Influence of the proximity effect on the shot noise in a diffusive metallic wire

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In frame of the Keldysh formalism, we calculate the low frequency current noise in a metallic diffusive wire connected to a normal and a superconducting reservoir and we show how it is related to the reentrance of the conductance.

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Measuring the equilibrium current noise $S_I = 2\int dt \langle \delta I(t)\delta I(0)\rangle$ in a device does not bring additional information compared to the differential conductance at temperature T, $G(T)=(\partial I/\partial V)_{V=0}$—$I$ is the mean current, V is the bias voltage, according to the fluctuation-dissipation relation $S_I = 4TG(T)$. On the contrary, out-of-equilibrium noise measurements have deserved a lot of attention in mesoscopic devices as these can reveal fundamental properties of the type of charge carriers and the nature of the transport.[1] In particular, the zero-frequency noise in a normal diffusive metallic wire with noninteracting electrons was calculated in the presence of dc current and highly transparent interfaces with the leads, and it was shown to have the universal value $S_I^N=(1/3)2eGV$—$(-e)$ is the charge of the electron—at low temperatures $T \ll eV$.[2,3] This is called the shot noise and it is reduced to a factor 1/3 (the Fano factor) of the Schottky noise through a tunnel barrier. When one of the leads connecting the wire is brought to a superconducting state, the shot noise at $eV \ll \Delta$ doubles its value.[4,5] The additional factor 2 arises with the Andreev reflection, that is, a subgap electron with the energy ε coming from the normal side is reflected into a hole with the same energy while a Cooper pair is created on the superconducting side, thus leading to the replacement of the charge $e \rightarrow 2e$ in the formula for S_I in the normal system.

Due to their wave vector mismatch, the electron and the hole get dephased at the distance L_ε from the interface. If the normal metal is diffusive, $L_\varepsilon = \sqrt{\hbar D/\varepsilon}$, where D is its diffusion constant. So in the wire with the length L at energies below $E_{Th}=\hbar D/L^2$, called the Thouless energy, an electron and the Andreev-reflected hole are coherent in the whole wire. The available calculations on the noise concentrate either on the fully incoherent electron transport ($E_{Th} \ll T, eV$) with the help of the Boltzmann-Langevin equation,[5] or on the fully coherent case ($T, eV \ll E_{Th} < \Delta$) in the ballistic scattering theory of transport.[4,6] In both cases, the crossover from shot noise to thermal noise in the regime ($T, eV \ll \Delta$) is given by

$$S_I^{NS} = \frac{8T}{3}G + \frac{4eGV}{3}\coth\frac{eV}{T}, \tag{1}$$

where $G=G_N$ is the normal conductance of the wire. It is the same formula as for the normal diffusive wire[3] still after making the substitution $e \rightarrow 2e$. Experimental results were shown to be remarkably accounted with Eq. (1).[7,8]

The intermediate regime when $T, eV \sim E_{Th}$ should also deserve some attention because it is the region where the conductance deviates from G_N. At decreasing temperature T, the conductance $G(T)$ starts to increase and differs from G_N. It corresponds to the reduction of the effective length of the wire by L_T due to the proximity effect. Then it decreases and finally reaches exactly its normal state value $G(T\rightarrow 0) = G_N$.[9,10] In the wire with the length $L \gg \xi_0$, where $\xi_0 = \sqrt{\hbar D/\Delta}$ is the coherence length, the maximum of the conductance occurs when $T \sim 4E_{Th} \ll \Delta$. This phenomenon is called the reentrance and it also manifests in nontrivial differential conductance versus voltage at fixed T. It was only recently that progress in microtechnology allowed to test this prediction quite successfully.[11]

In the present paper, we argue that the formula similar to Eq. (1),

$$S_I^{NS} = \frac{8T}{3}\frac{\partial I}{\partial V} + \frac{4eI}{3}\coth\frac{eV}{T}, \tag{2}$$

still applies at any $T, eV \ll \Delta$. Here, the current-voltage characteristic $I=I(V)$ displays the reentrance and is given below in Eq. (16). The formula (2) gives the precise way the differential resistance should be introduced in order to compare the noise with resistive measurements.

This problem was first considered in the frame of the full counting statistics.[12] A numerical calculation was done which allowed to reproduce at least semiquantitatively, but without any fitting parameter, a small bump in the experimental noise at low applied voltages.[8] A small disagreement was attributed to heating effects much beyond the calculations.[12] More recently, this approach was also shown to provide satisfactory explanation of phase sensitive shot noise in an Andreev interferometer.[13] Our noise calculations based on Eq. (2) reproduce nearly the same curves plotted in Ref. 12, except for a small disagreement that cannot be explained at this stage.

