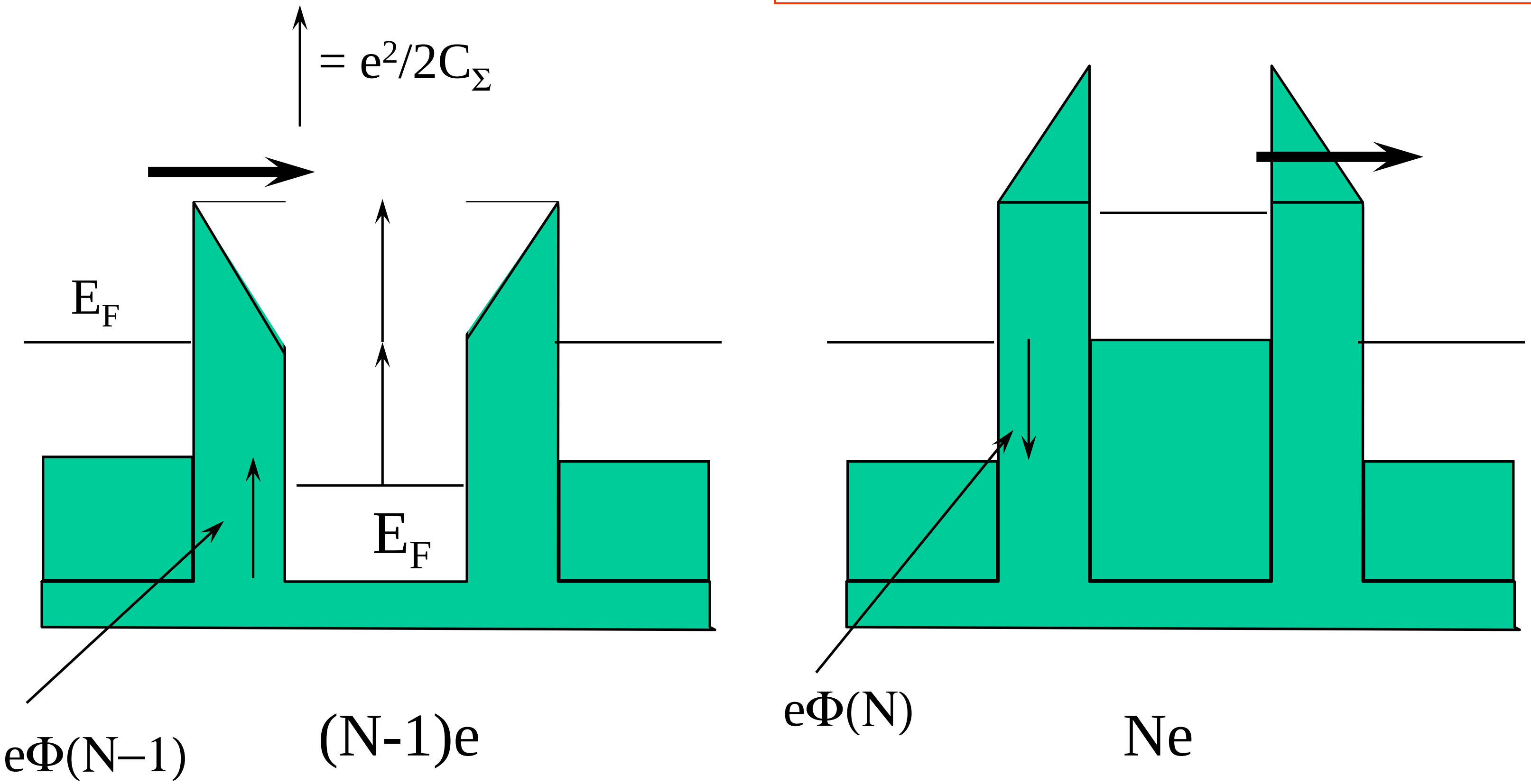


complex Coulomb blockade when :

ι) Δ is non negligible (« quantum capacitance »)

ii) when the size becomes comparable to the mean distance between carriers.

