Quantum physics in one dimension using Josephson-junction arrays.

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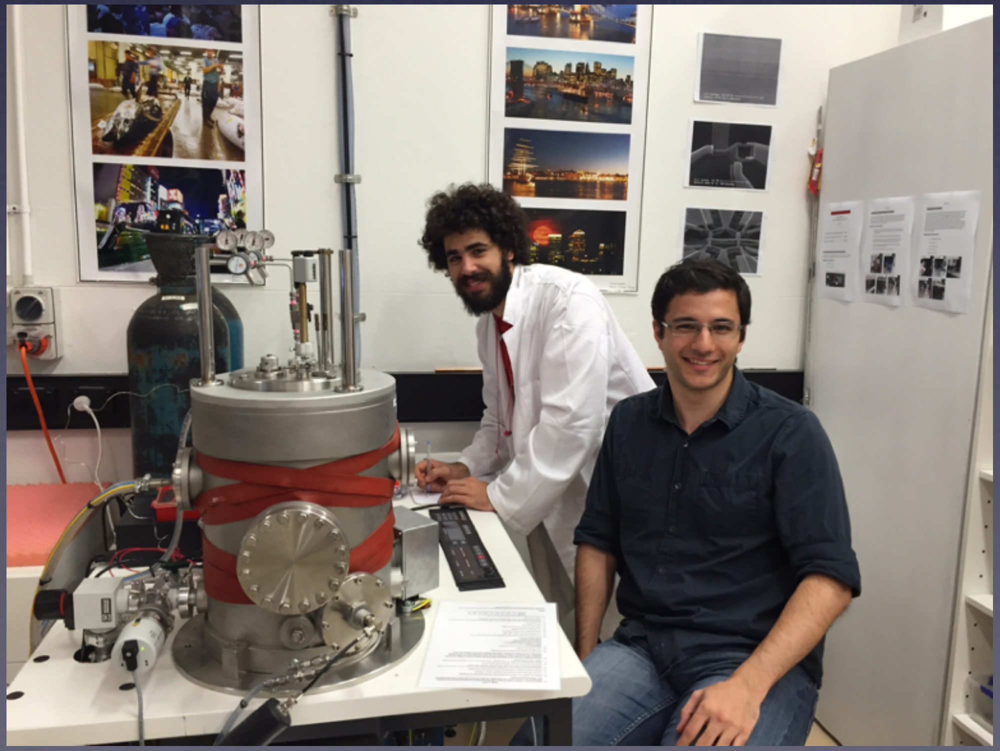


Josephson-junction arrays in the charge regime

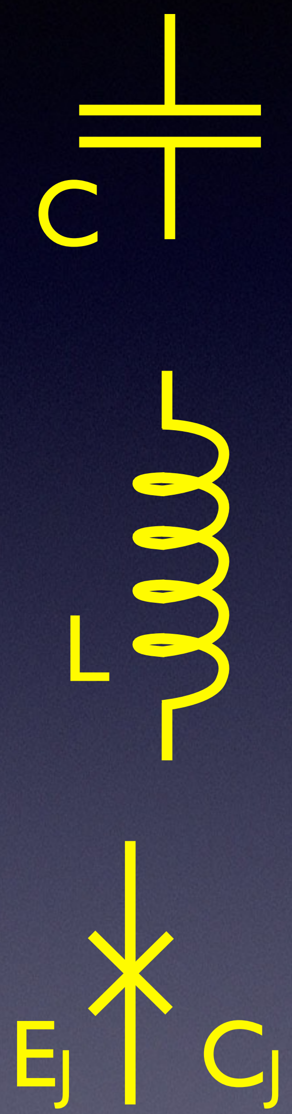
- **Enable Synthetic quantum materials**
  experimental platform to study QPT's, the AdS-CFT conjecture, novel quantum phases, *etc*…

Like optical lattices or trapped ions, but with *strong interactions*, *significant intrinsic disorder*, and possibility of *novel electronic devices*

- **Quantum standards metrology**
  with 1D chains, possibility of ***dual Josephson effect***
  → "Shapiro" current steps - a primary current standard of unrivalled accuracy

Significant for *redefinition* of the *SI ampere and kilogram*, but not yet successful. Why not?

Using JJ's, circuits can behave quantum-mechanically



Electrical	Mechanical
charge Q	momentum p
voltage $V=Q/C$	velocity $v=p/M$
capacitance C	mass M
phase θ	coordinate x
$[\theta,Q]=ie$	$[x,p]=i\hbar$
inductance L	spring constant k
LC-circuit	harmonic oscillator

phase

$$\theta(t) = \frac{e}{\hbar}\int_{-\infty}^{t} dt' V(t')$$

in superconducting leads

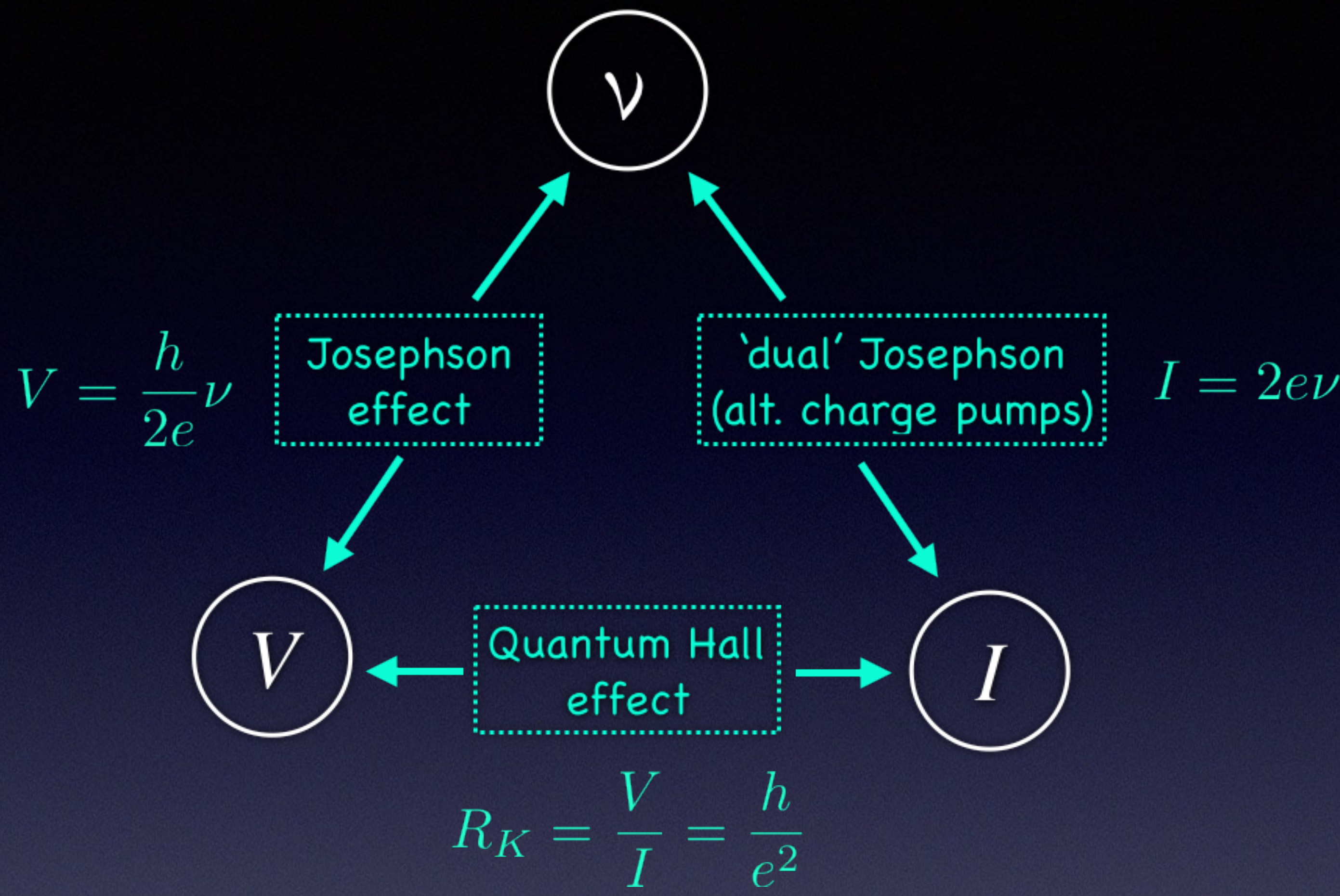
$$\psi = |\psi|\, e^{i\theta}$$

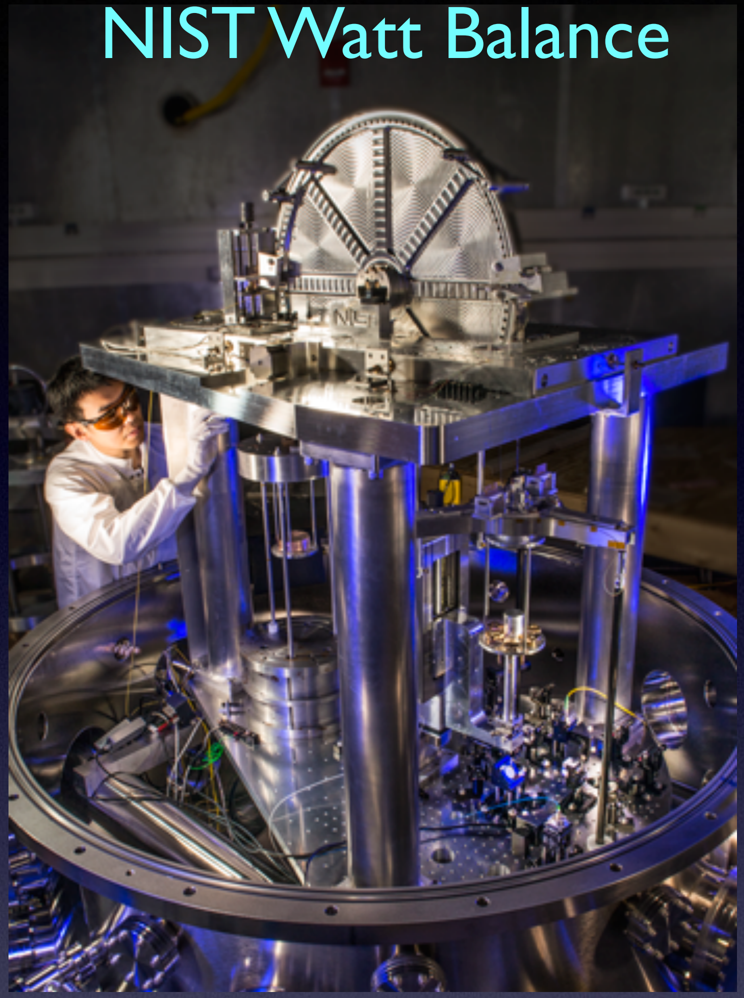
wave-function for Cooper pairs

The Josephson junction is the only circuit element that is non-linear and non-dissipative

a few circuit elements → lots of possibilities!
(can emulate lots of possible Hamiltonians)