The main difficulty of the microscopic calculation of the noise in a disordered system consists in evaluating an averaged two-point Green's functions that form the current-current correlation function, whereas most resolved problems in quantum field theory are concerned with one-point Green's functions. This problem was solved in normal metals,[14] which provided a firm basis for extending the noise calculation to finite frequency and the effect of electron-electron interactions.[15] The question is much more difficult when superconducting correlations are present. The only attempt, to our knowledge, was made in Ref. 16 for S/c/N constrictions. Generalizing the ideas of Refs. 14,16 are the principal steps of the derivation that is presented below. It appears that a full theory of nonequilibrium superconductivity still remains to be formulated as soon as the correlation functions are considered. Some steps are extrapolated in using reasonable physical assumptions and analogies with the normal case.[14]

The method for calculating microscopically the current and the correlation function for its fluctuations starts with the Keldysh formulation for the nonequilibrium electronic Green's functions[17]

$$\check{G}=\begin{pmatrix}\hat{G}^R & \hat{G}^K\\ 0 & \hat{G}^A\end{pmatrix}, \tag{3}$$

where R,A, and K stand, respectively, for the retarded, advanced and Keldysh Green's functions, which are the Gorkov-Nambu matrices with proximity-induced anomalous components. By fixing the voltage equal to zero in the superconducting reservoir as a convention, the Green's functions depend only on the difference of their time arguments. In the Wigner representation, they obey

$$\left[\varepsilon\check{\tau}_z-\xi_{\mathbf{p}}+\frac{i}{2m}\mathbf{p}\partial_{\mathbf{r}}-\check{\Sigma}_\varepsilon(\mathbf{r})\right]\langle\check{G}_\varepsilon(\mathbf{r},\mathbf{p})\rangle=1, \tag{4}$$

where the brackets stand for disorder averaging, $\check{\tau}_z$ acts as unity in the Keldysh space and as the Pauli matrix $\hat{\tau}_z$ in Nambu space,

$$\xi_{\mathbf{p}}=\frac{p^2}{2m}-\frac{p_F^2}{2m},\quad \check{\Sigma}_\varepsilon(\mathbf{r})=\frac{1}{2\pi N_0\tau}\int\frac{d^3p}{(2\pi)^3}\langle\check{G}_\varepsilon(\mathbf{r},\mathbf{p})\rangle. \tag{5}$$

Here $p_F=mv_F$ is the Fermi wave vector, m is the electron mass, $N_0=mp_F/(2\pi^2)$ is the density of states at Fermi level, and $\ell=v_F\tau$ is the mean free path. The transport properties are due to the conduction electrons near the Fermi surface. These can be described by introducing the quasiclassical approximation for the energy-averaged Green's functions

$$\check{g}_\varepsilon(\mathbf{r},\hat{p})=\frac{i}{\pi}\int d\xi_{\mathbf{p}}\langle\check{G}_\varepsilon(\mathbf{r},\mathbf{p})\rangle, \tag{6}$$

which obey the normalization condition $\check{g}_\varepsilon^2=1$.[17] In the diffusive limit $\ell\ll\xi_0$, these can be decomposed into an isotropic part and a small correction:[17]

$$\check{g}_\varepsilon(\mathbf{r},\hat{p})=\check{g}_\varepsilon(\mathbf{r})+\hat{p}\check{\mathbf{g}}_\varepsilon(\mathbf{r}),\quad \check{\mathbf{g}}_\varepsilon=-\ell\check{g}_\varepsilon\partial_{\mathbf{r}}\check{g}_\varepsilon, \tag{7}$$

$$D\partial_{\mathbf{r}}(\check{g}_\varepsilon\partial_{\mathbf{r}}\check{g}_\varepsilon)=-i\varepsilon[\check{\tau}_z,\check{g}_\varepsilon],\quad D=\ell v_F/3. \tag{8}$$

We first consider the mean density of current

$$\mathbf{j}(\mathbf{r},t)=-\frac{ep_F^2}{4\pi}\int\frac{d\varepsilon}{2\pi}\int\frac{d\Omega_{\hat{p}}}{4\pi}\hat{p}\,\mathrm{Tr}[\hat{\tau}_z\hat{g}_\varepsilon^K(\mathbf{r},\hat{p})]. \tag{9}$$