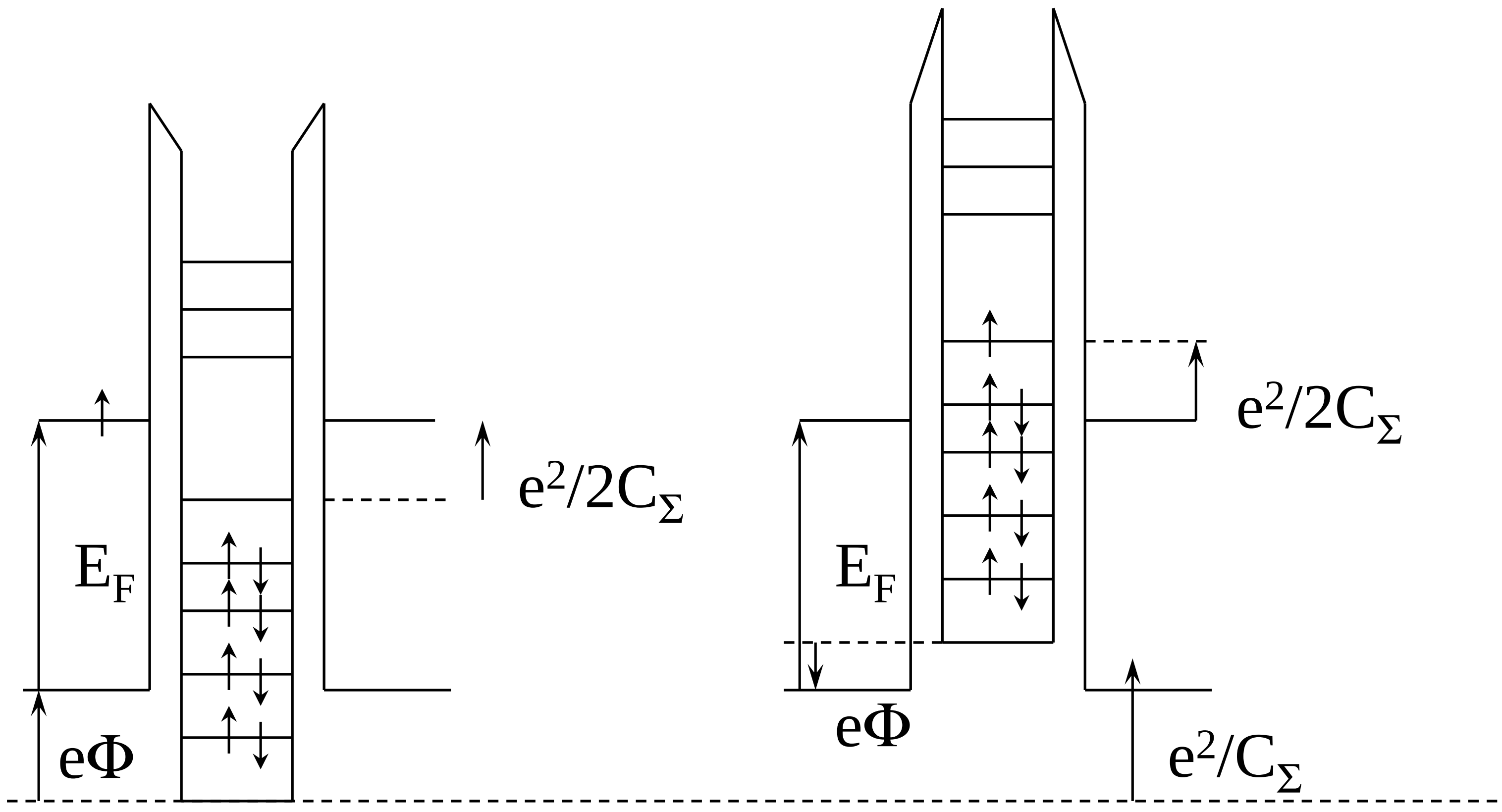
$$\boxed{\frac{e^2}{2C_\Sigma} + E_F = E_F + e\phi\left(N-1\right)} \quad \boxed{-\frac{e^2}{2C_\Sigma} + E_F = E_F + e\phi\left(N\right)}$$



$$\Phi(N) = -eN/C_{\sum} + \Phi_{ext}$$

$$\Phi_{ext} = \frac{C_G V_G}{C_{\sum}}$$

(e>0)



$$\boxed{\frac{e^2}{2C_\Sigma} + K_N = E_F + e\phi\left(N-1\right)}$$

$$\boxed{-\frac{e^2}{2C_\Sigma} + K_N = E_F + e\phi\left(N\right)}$$

$$\Phi(N) = -eN/C_{\sum} + \Phi_{ext} \qquad \text{e>0}$$

orthodox model:

$$E(N) - E(N-1) = 0 \qquad \left(N - \frac{1}{2}\right)\frac{e^2}{C_\Sigma} = e\phi_{ext}$$