The Quantum Metrological Triangle → quantum kilogram

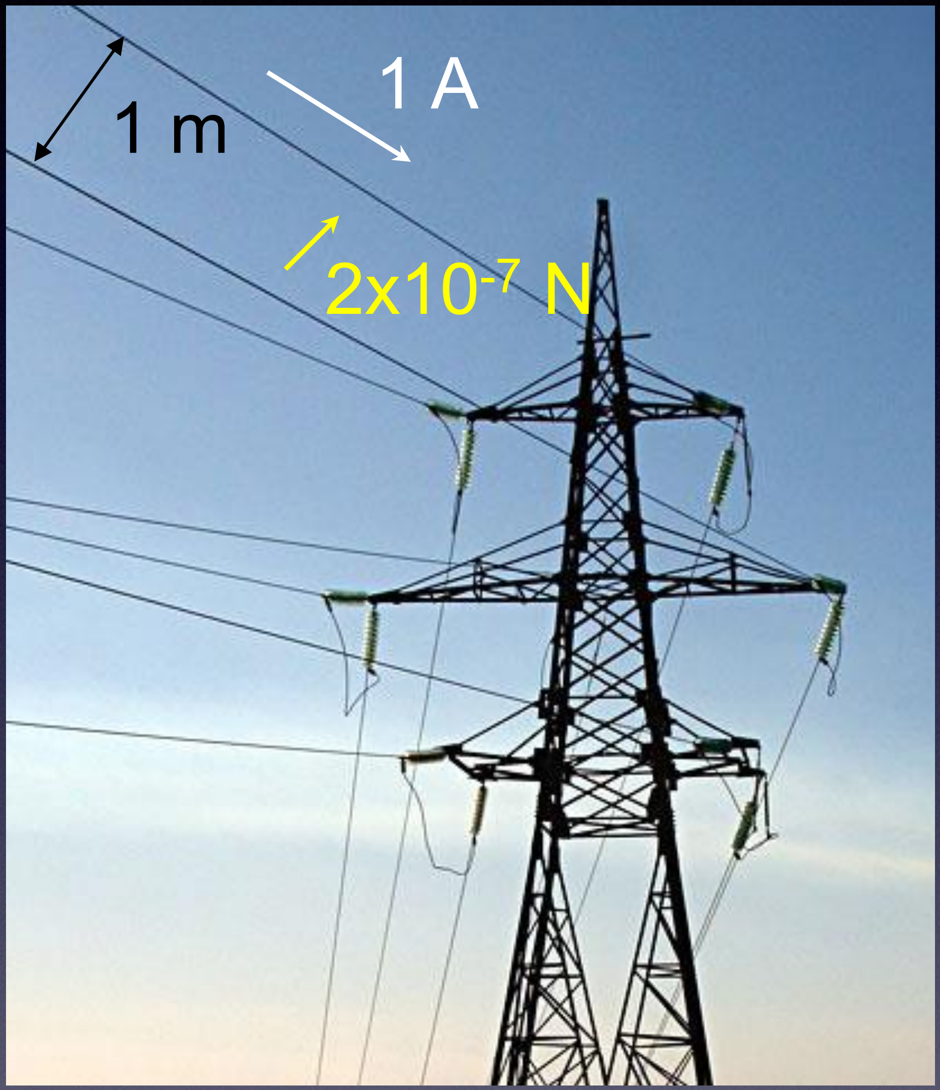




- JE frequency-to-voltage *h*/2*e* - arrays at 1 Volt accurate to 2 parts in 10^{17}
- Josephson effect and (integer) quantum Hall effect fix *h* and e
- Redefine SI kilogram by link to *h*, using a Watt balance.
- Desirable to test consistency of JE and QHE independently using a primary current standard , *i.e.* complete the QMT
- 'dual' Josephson effect enables novel quantum devices

The Ampere

presently defined *via* mechanical force



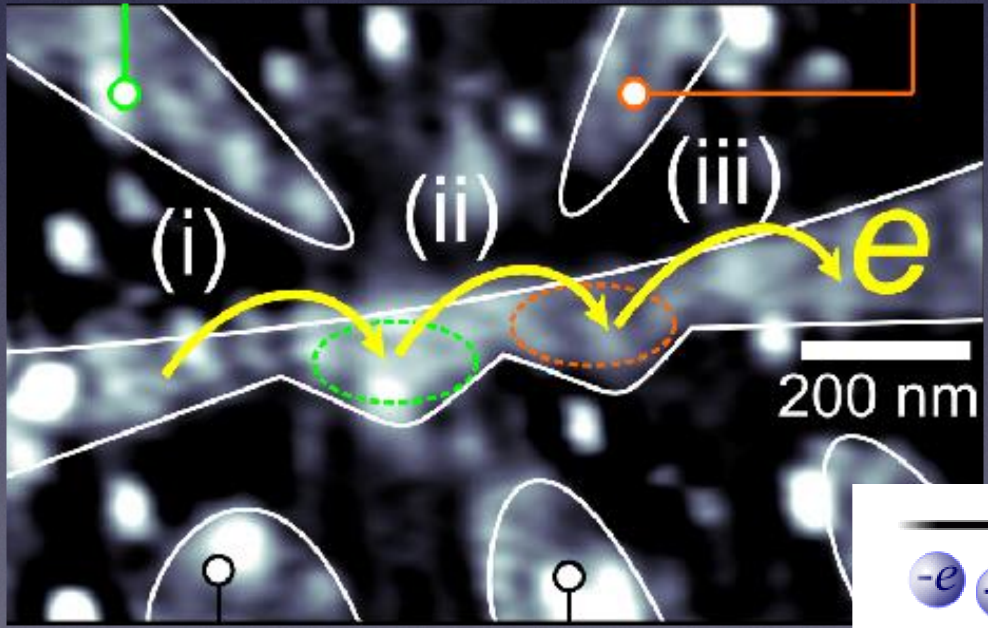
versus

A. 'quantum' Ohm's law

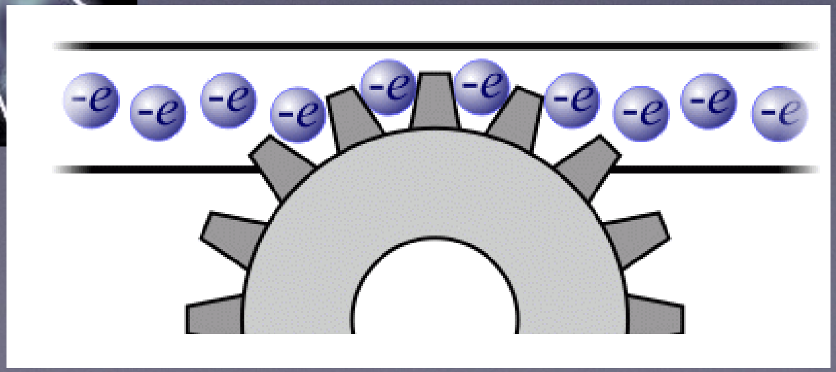
$$I = \frac{V}{R} = \frac{\text{Josephson}}{\text{Quantum Hall}}$$
$$= \frac{h\nu/2e}{h/e^2} = e\nu/2$$

B. single charge transport

e.g. graphene QD — Connolly *et al.* Nature Nano (2013)



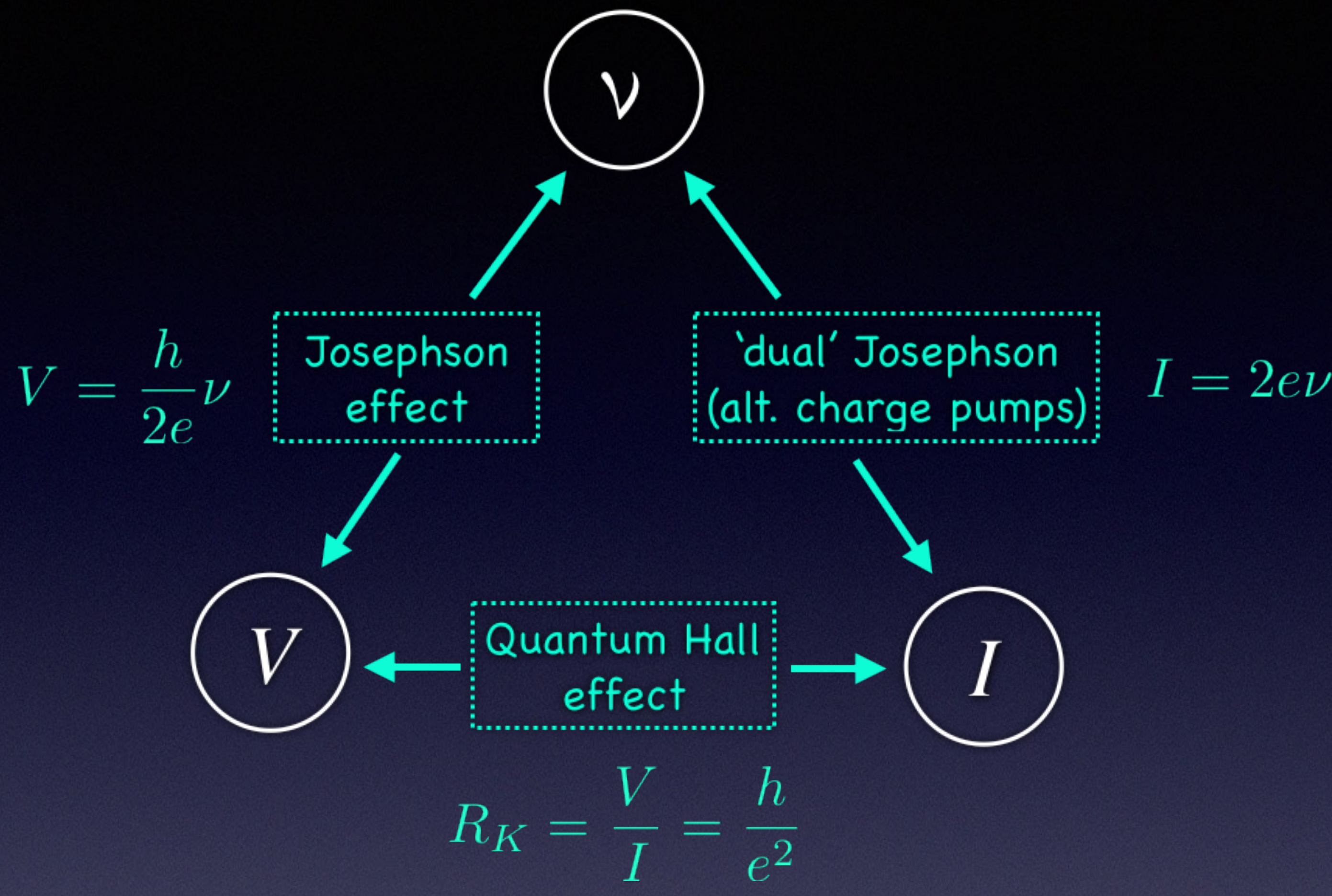
$$I = e\nu$$

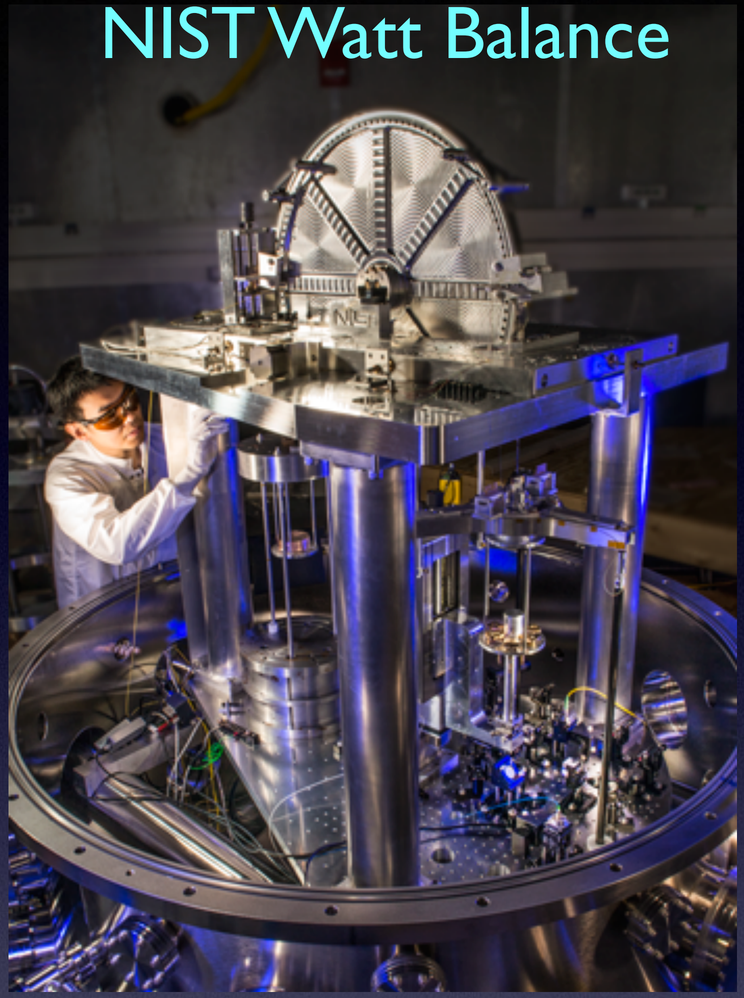


i.e. a charge pump

but rel. accuracy too large ($\sim10^{-6}$)