In the absence of supercurrent, we use the parametrization $\hat{g}^R=\hat{\tau}_z\cosh\theta+i\hat{\tau}_y\sinh\theta$, where $\theta=\theta_1+i\theta_2$, $\hat{g}^A=-\hat{\tau}_z(\hat{g}^R)^\dagger\hat{\tau}_z$, $\hat{g}^K=\hat{g}^R\hat{f}-\hat{f}\hat{g}^A$, and the distribution function matrix

$$\hat{f}=f_L+\hat{\tau}_zf_T. \tag{10}$$

From Eq. (8), $\hat{g}^R$ and $\hat{g}^A$ are determined by

$$D\partial_{\mathbf{r}}^2\theta+2i\varepsilon\sinh\theta=0, \tag{11}$$

and by the boundary conditions[18] at highly transparent interfaces which impose the continuity of θ in the normal reservoir ($\theta^N=0$) and in the superconducting reservoir ($\theta^S=-i\pi/2$ for $\varepsilon\ll\Delta$). The kinetic equations read

$$\partial_{\mathbf{r}}(\cos^2\theta_2\partial_{\mathbf{r}}f_L)=0, \tag{12}$$

$$\partial_{\mathbf{r}}(\cosh^2\theta_1\partial_{\mathbf{r}}f_T)=0. \tag{13}$$

Equation (13) is the current conservation law. In equilibrium, the distribution functions reduce to $f_L(\varepsilon)=\tanh[\varepsilon/2T]$ and $f_T(\varepsilon)=0$. When a voltage is applied, f_L stands equal to its value in the normal reservoir $f_L^N(\varepsilon)=\frac{1}{2}\{\tanh[(\varepsilon+eV)/(2T)]+\tanh[(\varepsilon-eV)/(2T)]\}$, whereas f_T is also continuous in the reservoirs, and $f_T^S=0$, $f_T^N(\varepsilon)=\frac{1}{2}\{\tanh[(\varepsilon+eV)/2T]-\tanh[(\varepsilon-eV)/(2T)]\}$. Moreover, the symmetry properties

$$\theta_{-\varepsilon}=-\theta_\varepsilon^*,\quad f_L(-\varepsilon,\mathbf{r})=-f_L(\varepsilon,\mathbf{r}),\quad f_T(-\varepsilon,\mathbf{r})=f_T(\varepsilon,\mathbf{r}) \tag{14}$$

hold in the absence of spin polarization. Then the current density (9) reduces to the quasiparticle current density

$$\mathbf{j}^{qp}=\frac{\sigma_N}{e}\int_0^\infty d\varepsilon\cosh^2\theta_1\partial_{\mathbf{r}}f_T, \tag{15}$$

where $\sigma_N=2e^2DN_0$ is the conductivity in the normal state.

In a thin wire, the current-voltage relation can now be established from Eqs. (13) and (15) as

$$I=\frac{G_N}{2e}\int_0^\infty d\varepsilon\mathfrak{D}(\varepsilon)\left[\tanh\frac{\varepsilon+eV}{2T}-\tanh\frac{\varepsilon-eV}{2T}\right], \tag{16}$$

where

$$\mathfrak{D}(\varepsilon)=\left(\frac{1}{L}\int_0^L dx\frac{1}{\cosh^2\theta_1(\varepsilon,x)}\right)^{-1} \tag{17}$$

is the spectral conductance that governs the reentrance.[10,11] Here we have used standard relations for current $I=jA$ and conductance $G_N=\sigma_NA/L$, where A is the wire cross section area.

In order to calculate the current noise, we consider the current density operator

$$\hat{\mathbf{j}}(\mathbf{r},t)=\frac{ie}{2m}\lim_{\mathbf{r}'\to\mathbf{r}}(\partial_{\mathbf{r}}-\partial_{\mathbf{r}'})\sum_{\sigma=\uparrow,\downarrow}\hat{\psi}_\sigma^\dagger(\mathbf{r}',t)\hat{\psi}_\sigma(\mathbf{r},t) \tag{18}$$

and the correlation function of its fluctuations $\delta\hat{\mathbf{j}}=\hat{\mathbf{j}}-\langle\langle\hat{\mathbf{j}}\rangle\rangle$:

$$S_{\alpha\beta}(\mathbf{r},t,\mathbf{r}',t') = \langle\langle \delta\hat{j}_{\alpha}(\mathbf{r},t)\delta\hat{j}_{\beta}(\mathbf{r}',t') + \delta\hat{j}_{\beta}(\mathbf{r}',t')\delta\hat{j}_{\alpha}(\mathbf{r},t)\rangle\rangle, \qquad (19)$$

where double angular brackets mean quantum statistical averaging and averaging over disorder. It depends only on the difference of its time arguments, and its spectral density is

$$S_{\alpha\beta}(\mathbf{r},\mathbf{r}',\omega) = \int d(t-t')e^{i\omega(t-t')}S_{\alpha\beta}(\mathbf{r},t,\mathbf{r}',t'). \quad (20)$$