with an E_N spectrum

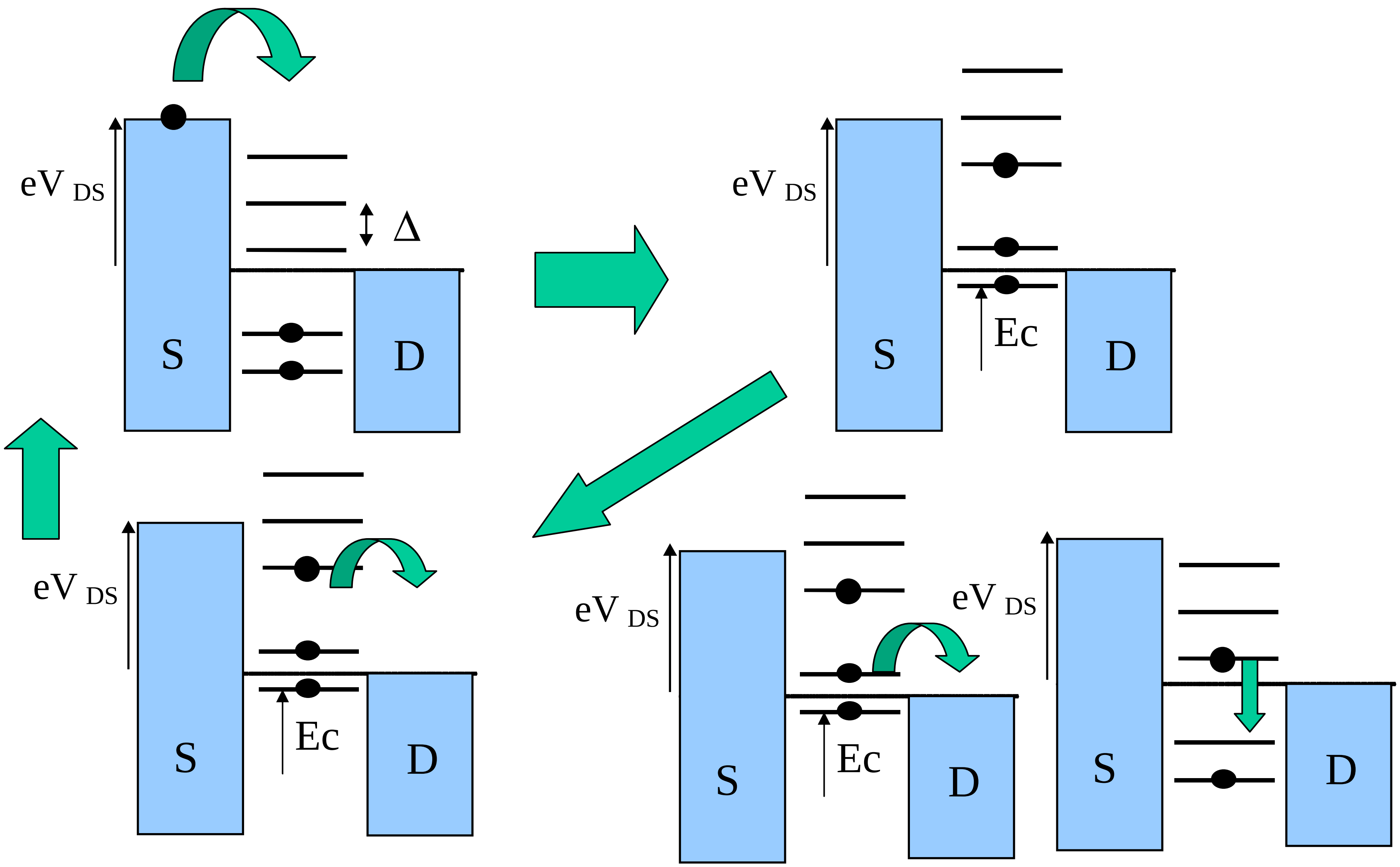
$$E(N) - E(N-1) = E_F - K_N$$

$$\left(N - \frac{1}{2}\right)\frac{e^2}{C_\Sigma} + K_N = E_F + e\phi_{ext}$$

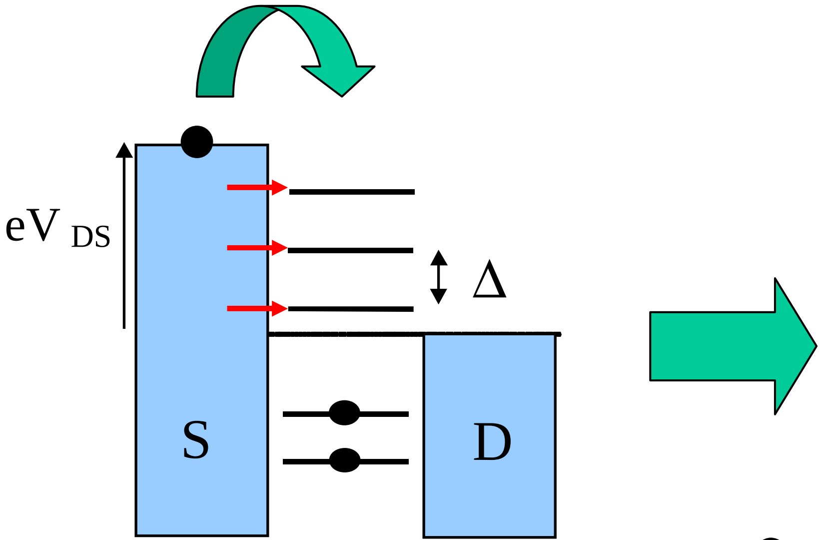
Hypothesis: E_F does not vary with ϕ_{ext}

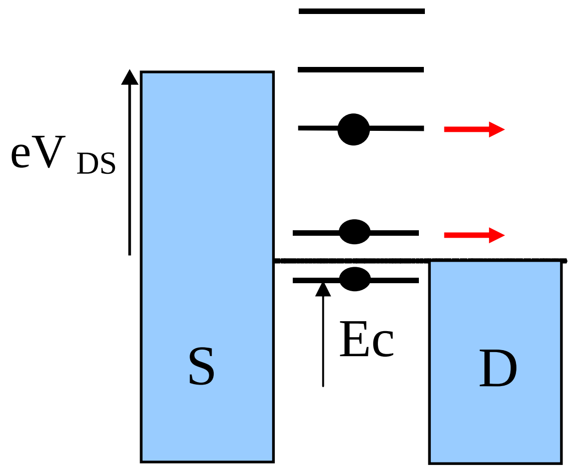
$$\frac{e^2}{C_\Sigma} + \Delta K_N = e\Delta\phi_{ext} = e\frac{C_G}{C_\Sigma}\Delta V_G$$