The Quantum Metrological Triangle → quantum kilogram





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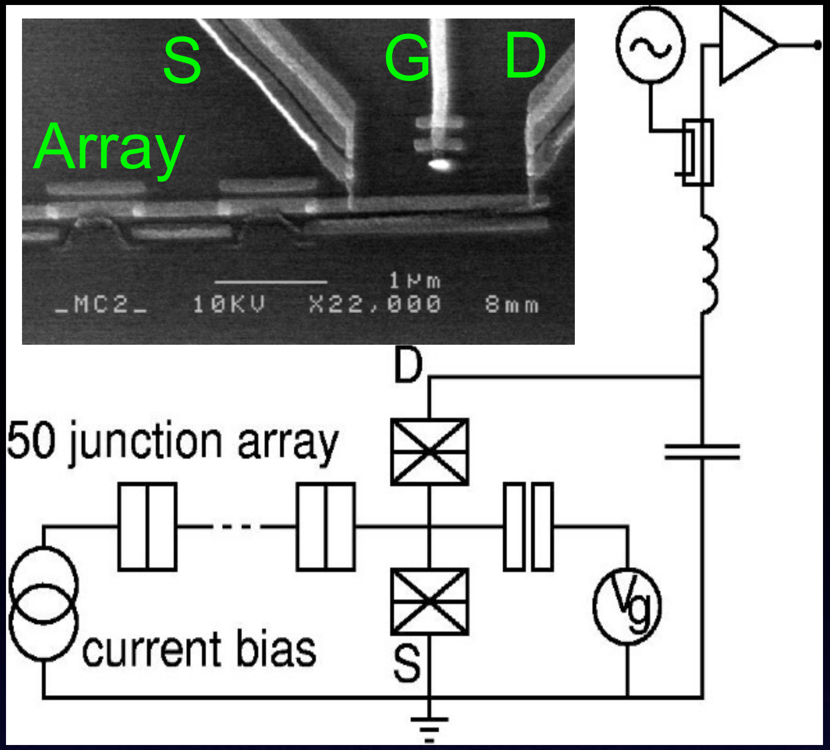
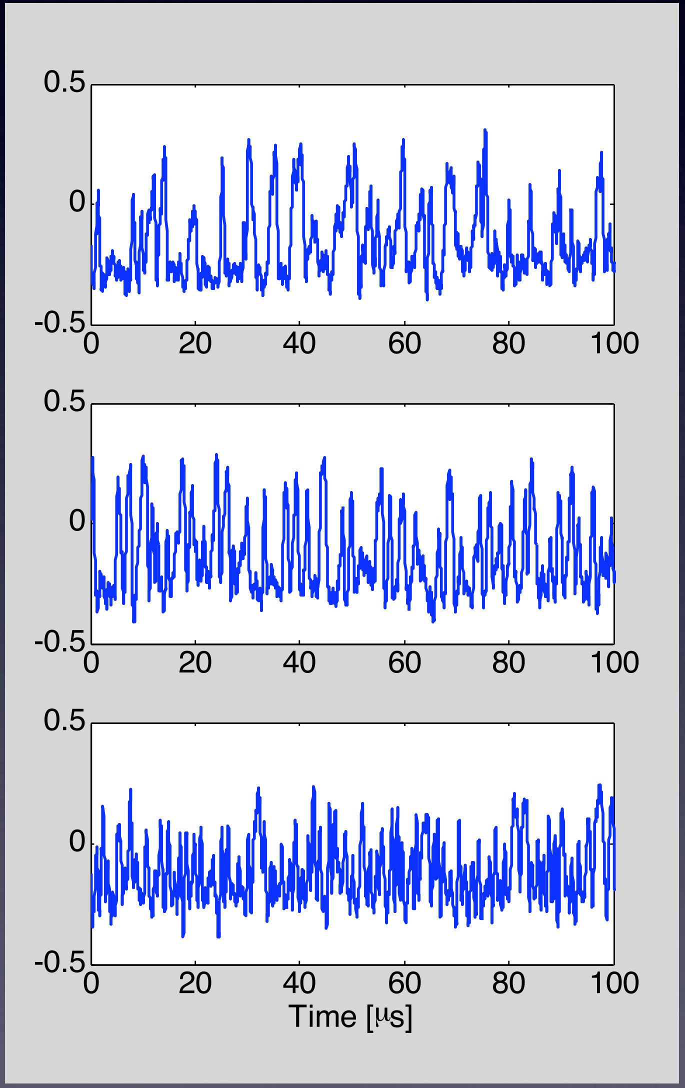
A Single-Electron 'Clock'

Bylander, Duty and Delsing, **Nature**, 2005

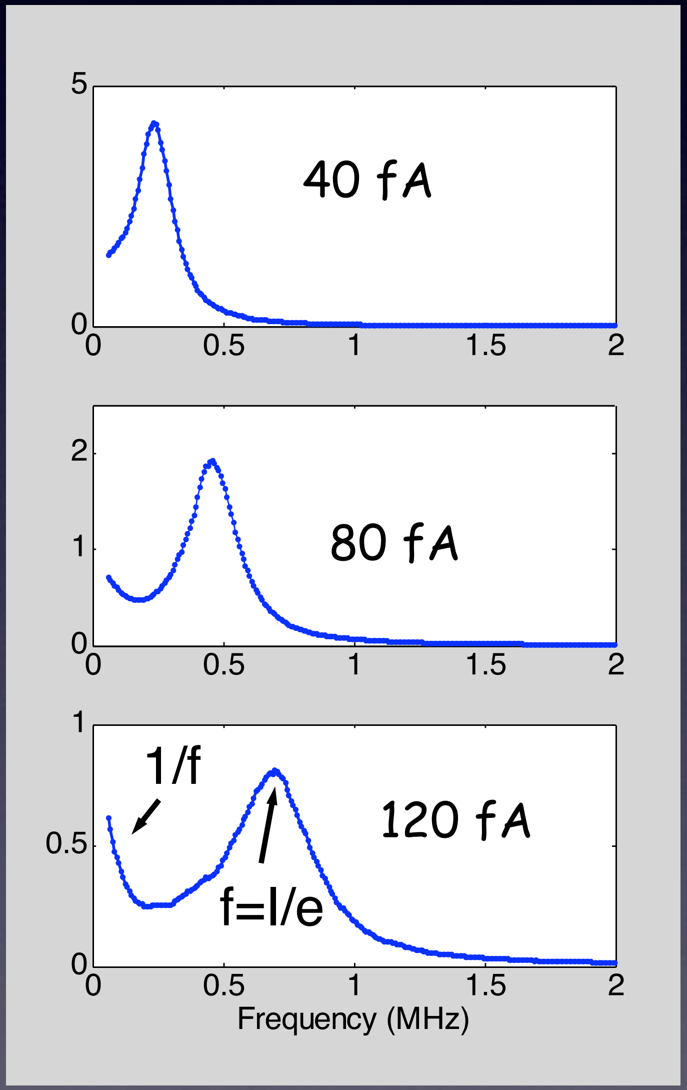
50-junction chain injected into RF-SET

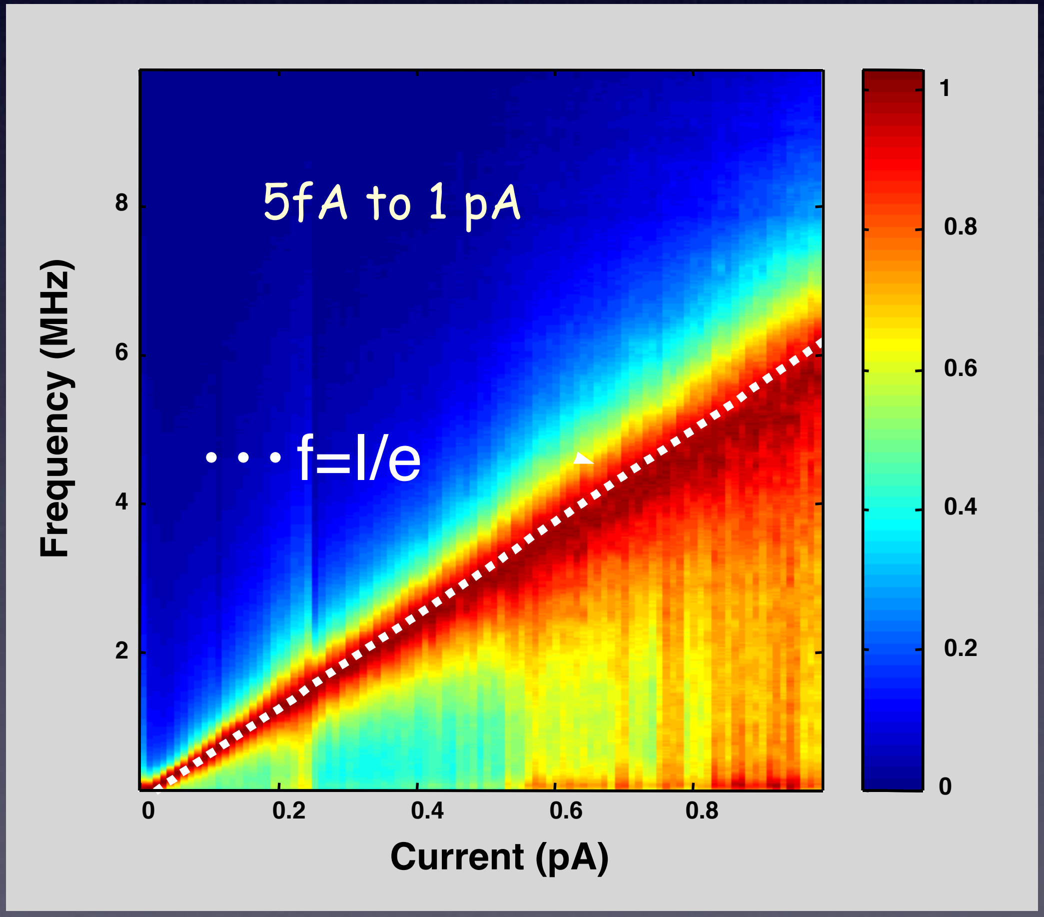


In real time....



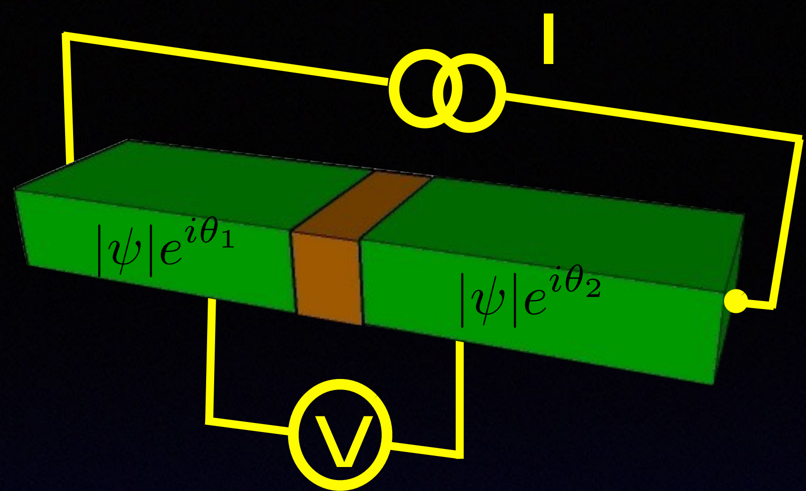
or with spectrum analyzer





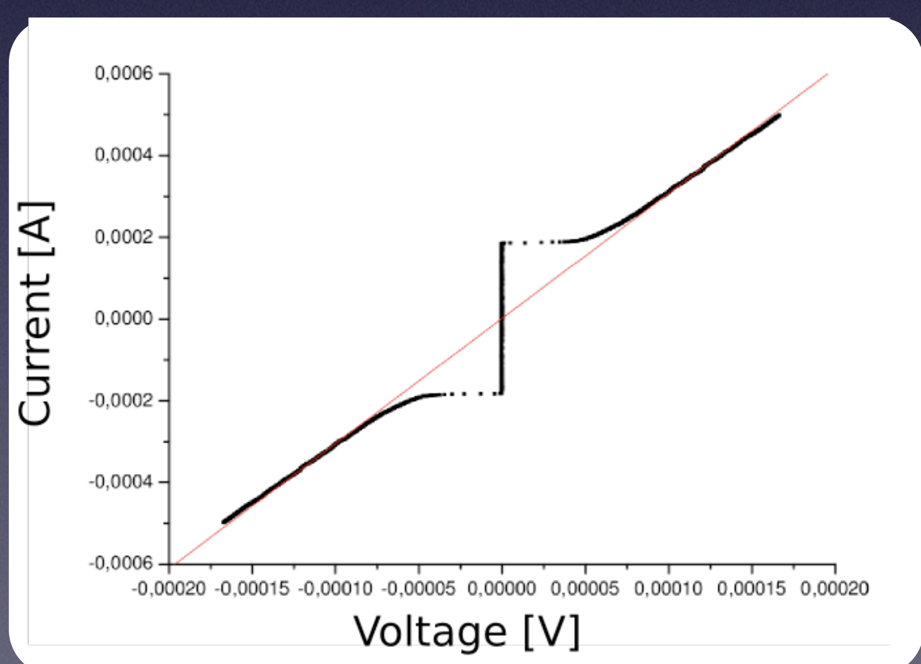
time-correlated but incoherent tunnelling

→ accuracy too large ($\sim 10^{-6}$)