Using the Wick's theorem, we transform Eq. (19) into

$$\begin{aligned} S_{\alpha\beta}(\mathbf{r},\mathbf{r}',\omega) = &-\frac{e^2}{8m^2}\lim_{\substack{\mathbf{r}_1'\to\mathbf{r}'\\ \mathbf{r}_1\to\mathbf{r}}}(\partial_{\mathbf{r}_1}-\partial_{\mathbf{r}})_{\alpha}(\partial_{\mathbf{r}_1'}-\partial_{\mathbf{r}'})_{\beta} \\ &\times\int\frac{d\varepsilon}{2\pi}\mathrm{Tr}\{\langle \hat{G}^K_{\varepsilon-\omega/2}(\mathbf{r}_1',\mathbf{r})\hat{\tau}_z\hat{G}^K_{\varepsilon+\omega/2}(\mathbf{r}_1,\mathbf{r}')\hat{\tau}_z \\ &-[\hat{G}^R_{\varepsilon-\omega/2}(\mathbf{r}_1',\mathbf{r})-\hat{G}^A_{\varepsilon-\omega/2}(\mathbf{r}_1',\mathbf{r})]\hat{\tau}_z \\ &\times[\hat{G}^R_{\varepsilon+\omega/2}(\mathbf{r}_1,\mathbf{r}')-\hat{G}^A_{\varepsilon+\omega/2}(\mathbf{r}_1,\mathbf{r}')]\hat{\tau}_z\rangle\}. \end{aligned} \quad (21)$$

Instead of calculating the disorder average of the product of two Green's functions in this equation, we make an analogy with the results of Ref. 14, where S is decomposed into local and nonlocal contributions. In the first local component $S^{(0)}_{\alpha\beta}$, terms of the type $\langle \hat{G}\hat{\tau}_z\hat{G}\hat{\tau}_z\rangle$ are replaced by $\langle \hat{G}\rangle\hat{\tau}_z\langle\hat{G}\rangle\hat{\tau}_z$. The nonlocal component includes the diffusion induced vertex corrections. The procedure corresponds to the two-scale decomposition of the total current density fluctuations

$$\delta\mathbf{j} = \delta\mathbf{j}^{int} + \delta\mathbf{j}^{qp}. \quad (22)$$

The intrinsic fluctuating current $\delta\mathbf{j}^{int}$ is due to the random scattering of electrons on impurities which produces a noise on the short length scale of the mean free path ℓ. The fluctuating current $\delta\mathbf{j}^{qp}$ comes from the propagation of $\delta\mathbf{j}^{int}$ into the medium by the same mechanism responsible for the transport. It is related to the fluctuations of the distribution function δf_T through

$$\delta\mathbf{j}^{qp} = \frac{\sigma_N}{e}\int_0^{\infty} d\varepsilon \cosh^2\theta_1\partial_{\mathbf{r}}\delta f_T \quad (23)$$

in analogy with Eq. (15).

Formula (15) for the current shows that it is made up of spectral components that are added in parallel, each one having a position- and energy-dependent dimensionless "conductivity." Noise occurs due to random elastic processes. Thus, it cannot mix spectral currents of different energies. Only the mixing of opposite energies happens due to the Andreev reflection. Then, each spectral component of the fluctuating current densities can be considered separately:

$$\delta\mathbf{j}(\mathbf{r},\varepsilon) = \delta\mathbf{j}^{int}(\mathbf{r},\varepsilon) + \delta\mathbf{j}^{qp}(\mathbf{r},\varepsilon), \quad (24)$$

$$\delta\mathbf{j}^{qp}(\mathbf{r},\varepsilon) = \frac{\sigma_N}{e}\cosh^2\theta_1(\mathbf{r},\varepsilon)\partial_{\mathbf{r}}\delta f_T(\varepsilon,\mathbf{r}). \quad (25)$$