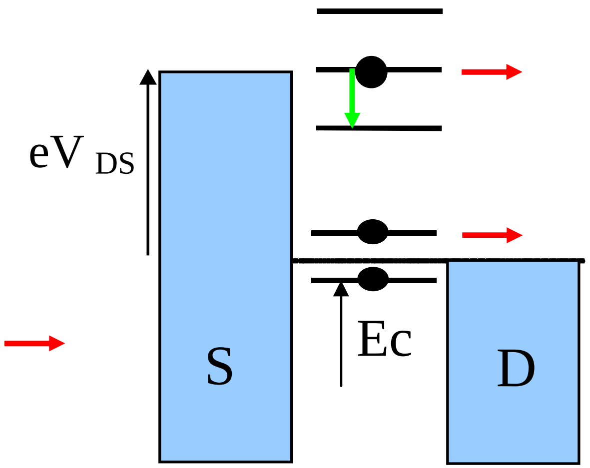
Some effects of finite Δ (at finite bias)

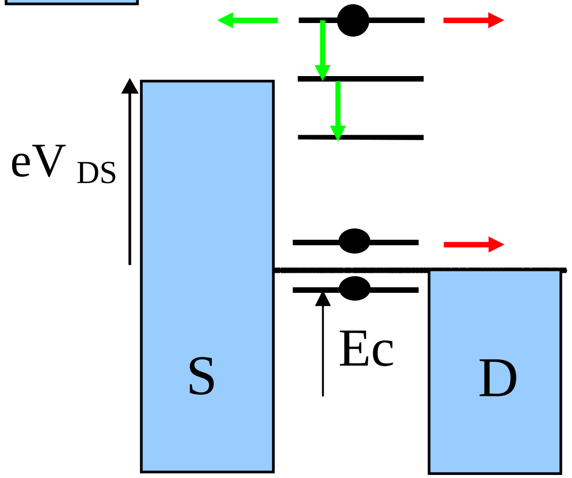


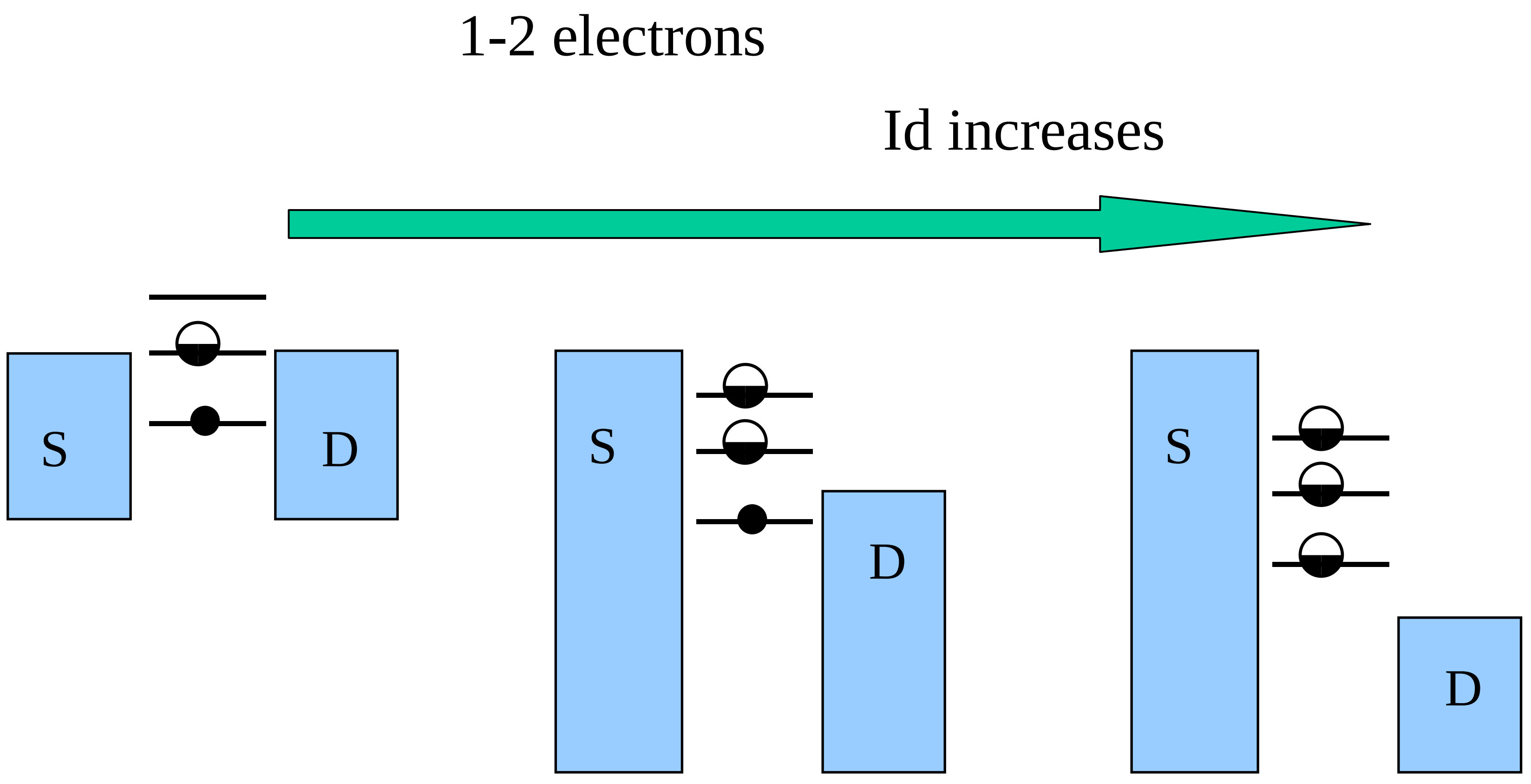
Count the available states:
example : 3 IN- 2 OUT