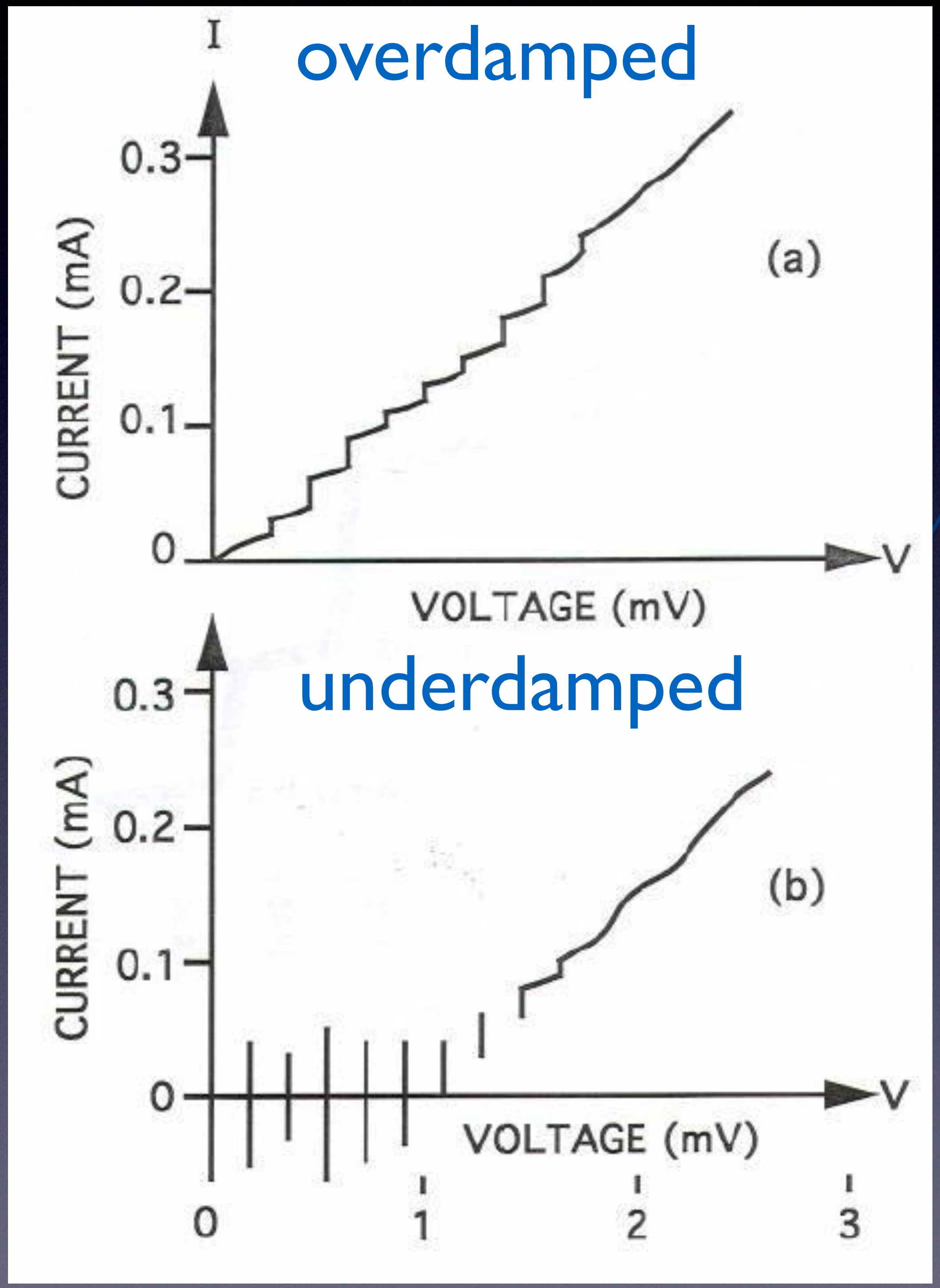
The DC Josephson effect

$$I = I_0 \sin\theta$$



$$V = \frac{\hbar}{2e}\frac{d\theta}{dt}$$

AC Josephson effect



$$V = n\frac{h}{2e}\nu$$

Shapiro Steps

- Josephson junction ac-biased with microwaves

$$\hbar \frac{d}{dt}\theta = 2eV + 2ev\cos(\omega t + \delta)$$

find $\quad \theta(t) = \frac{2eV}{\hbar}t + \frac{2ev}{\hbar\omega}\sin(\omega t + \delta) + \theta_0$

with $I = I_0 \sin\theta$, we get

$$I = I_0 \sum_{n=-\infty}^{+\infty} (-1)^n J_n\left(\frac{2ev}{\hbar\omega}\right) \sin\left[\left(\frac{2eV}{\hbar} - n\omega\right)t - n\delta + \theta_0\right]$$

$J_n(x)$ is the Bessel function of order *n*

extra DC super-current for $2eV = n\hbar\omega$

constant-voltage current step

$$I_n^{\mathrm{step}} = 2I_0\, J_n\left(\frac{2ev}{\hbar\omega}\right)$$

zero voltage super-current suppressed as

$$I_0^{\mathrm{step}} = 2I_0\, J_0\left(\frac{2ev}{\hbar\omega}\right)$$

$E = h\nu = 2eV \longrightarrow 1\ \mu\mathrm{eV} = 484\ \mathrm{MHz}$

Shapiro Steps

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find $\quad \theta(t) = \frac{2eV}{\hbar}t + \frac{2ev}{\hbar\omega}\sin(\omega t + \delta) + \theta_0$

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extra DC super-current for $2eV = n$

constant-voltage current step

$$I_n^{\text{step}} = 2I_0\, J_n\left(\frac{2ev}{\hbar\omega}\right)$$

$E = h\nu = 2eV \longrightarrow$

169 MILLIMETER-WAVE MIXING WITH JOSEPHSON JUNCTIONS 401

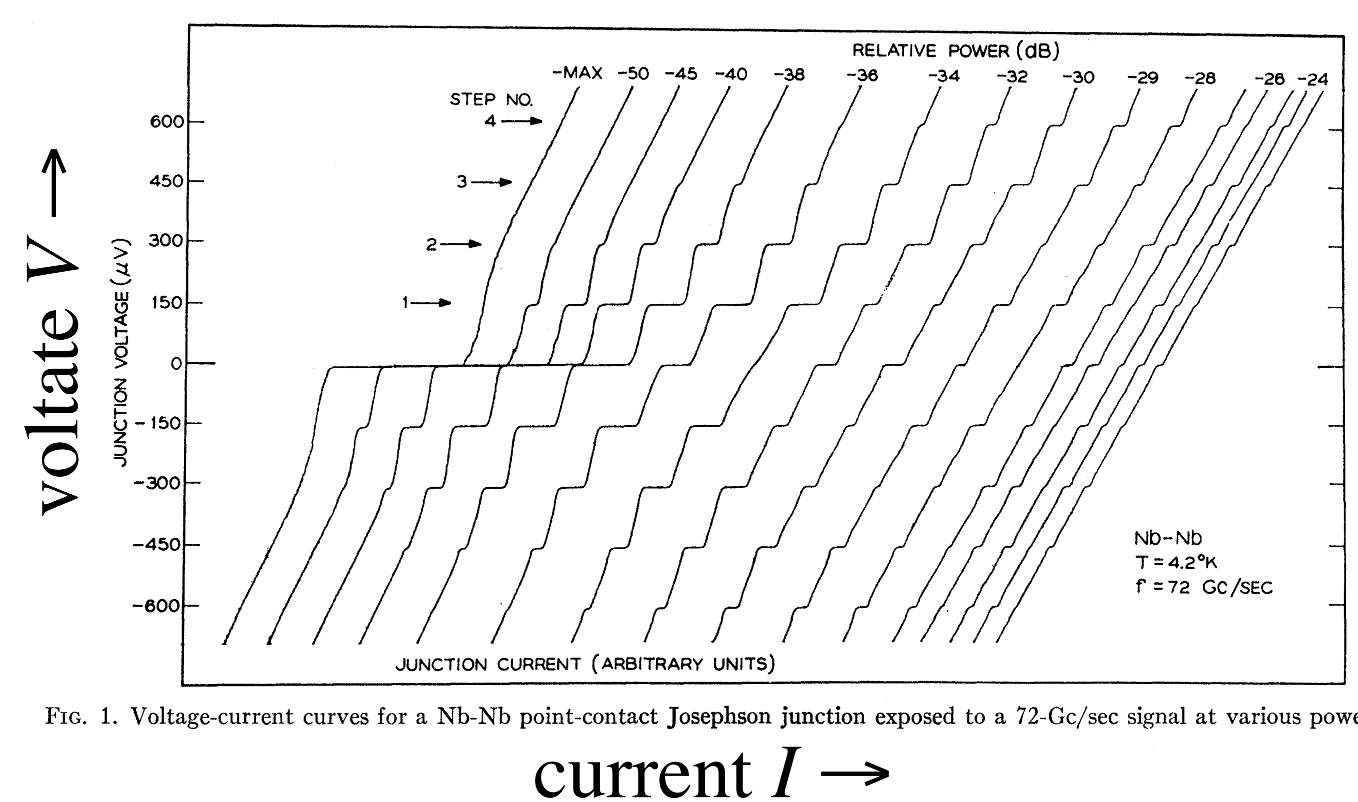
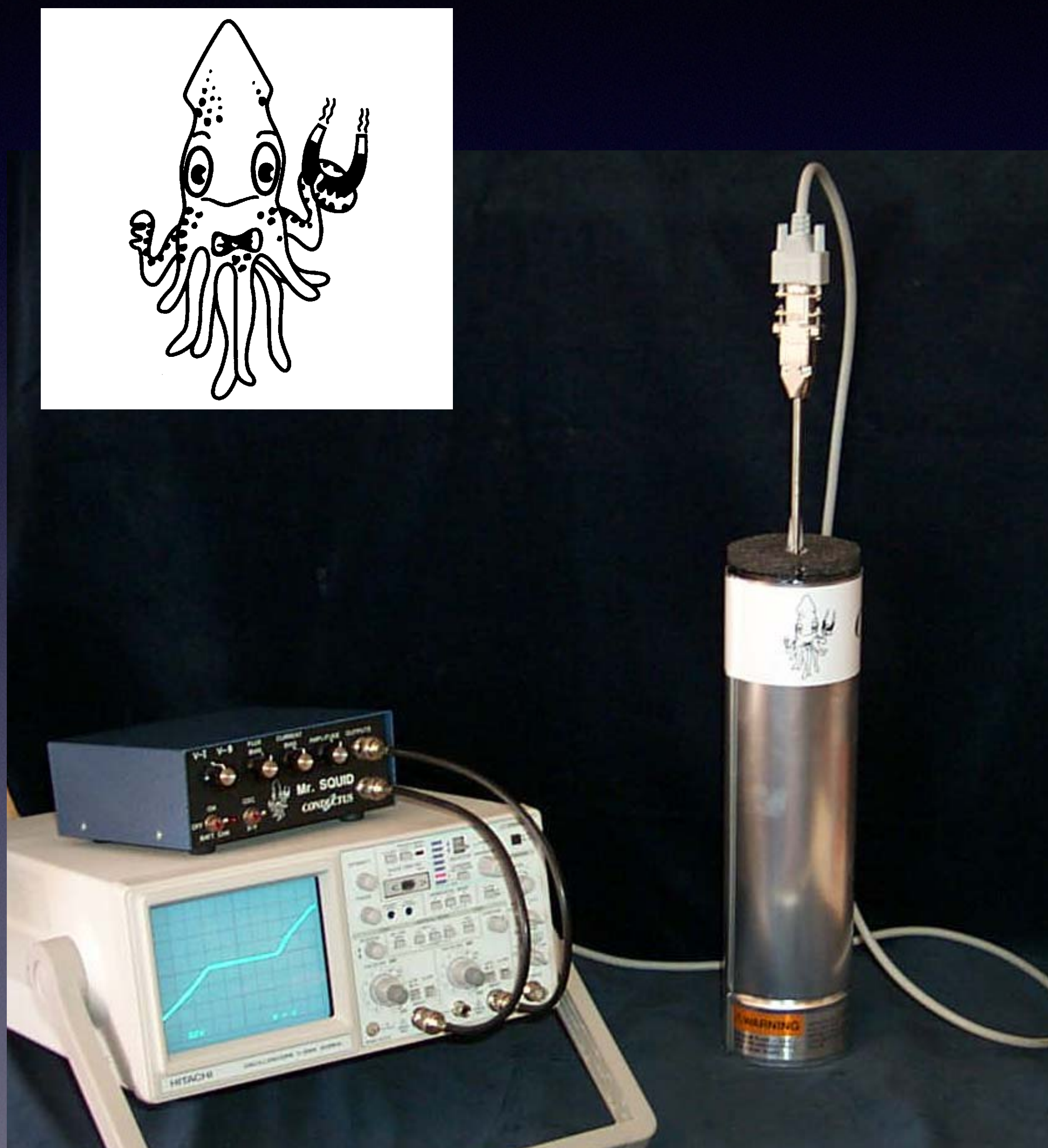
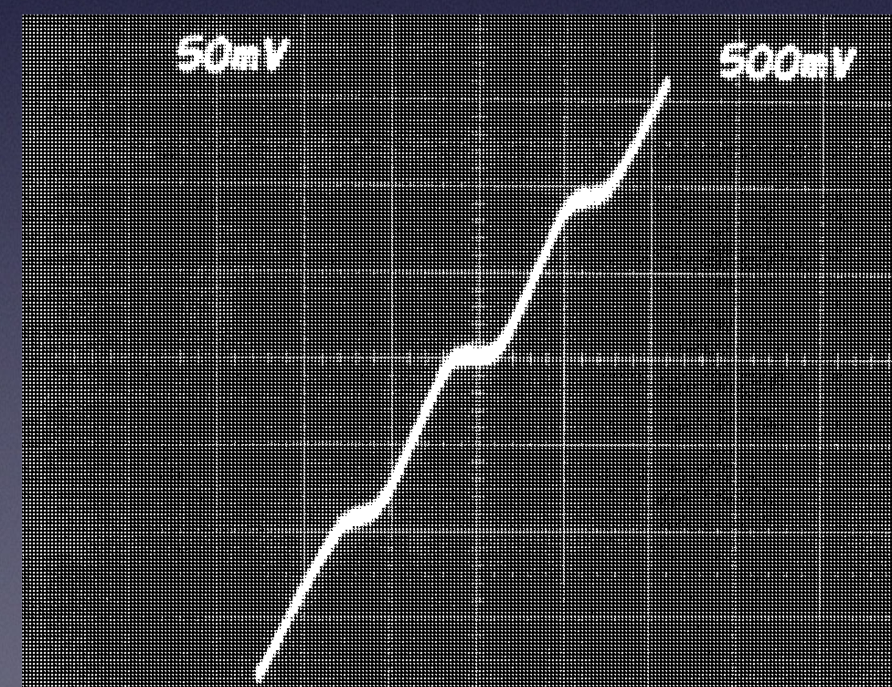


FIG. 1. Voltage-current curves for a Nb-Nb point-contact Josephson junction exposed to a 72-Gc/sec signal at various power levels.