The local fluctuations of spectral current

$$\langle \delta j^{int}_{\alpha}(\mathbf{r},\varepsilon)\delta j^{int}_{\beta}(\mathbf{r}',\varepsilon)\rangle = \delta_{\alpha\beta}\delta(\mathbf{r}-\mathbf{r}')\Lambda(\mathbf{r},\varepsilon). \quad (26)$$

produce the local noise component

$$S^{(0)}_{\alpha\beta}(\mathbf{r},\mathbf{r}',\omega=0) = 2\int_0^{\infty} d\varepsilon\langle \delta j^{int}_{\alpha}(\mathbf{r},\varepsilon)\delta j^{int}_{\beta}(\mathbf{r}',\varepsilon)\rangle_{\omega=0}, \quad (27)$$

Now we use the same procedure as in Ref. 19, starting from $\int d^3r\,\partial_{\mathbf{r}} f_T\delta\mathbf{j}$, using the conservation of the current $\partial_{\mathbf{r}}\delta\mathbf{j}=0$, and assuming that the distribution functions do not fluctuate in the reservoirs. We get the total spectral current fluctuation

$$\delta I(\varepsilon) = \int_{N\mathrm{side}} d\mathbf{s}\,\delta\mathbf{j}(\mathbf{r},\varepsilon) = \frac{1}{f^N_T(\varepsilon)}\int d^3r\,\delta\mathbf{j}^{int}(\mathbf{r},\varepsilon)\partial_{\mathbf{r}} f_T(\varepsilon,\mathbf{r}), \quad (28)$$

and then the noise

$$\begin{aligned} S^{NS}_I &= 2\int_0^{\infty} d\varepsilon\langle \delta I(\varepsilon)\delta I(\varepsilon)\rangle_{\omega=0} \\ &= \int_0^{\infty} d\varepsilon\int d^3r\,\Lambda(\mathbf{r},\varepsilon)\left(\frac{\partial_{\mathbf{r}} f_T(\varepsilon,\mathbf{r})}{f^N_T(\varepsilon)}\right)^2. \end{aligned} \quad (29)$$

$\Lambda(\mathbf{r},\varepsilon)$ should be evaluated from Eq. (21) where, as was mentioned already, terms of the type $\langle \hat{G}\hat{\tau}_z\hat{G}\hat{\tau}_z\rangle$ are replaced by $\langle\hat{G}\rangle\hat{\tau}_z\langle\hat{G}\rangle\hat{\tau}_z$. The region of integration over ε is always effectively limited by max(T,eV) that is much smaller than T_c and thus than $1/\tau$. Hence, we may use the following representation for the average Green's functions:

$$\langle \check{G}_{\varepsilon}(\mathbf{r},\mathbf{p})\rangle \simeq -\frac{\xi_{\mathbf{p}}+\dfrac{i}{2\tau}\check{g}_{\varepsilon}(\mathbf{r})}{\xi^2_{\mathbf{p}}+\dfrac{1}{4\tau^2}}, \quad (30)$$

and consequently

$$\langle \hat{G}^K_{\varepsilon}(\mathbf{r},\mathbf{p})\rangle = \langle \hat{G}^R_{\varepsilon}(\mathbf{r},\mathbf{p})\rangle\hat{f}_{\varepsilon}(\mathbf{r},\hat{p}) - \hat{f}_{\varepsilon}(\mathbf{r},\hat{p})\langle\hat{G}^A_{\varepsilon}(\mathbf{r},\mathbf{p})\rangle, \quad (31)$$

and we get for the low frequency local spectral noise density

$$\begin{aligned} \Lambda(\mathbf{r},\varepsilon) = 2\sigma_N\{&\cosh^2\theta_1(\mathbf{r},\varepsilon)[1-f_L(\varepsilon,\mathbf{r})^2 - f_T(\varepsilon,\mathbf{r})^2] \\ &+\mathrm{Re}[\sinh^2\theta(\mathbf{r},\varepsilon)][1-f_L(\varepsilon,\mathbf{r})^2]\}. \end{aligned} \quad (32)$$

We note that the first term in Eq. (32) originates from G^RG^A products. It exactly corresponds to the local density of current noise which is obtained[20] by introducing Langevin sources into the collision integrals of Larkin-Ovchinnikov kinetic equations.[17] The second term originates from G^RG^R and G^AG^A products. It is absent in a normal metal where θ