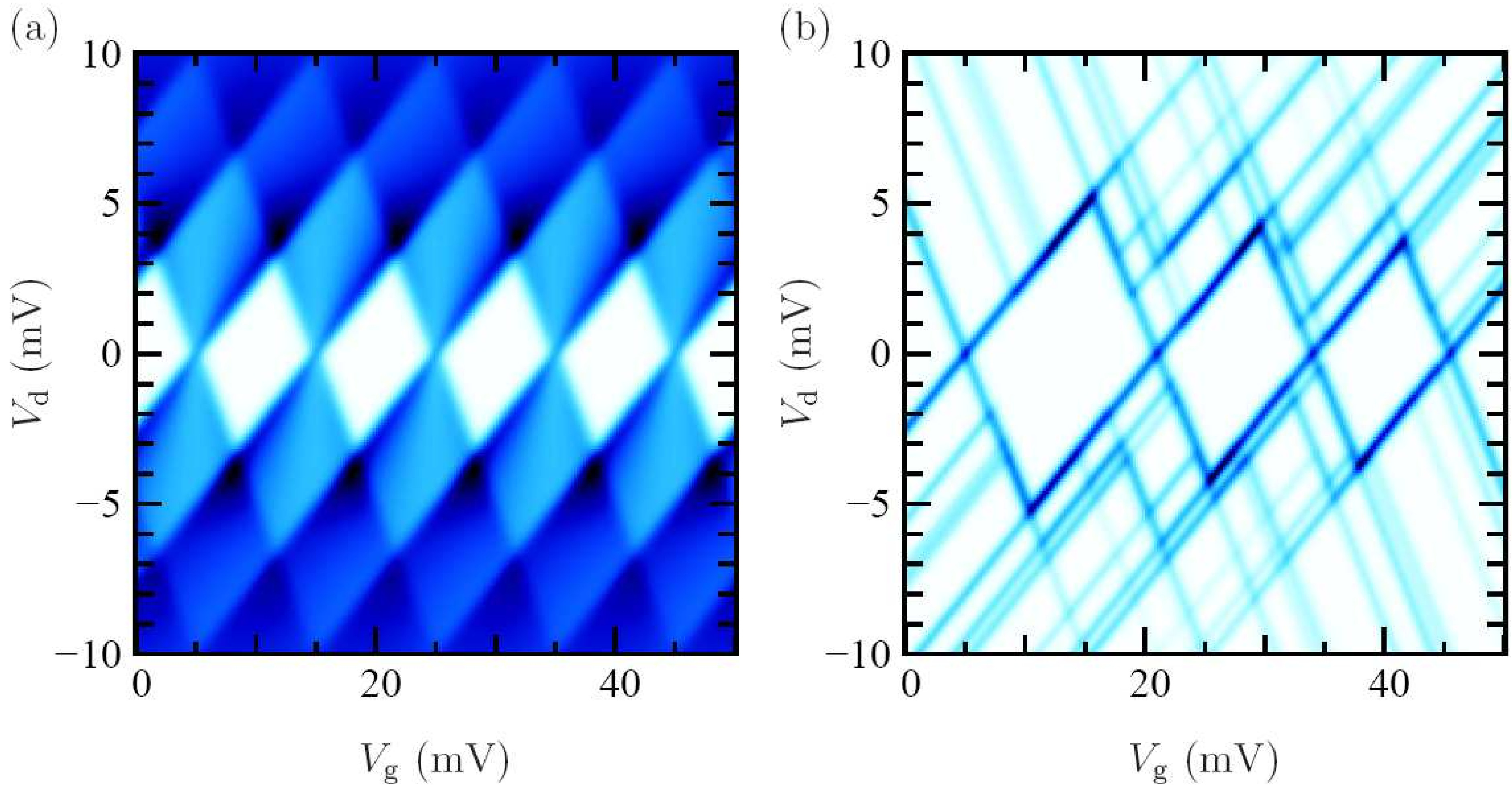




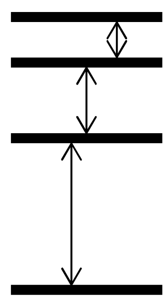





1-2 electrons
Id increases
S
D
S
D
S
D



calculated spectrum : T=1K, Cs=Cd=Cg=100 e/V for both cases



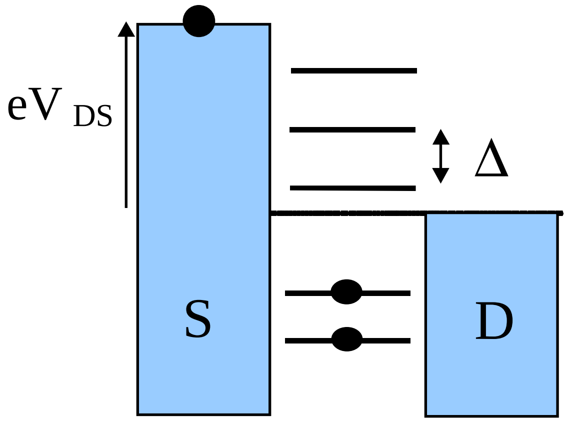
left =continuum

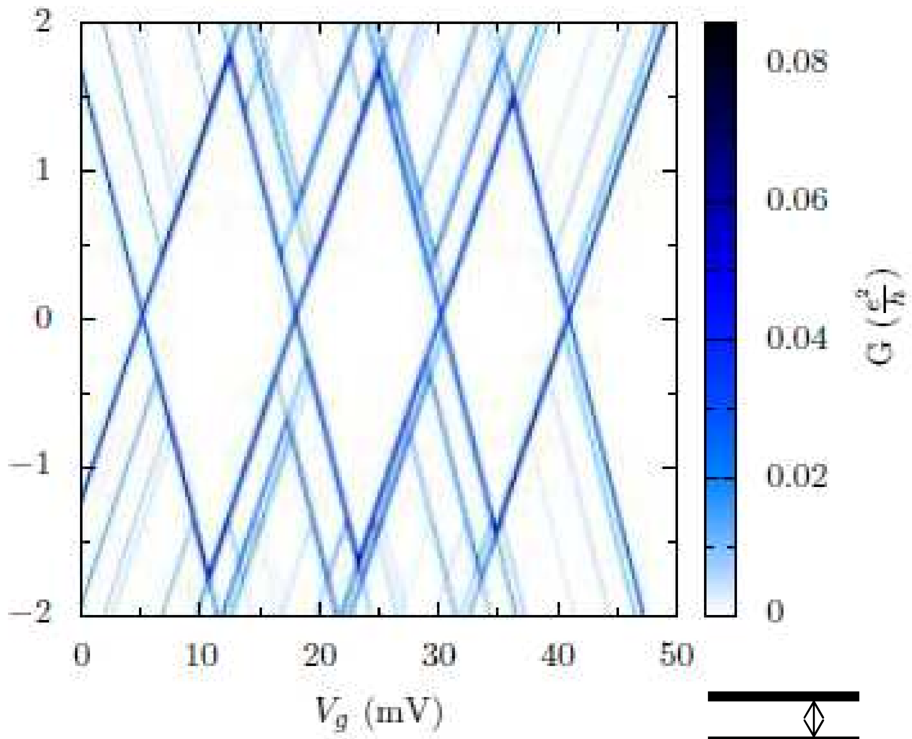
Δ=2, 1, 0.5 meV

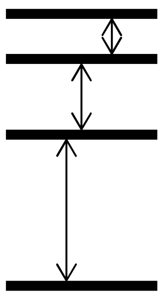
PhD Max Hofheinz 2006