Grimes and Shapiro **Phys. Rev. 169**, 397 (1968)

Mr. SQUID® - A High-Tc Superconductor SQUID System for Undergraduate Laboratories





Can use Mr SQUID to determine *h*/*e* to 0.8% in an undergraduate lab!

The Josephson Voltage Standard

Volume 51, Number 4 PHYSICAL REVIEW LETTERS 25 July 1983

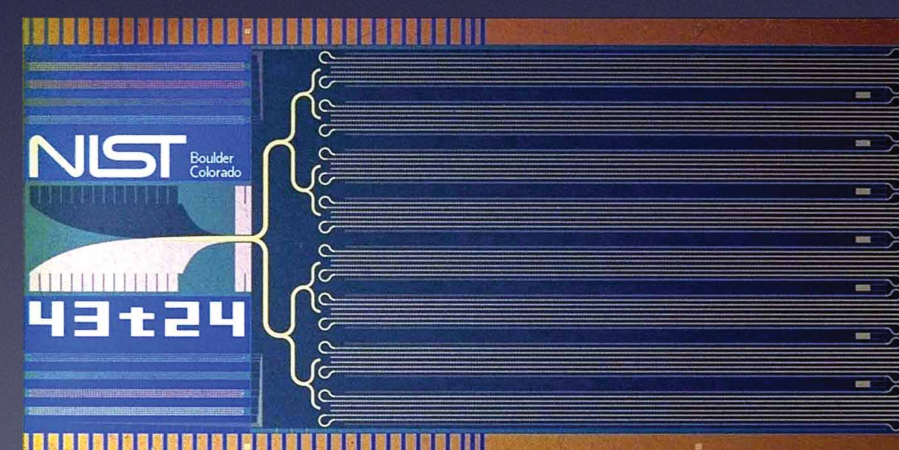
High-Precision Test of the Universality of the Josephson Voltage-Frequency Relation

Jaw-Shen Tsai,[a] A. K. Jain, and J. E. Lukens
Department of Physics, State University of New York at Stony Brook, Stony Brook, New York 11794
(Received 11 May 1983)

The Josephson voltage-frequency relation has been compared between two quite different (and nonideal) types of Josephson junctions—an indium microbridge and a planar normal-metal barrier junction of niobium with a copper normal region. It is found that the constant of proportionality between voltage and frequency is the same in both the junctions to at least 2 parts in 10^{16}.

Voltage-frequency conversion tested for

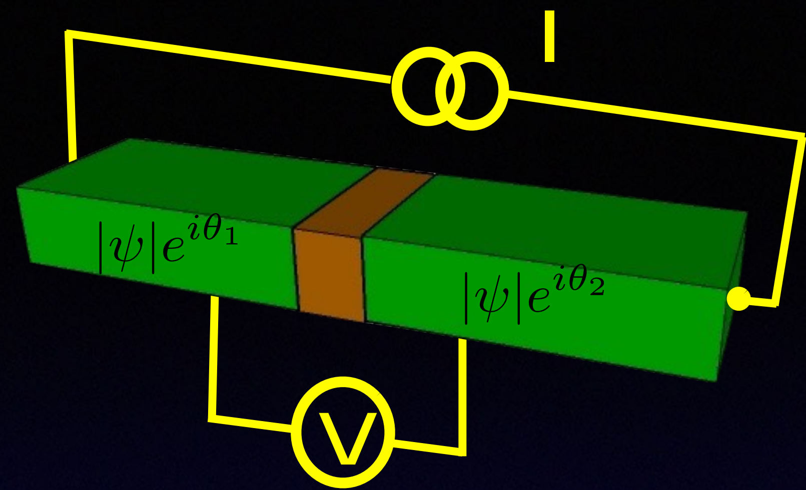
- dissimilar junctions: to 2 parts in 10^{16}
- similar junctions: to 3 parts in 10^{19}
- arrays at 1 V: to 2 parts in 10^{17}



10-volt Array
20,208 Nb-AlO-Nb junctions
NIST, Boulder, 1992
10x20 mm chip

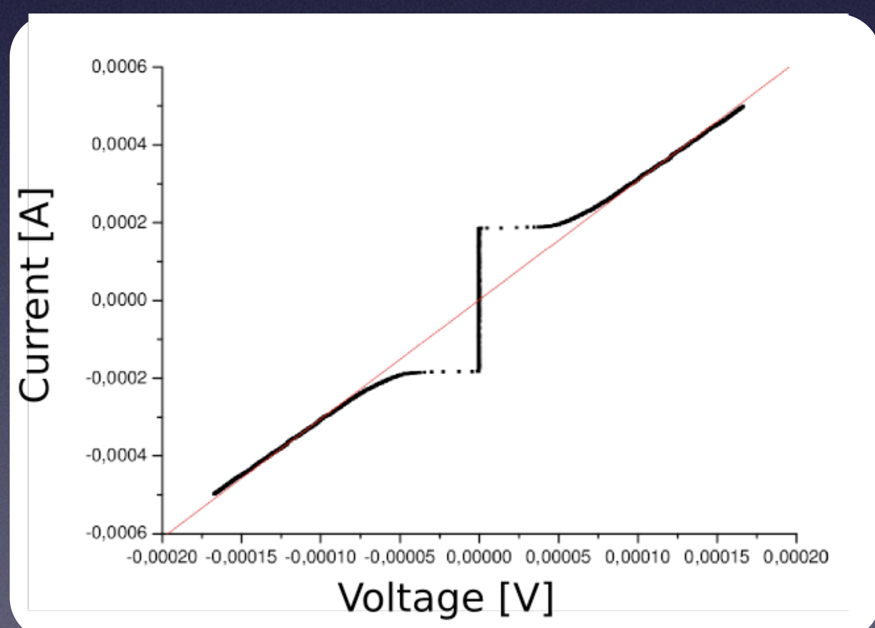
see review by R. L Kautz
Rep. Prog. Phys. **59** (1996) 935.

or "Josephson Voltage Standard" on Wikipedia



The DC Josephson effect

$$I = I_0 \sin\theta$$

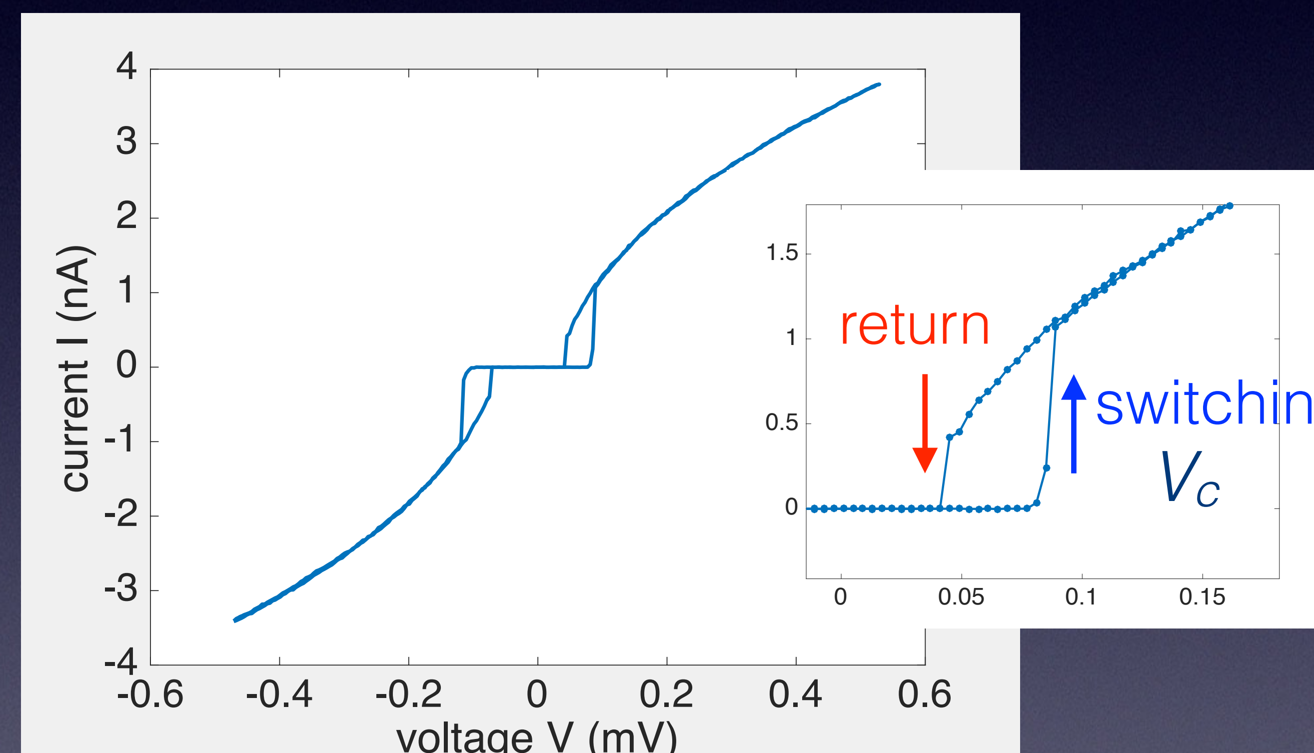


$$V = \frac{\hbar}{2e}\frac{d\theta}{dt}$$

data from UNSW j.j. chain

A Dual Josephson effect?