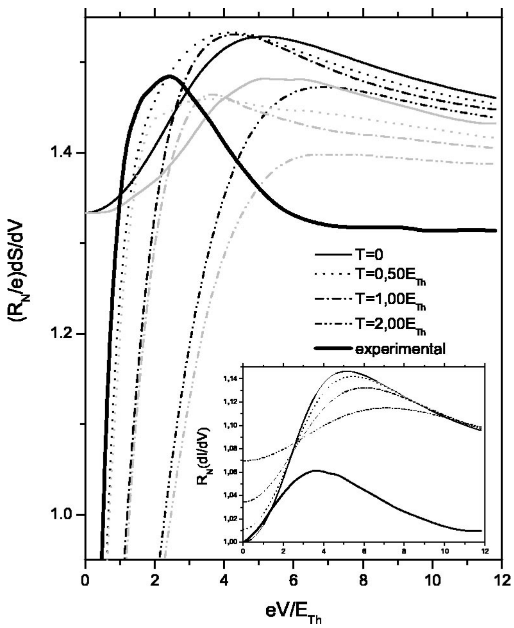

FIG. 1. The main plot is the derivative of the noise vs the voltage at different temperatures. The inset shows the differential conductance vs the voltage at the same temperatures. As a comparison, the noise obtained in Ref. 12 is drawn in gray for the same parameters. The thick line interpolates experimental data Ref. 8,12, the experimental setup corresponds to $T \approx 0.2 E_{Th}$ (Ref. 8).

$=0$. However, it breaks the necessary correspondence of the thermal noise to fluctuation-dissipation theorem, as well as the shot noise in the previously considered coherent and incoherent regimes.[4,5] That is why we neglect it here assuming that in result of complete calculation of diffusion induced vortex corrections (we plan to do it in future), the latter integral is cancelled out. Thus for the local spectral density of noise Λ, we take

$$\Lambda(\mathbf{r},\varepsilon) = 2\sigma_N \cosh^2\theta_1(\mathbf{r},\varepsilon)[1 - f_L(\varepsilon,\mathbf{r})^2 - f_T(\varepsilon,\mathbf{r})^2], \tag{33}$$

instead of Eq. (32). Performing integration in Eq. (29) over ε and x we get Eq. (2) finally.

In Fig. 1, $\partial S_I/\partial V$ is plotted as a function of V for different temperatures. As it comes from Eq. (2), the expected behavior of the noise is recovered both in the coherent and incoherent regimes. In particular, the decrease of $\partial S_I/\partial V$ at low voltages and finite temperatures is a consequence of the fluctuation-dissipation theorem. The numerical results are very similar to those from Ref. 12, also represented in Fig. 1 for a comparison, with the bump at voltages $eV \sim E_{Th}$. However, the small quantitative disagreement [the noise from Eq. (2) is always higher, but not more than 5%] with Ref. 12 is not explained at this stage. In particular, we get $S_I(V) = (4e/3)I(V)$ at $T=0$ in contradiction with the numerical results of Ref. 12 at $eV \sim E_{Th}$. In Ref. 13 the authors interpret this as a deviation of the effective charge from its value $q_{eff} = 2e$, when writing $S_I(V) = [2q_{eff}(V)/3]I(V)$. This would reflect the suppression of the noise due to correlated entry of Andreev pairs in the wire as the voltage is decreased at $eV > E_{Th}$. This seems to us a bit speculative at this stage. As concerns the comparison with the experimental results,[8] we note that the authors of Ref. 12 could explain qualitatively the bump in the conductance and the noise measured experimentally, at the right energy scale $eV \sim E_{Th}$, without any fitting parameter. However, they noticed that both, the conductance and the noise, display narrower bumps than as obtained theoretically. They attributed this to possible heating phenomena with a voltage-dependent effective temperature in the system. Our results do not compare better to the experiment.

As a conclusion, we present a microscopic calculation of the current noise in a normal/superconducting heterostructure. At the end, it looks like the Boltzmann-Langevin approach,[5] with the intrinsic fluctuating currents playing the role of the Langevin sources, but in our case the proximity effect is fully taken into account. We derive an analytical formula for the noise when the superconducting gap is large, but our method can be directly generalized to other situations, in particular for calculating the noise at any relation between T, eV, and Δ. In the present work, some steps had to be conjectured. This reflects the lack of a satisfactory theory for nonequilibrium correlation functions in nonhomogenous superconductors up to now. We hope that our work may play some role for stimulating future progress in this field.

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