*M. Pierre et al. Eur. Phys. J. B **70**, 475–481 (2009)*

Excitation spectrum of the dot

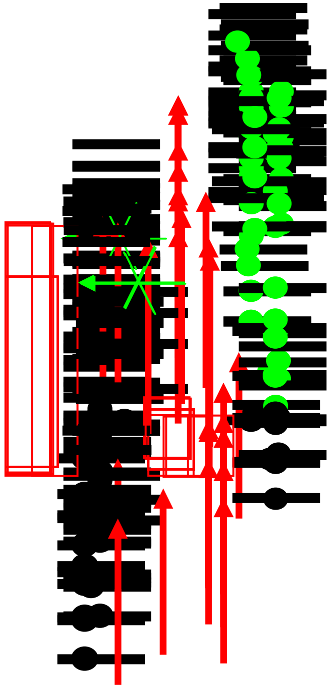


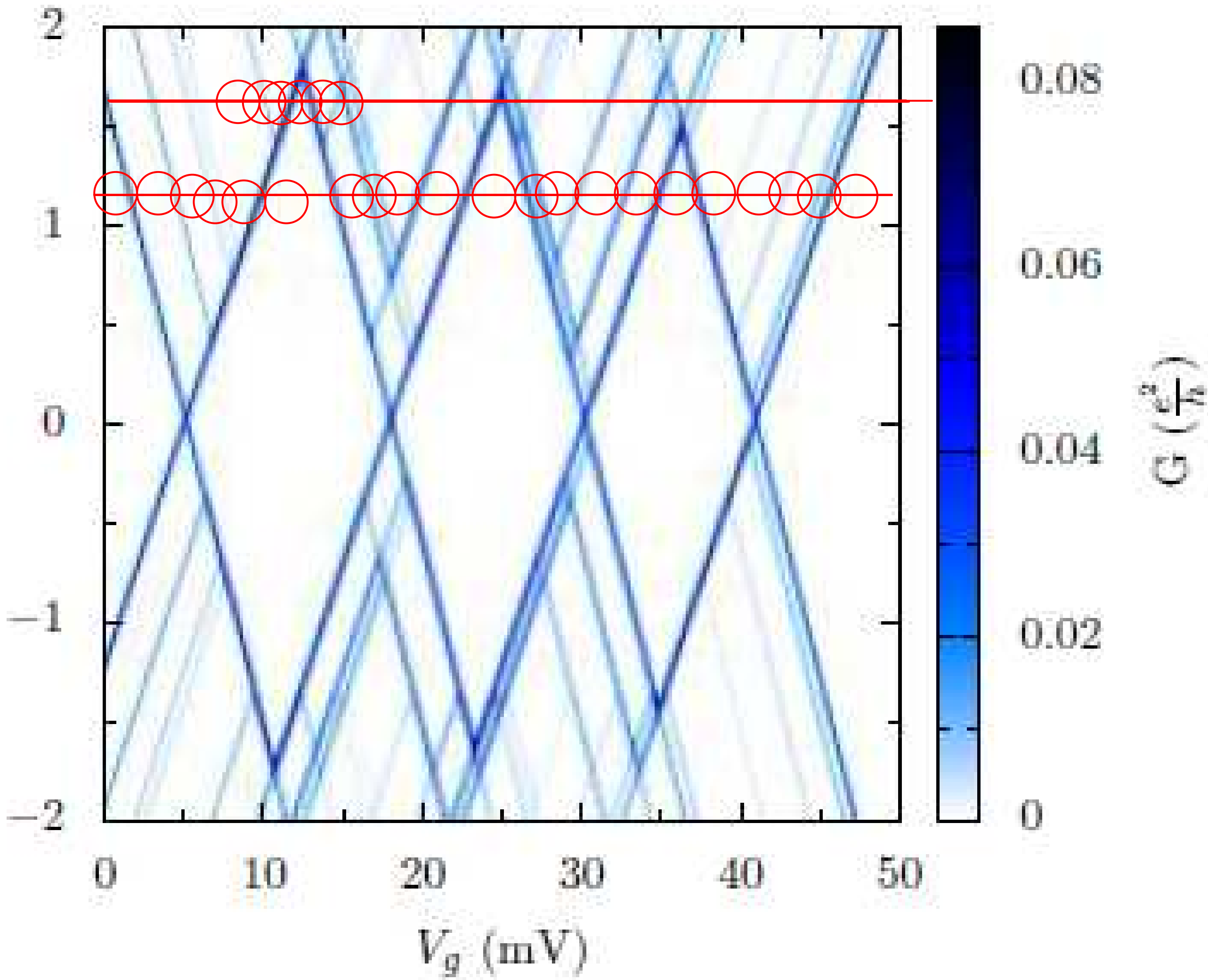




Ec increases as D^{-1}, Δ increases as D^{-2}

Δ= 0.4,0.3,0.1meV
Ec=1.38meV
T=0.1 K





2
1
0
−1
−2
0
10
20
30
40
50
V_g (mV)
0.08
0.06
0.04
0.02
0
$G\ (\frac{e^2}{h})$

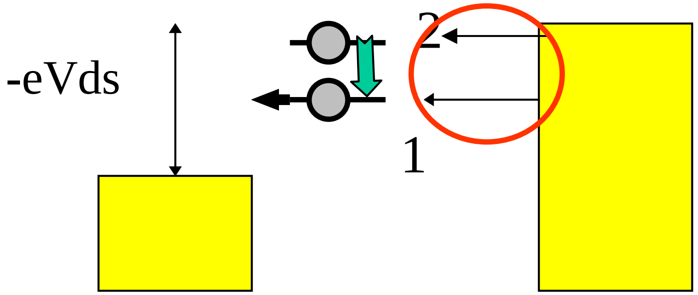