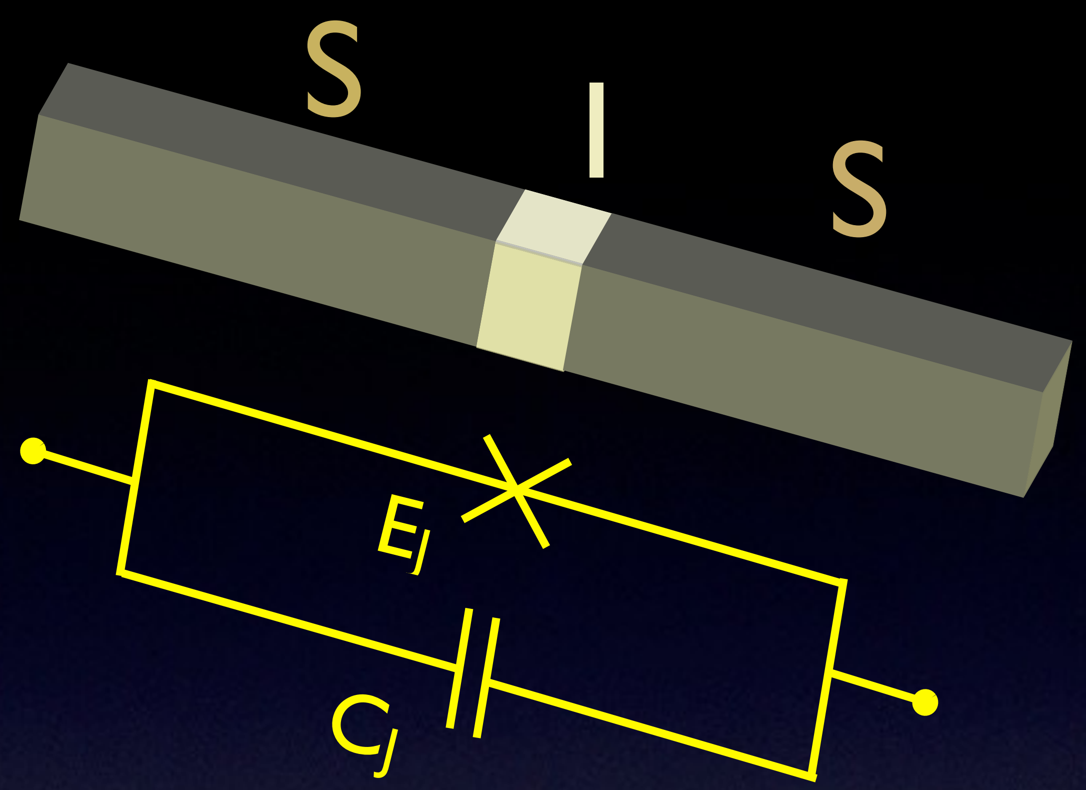
$$V = V_c \sin 2\pi q$$



$$I = 2e\frac{dq}{dt}$$

$$\left[\hat{q}, \hat{\theta}\right] = i$$

QM for the Josephson junction



$$H = E_Q \hat{q}^2 - E_J \cos\hat{\theta}$$

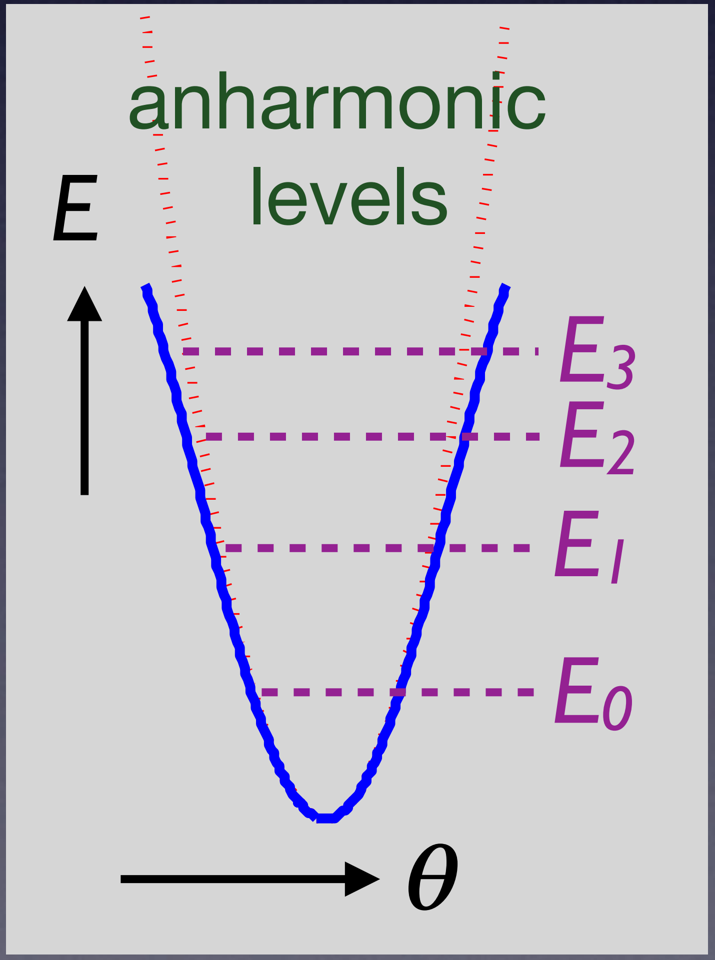
kinetic energy

'washboard' potential

$$\hat{q} = \hat{Q}/2e \qquad \left[\hat{q}, \hat{\theta}\right] = i$$

non-linear inductor when $E_Q \ll E_J$
embedded in a circuit such that
quantum fluctuations of θ are small,
e.g. phase, flux, and 'transmon' qubits,
SQUID magnetometers…

For $E_Q \gg E_J \implies$ single charge regime



Coulomb blockade: Josephson junction with large charging energy

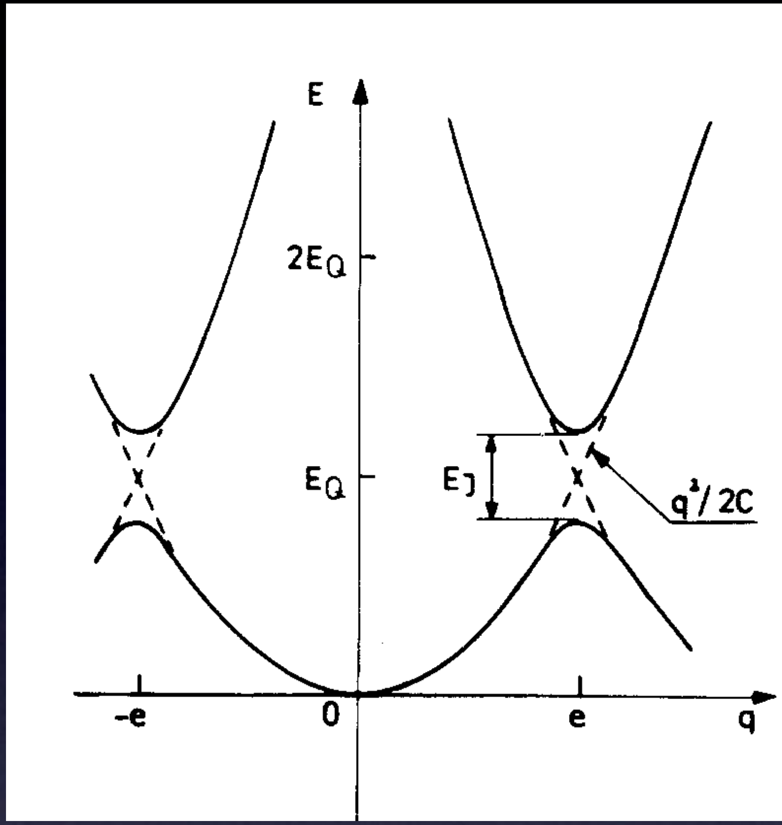
K. K. Likharev and A. B Zorin, JLTP 59, 347 (1984)

- 'θ-particle' in a periodic potential should have delocalised Bloch-wave eigenstates labeled by 'momenta' $k=2eq$

- q is the 'quasi-charge' or charge brought to the junction by the current source

- Can interpret voltage arising from `super-current' of flux quanta passing through the junction (*i.e.* quantum phase slips of order s.c. order parameter)

energy E →



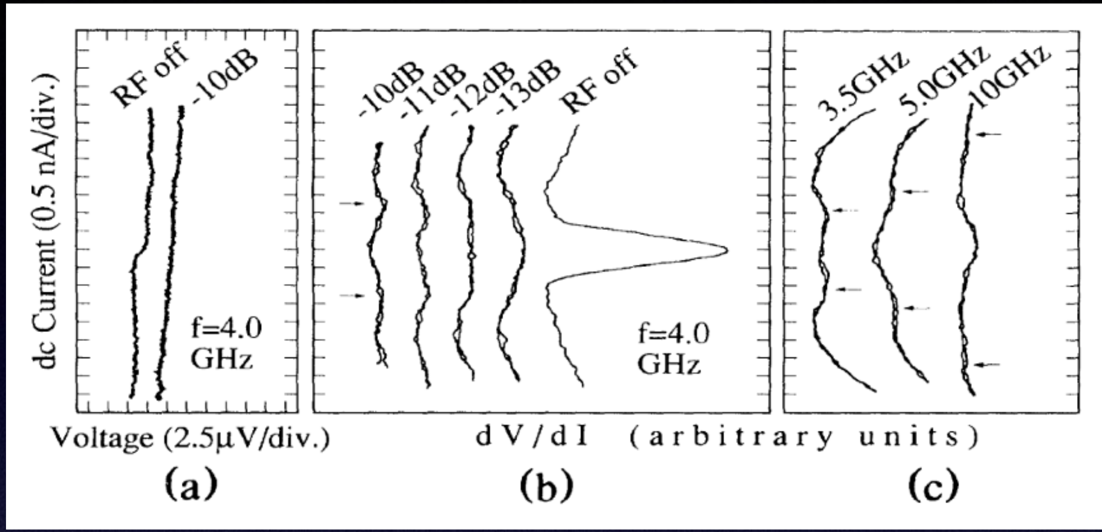
quasicharge q →

- Predicts 'dual' Josephson effects
- Nearly impossible to implement since one needs bias impedance at junction greater than $R_Q = 6.45\ k\Omega$
- Use voltage-biased chains of Josephson-junctions instead:
  junction in the middle see's high impedance from rest of the chain

Attempts to achieve dual Shapiro current steps

- Place large resistors (physically small) near single junction to get current bias with $Z_0 > R_Q$

Phase locking of Bloch oscillations
$I = 2ef$
Kuzmin and Haviland (1991)



- Use 1D Josephson-junction chain
- each junction is effectively current-biased by the rest of the chain

Andersson, Delsing and Haviland
Physica B (2001)
f = 30 MHz

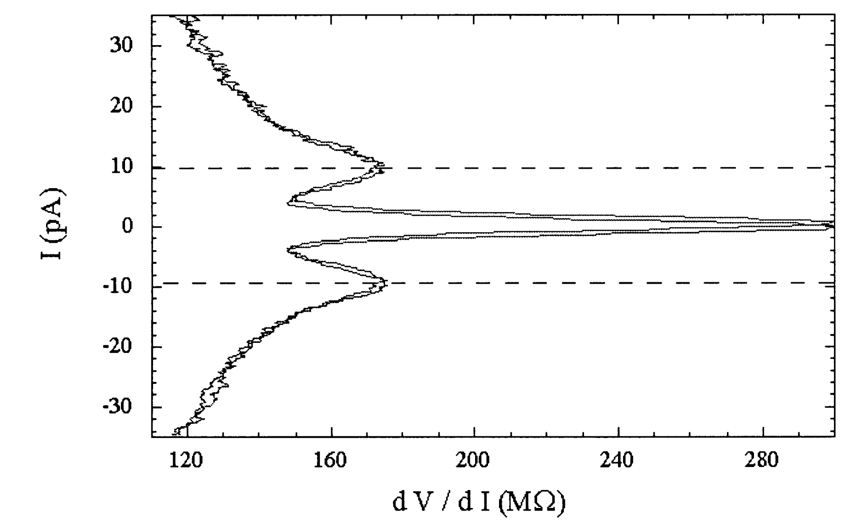
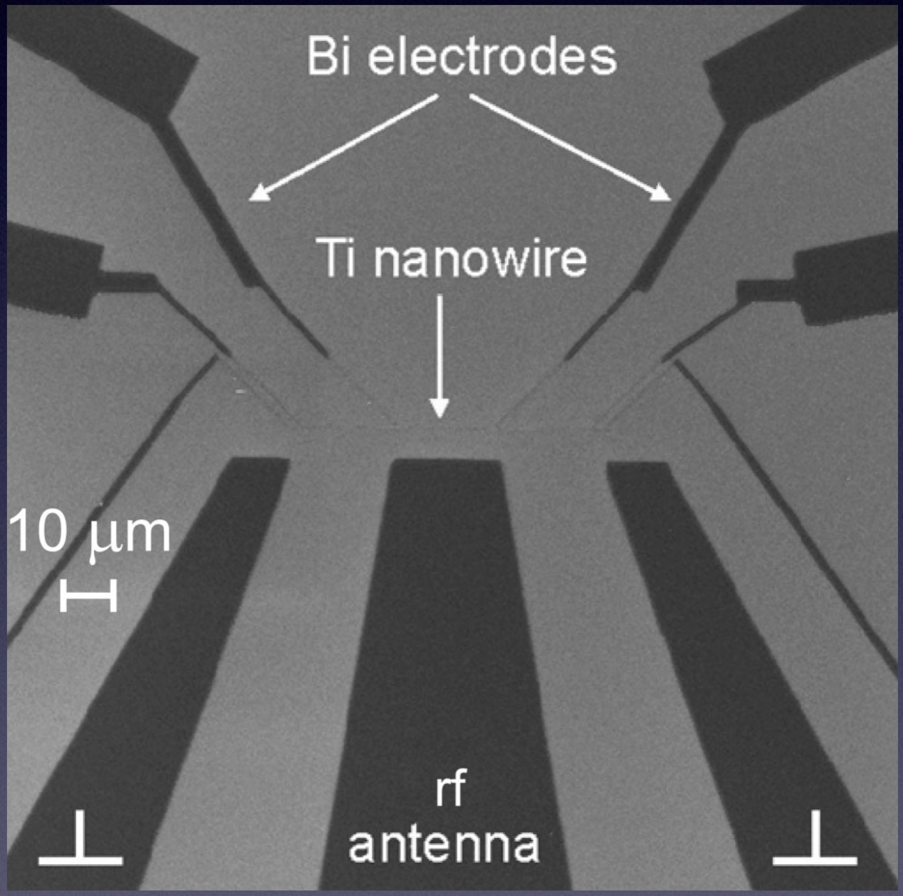