Current levels: (simplified) master rate equation for asymetric contacts

quick relaxation to the ground state #1 (compared to Γ's) :

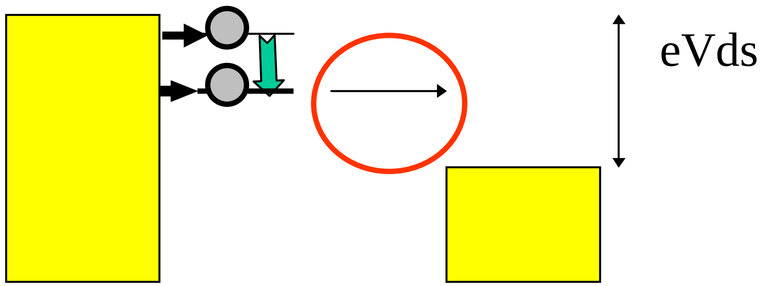
Sequential tunneling

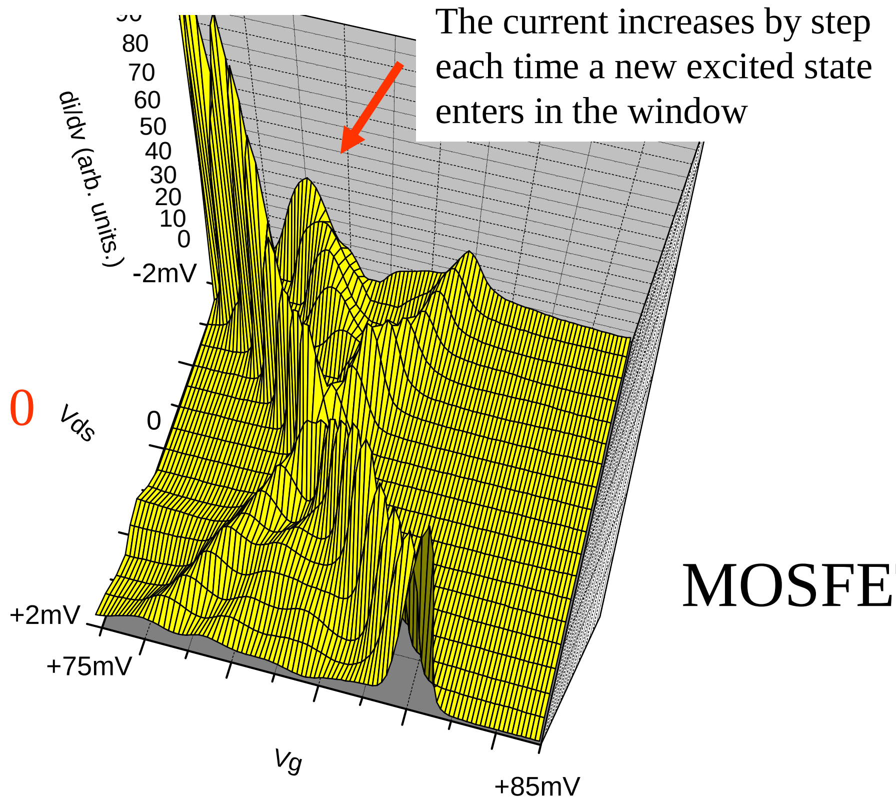
$$I = e\,\frac{(\Gamma_{\rm in}^1 + \Gamma_{\rm in}^2 + \cdots + \Gamma_{\rm in}^n)\Gamma_{\rm out}^1}{\Gamma_{\rm in}^1 + \Gamma_{\rm in}^2 + \cdots + \Gamma_{\rm in}^n + \Gamma_{\rm out}^1}$$