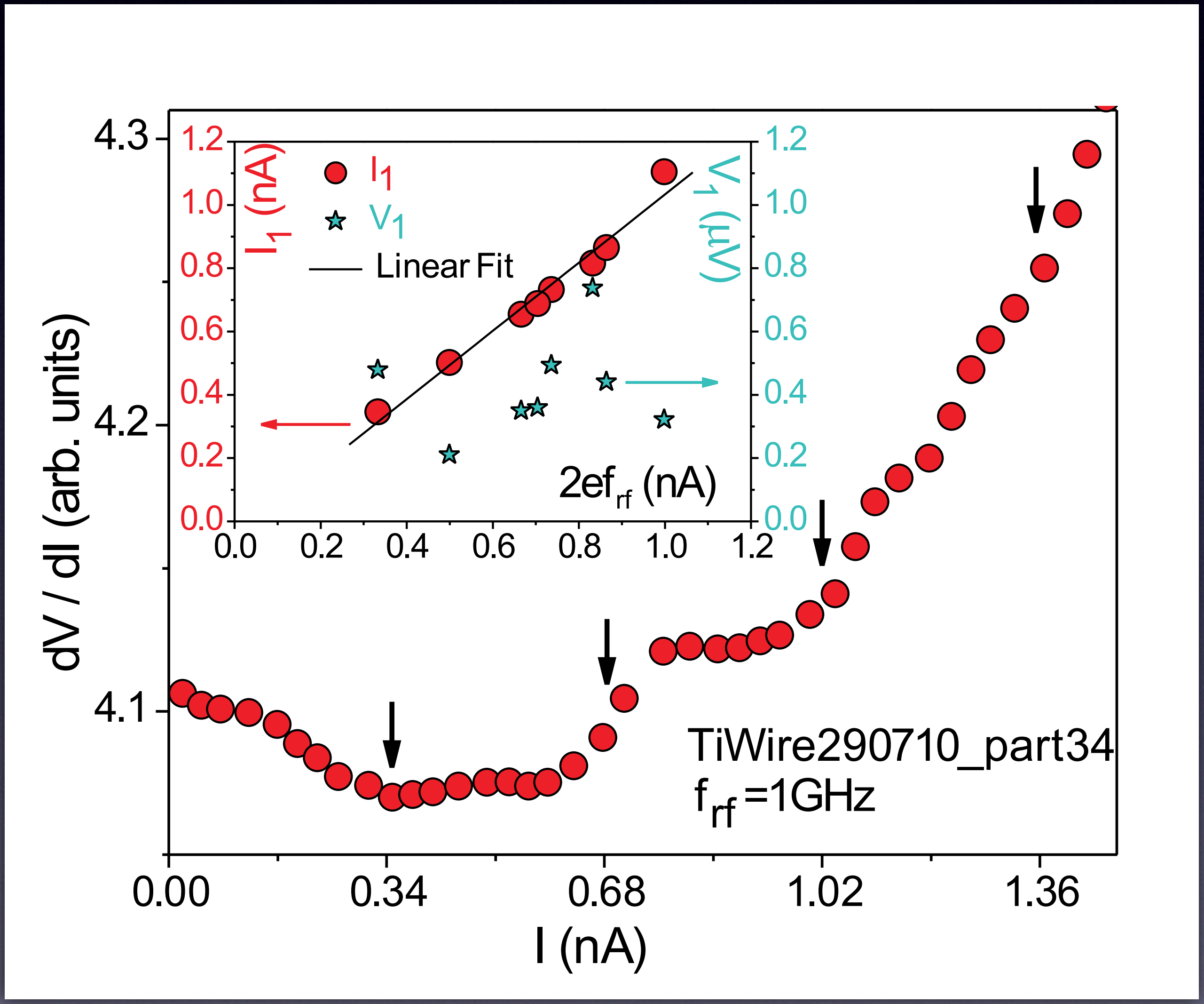
Fig. 2. The frequency of the signal to the gate is in this example 30 MHz and the dotted lines represent the value $I = \pm 2ef = 9.6$ pA. The magnetic field is 42 G.

But to this day….no really nice dual "Shapiro" current steps

Why not??

Using a superconducting nanowire (like a junction chain in the continuum limit)





Lehtinen *et al.* Phys. Rev. Lett. (2012)

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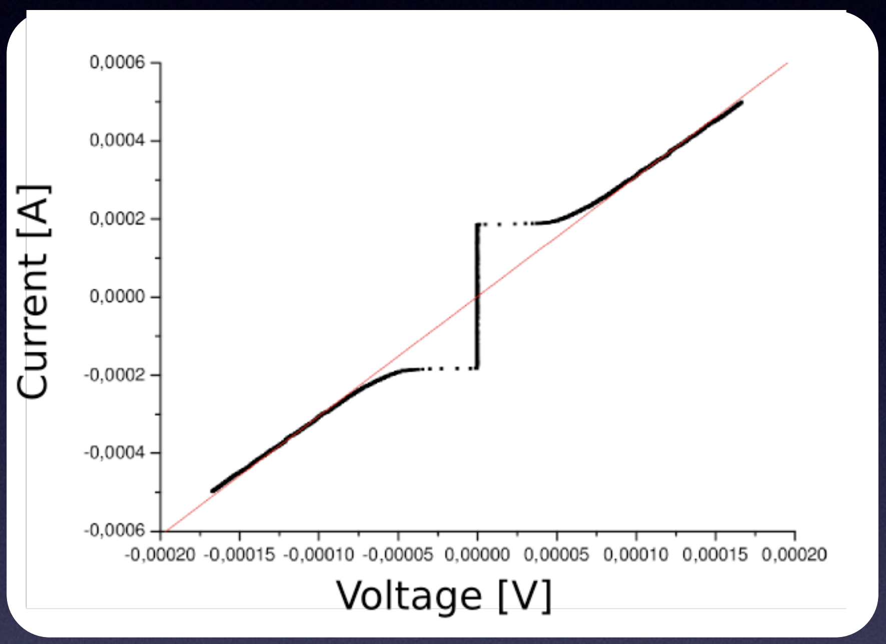
$$I = I_0 \sin\theta$$



$$V = \frac{\hbar}{2e}\frac{d\theta}{dt}$$

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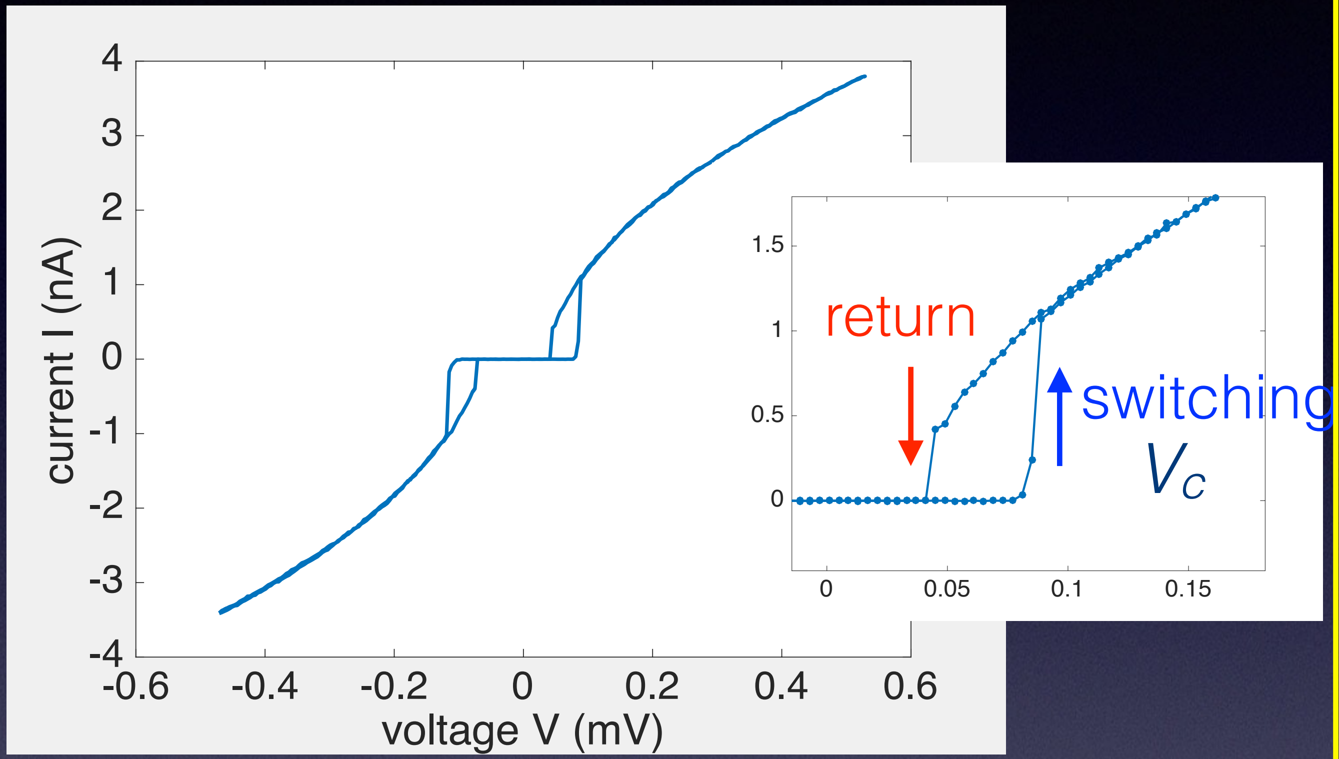


$$I = 2e\frac{dq}{dt}$$

- Looks like a 'dual' Josephson effect, but no one can't get nice synchronised current steps….why not?

MAYBE DO NOT UNDERSTAND WHAT IS REALLY GOING ON!

Arrays of Josephson junctions: quantum phase model

- Array of Josephson junctions is described by the **quantum phase model** (ignores q.p. excitations)

$$H = \sum_{i,j} n_i C^{-1}_{ij} n_j - E_J \sum_{<ij>} \cos\left(\theta_i - \theta_j\right)$$

simplest 1D model has $C^{-1}_{ij} \sim \Lambda E_Q e^{-|i-j|/\Lambda}$

e.m. screening length $\Lambda = \sqrt{C_J/C_0}$

- Can be mapped to **Bose-Hubbard model** with large filling, and interaction over Λ sites: superconductor-insulator **quantum phase transition** expected for

$$E_J \simeq E_0 = \Lambda^2 E_Q$$

- BH model, large <n> is the Quantum Phase Model…
- **Disorder:** has a **Bose Glass** (Anderson-like) insulating phase

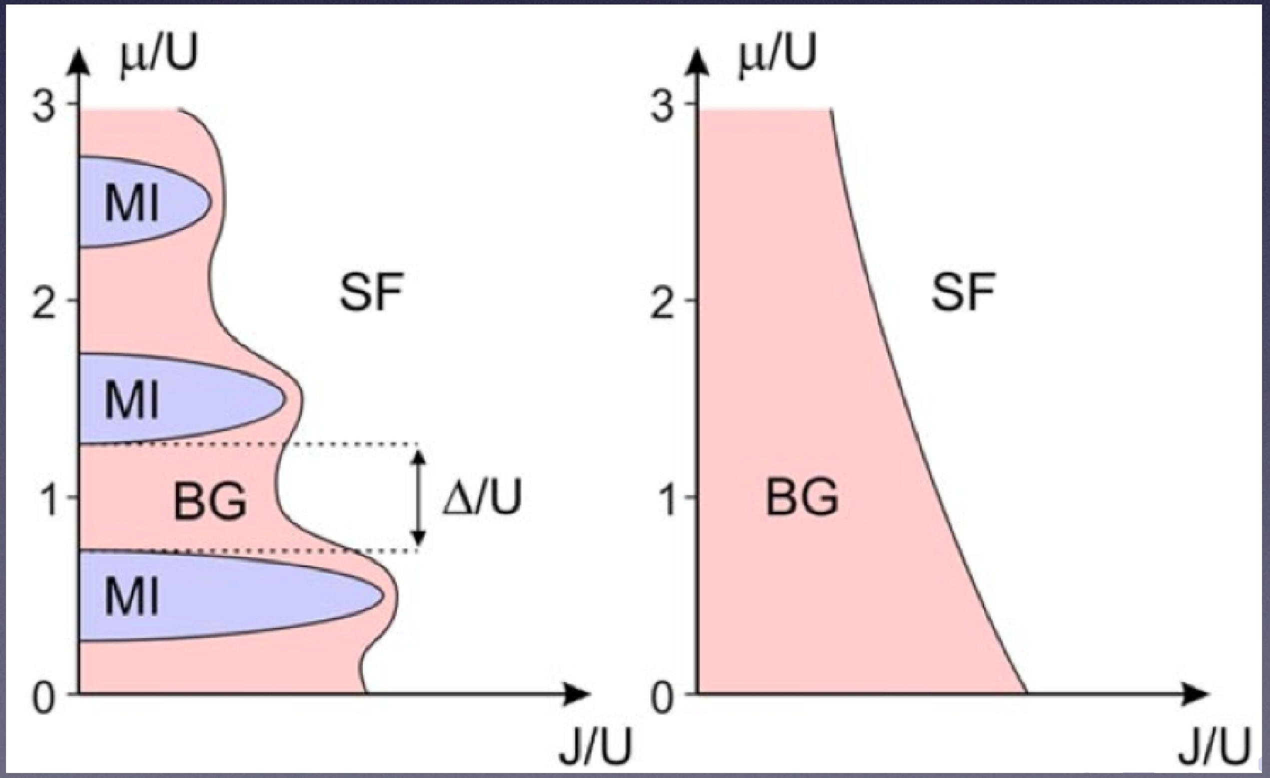
on-site Bose-Hubbard Model with disorder