$\Gamma_{in} << \Gamma_{out}$ for Vsd < 0 I=e $(\Gamma_{in}^{1} + \Gamma_{in}^{2})$ for Vsd < 0



$\Gamma_{in} >> \Gamma_{out}$ for Vsd > 0 I=e Γ_{out}^{1} for Vsd < 0

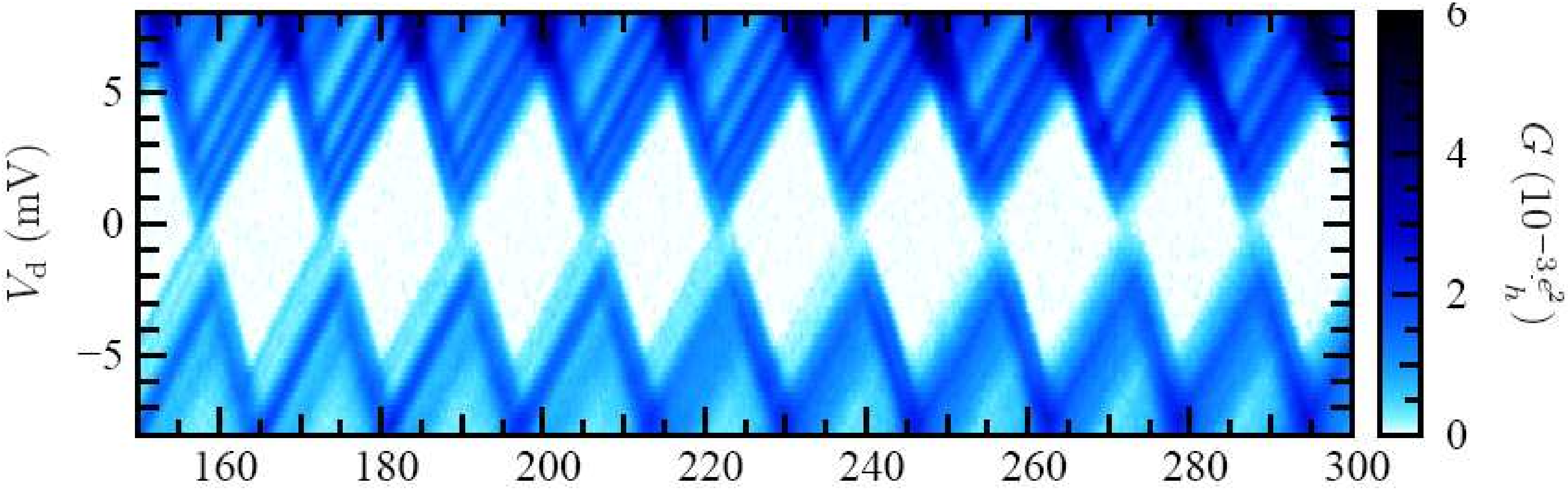




MOSFET

Stability diagram anomalies due to offset charges

(M. Hofheinz et al.2007):



Bibliography:

S. Datta: Electronic « Transport in Mesoscopic Systems »
Cambridge University Press 1995

C. W. J. Beenakker ”Theory of Coulomb-blockade oscillations in the conductance of a quantum dot”, Phys. Rev. B44, pp 1646-1656, 1991.

D. V. Averin, A.N. Korotkov and K. K. Likharev ”Theory of single-electron charging of quantum wells and dots”, Phys. Rev. B44, pp 6199-6211, 1991.

Y. Meir, N. S. Wingreen and P. A. Lee « transport through a Strongly Interacting Electron System: Theory of periodic Conductance Oscillations », Phys. Rev. Letters 66, 3048 (1991).

Y. Alhassid “The statistical theory of quantum dots” , Rev. of Modern Physics 72, 895 (2000)

E. Akkermans et G. Montambaux “Physique mésoscopique des électrons et des photons “ Collections Savoir actuels EDP Sciences 2005

T. Ando, A.B. Fowler and F. Stern,“Electronic properties of 2D systems” Review of Modern Physics 54, 437 (1982).

H. Grabert, M. Devoret, "Single charge tunneling Coulomb blockade phenomena in nanostructures", Vol. 294, NATO ASI Series B, Plenum Press, New York and London, physics edition , 1992