Fisher, Weichman, Grinstein and Fisher, Phys. Rev. B, **40**, 546 (1989)

$$H = -J \sum_{\langle i,j \rangle} \left(a_i^\dagger a_j + \mathrm{c.c} \right) + \frac{U}{2} \sum_i \hat{n}_i \left(\hat{n}_i - 1 \right) - \sum_i \left(\mu + \delta\mu_i \right) \hat{n}_i$$

'hopping' K.E. — on-site interaction — random chemical potential

- oversimplified but instructive
- $\delta\mu_i$ bounded by $\pm\Delta$
- Mott insulator completely vanishes for large disorder



Quasicharge description for 1D Josephson-junction chains

- use quasicharge instead of island charge
- quasicharge q_i is charge that has **passed through** junction i
- well familiar from theory of current-biased small junction (Likharev and Zorin 1985)
- Arrive at bosonic form of Luttinger liquid (charge mode) with backscattering interactions

$$\mathcal{L} = \frac{1}{2\pi K} \sum_i [\frac{\dot{q}_i^2}{v} - v\,(q_i - q_{i+1})^2 + \epsilon_0(q_i + f_i)]$$

$$K = \pi\sqrt{\frac{E_\mathrm{J}}{2E_0}} = \frac{\pi}{\Lambda}\sqrt{\frac{E_\mathrm{J}}{2E_Q}} \qquad v = \frac{2}{h}\sqrt{E_0 E_J}$$

offset charge disorder

- for large E_J/E_Q , $\epsilon_0(q) \simeq E_S \cos(q)$
  - → describes a sine-Gordon model (in the continuum limit)

Bosonization for Bosons

Effective Harmonic-Fluid Approach to Low-Energy Properties of One-Dimensional Quantum Fluids

F. D. M. Haldane
Department of Physics, University of Southern California, Los Angeles, California 90007,[a] *and Institut Laue-Langevin, F-38042 Grenoble-Cedex, France*
(Received 29 December 1980)

A universal description of the low-energy properties of one-dimensional quantum fluids, based on a harmonic theory of long-wavelength density fluctuations with use of renormalized parameters, is outlined. The structure of long-distance correlations of a spinless fluid is obtained, showing the essential similarity of one-dimensional Bose and Fermi fluids. The results are illustrated by application to the one-dimensional Bose fluid with δ-function interaction.

See book by T. Giamarchi, *Quantum Physics in One Dimension*

1D Josephson-junction chains as Luttinger Liquids

- Quantum 1D systems: 'Luttinger Liquids' specified by LL parameter K ($K \rightarrow 0$ with K/h fixed, classical density ordering)
- 'Phase slip' interactions (Coulomb blockade) leads to a Berezinskii-Kosterlitz-Thouless QPT at K_C=2 at SF-MI, K_C=3/2 SF-BG QPT's
- For JJ-chains

$$K = \pi\sqrt{\frac{E_{\mathrm{J}}}{2E_0}} = \frac{\pi}{\Lambda}\sqrt{\frac{E_{\mathrm{J}}}{2E_Q}}$$

But Chow, Delsing and Haviland (1998) observed SIT in SQUID chains at much lower K_c = 0.1 !

Can K_c really be so strongly renomalized?

Theory seems to rule this out! (Choi, Choi, Choi and Lee, 1998)

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$$K = \pi\sqrt{\frac{E_{\rm J}}{2E_0}} =$$

also $K = \frac{\pi}{\sqrt{2}}\frac{R_Q}{Z_{TL}}$

$$Z_{TL}^{\rm crit} = 1.1\,(1.5)\,R_Q$$

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- Summary: two big mysteries, one fundamental, one applied…..

Are these related? *i.e.* what is the connection (if any) between Luttinger liquids, the Bose glass and dual Josephson effect?

2016 Nobel Prize in Physics awarded to Thouless, Haldane, and Kosterlitz

"for theoretical discoveries of topological phase transitions and topological phases of matter"

Scientific Background on the Nobel Prize in Physics 2016

TOPOLOGICAL PHASE TRANSITIONS AND TOPOLOGICAL PHASES OF MATTER

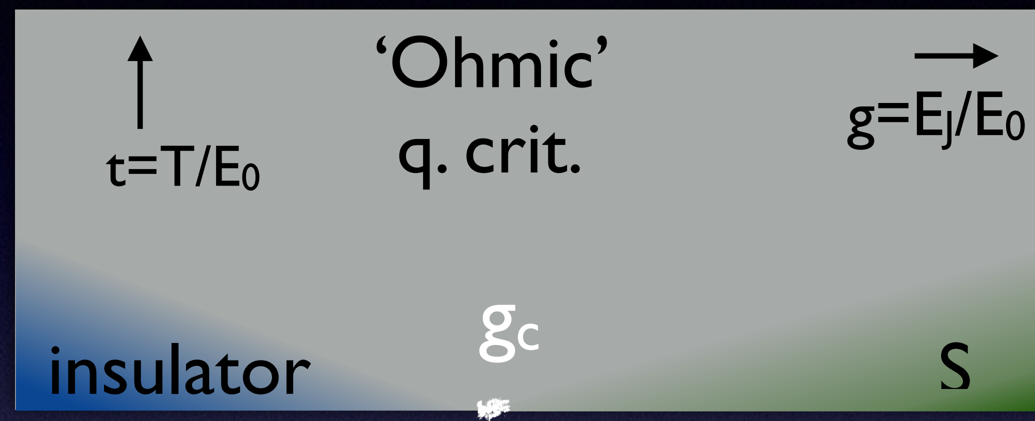
compiled by the Class for Physics of the Royal Swedish Academy of Sciences

6.2 Quantum simulations, and artificial states of matter

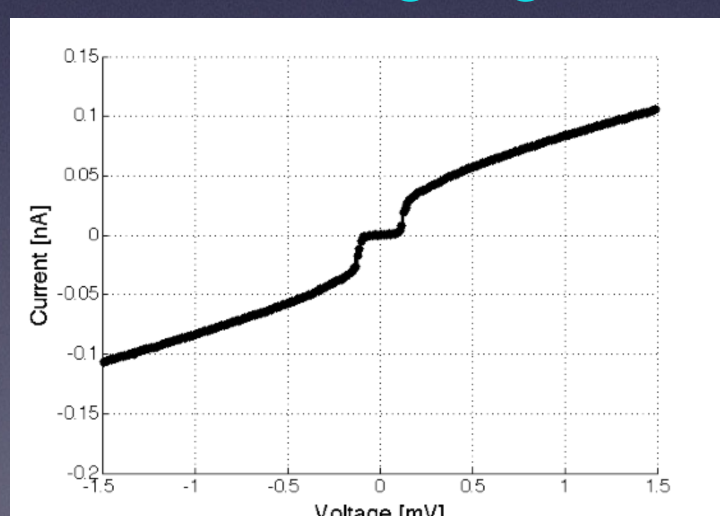
… the KT-transition forms the basis for understanding how a one-dimensional chain of Josephson tunnel junctions undergoes a zero-temperature transition from superconducting to insulating behaviour as the Josephson coupling between junctions is tuned [25].

Superconductor-insulator transition in JJ arrays

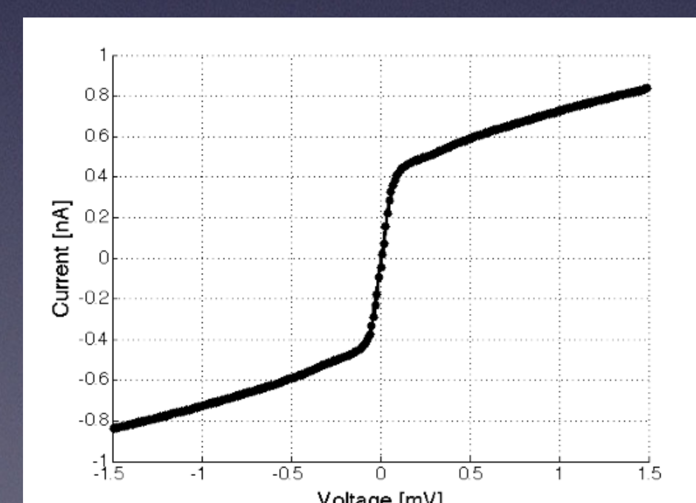
related to Mott-SF transition in optical lattices,
but can be probed using electrical measurements



Insulating regime

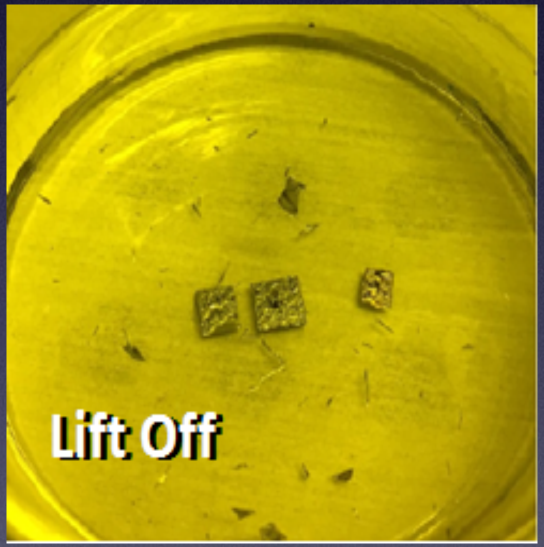


Supercurrent regime

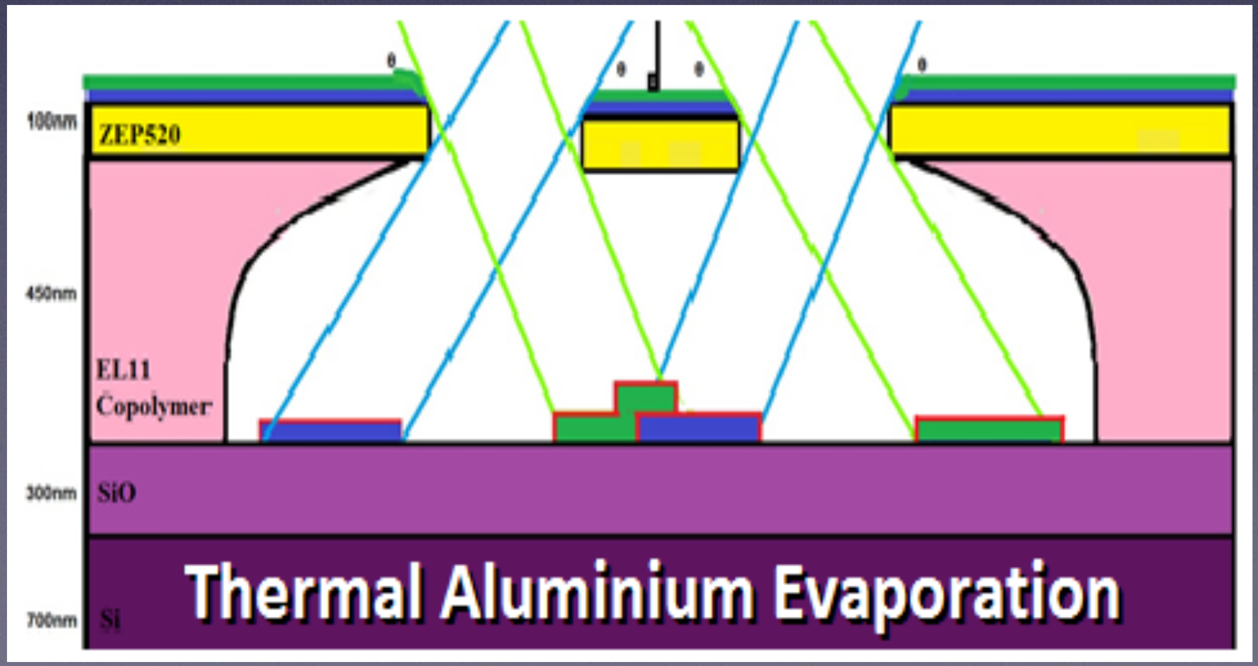


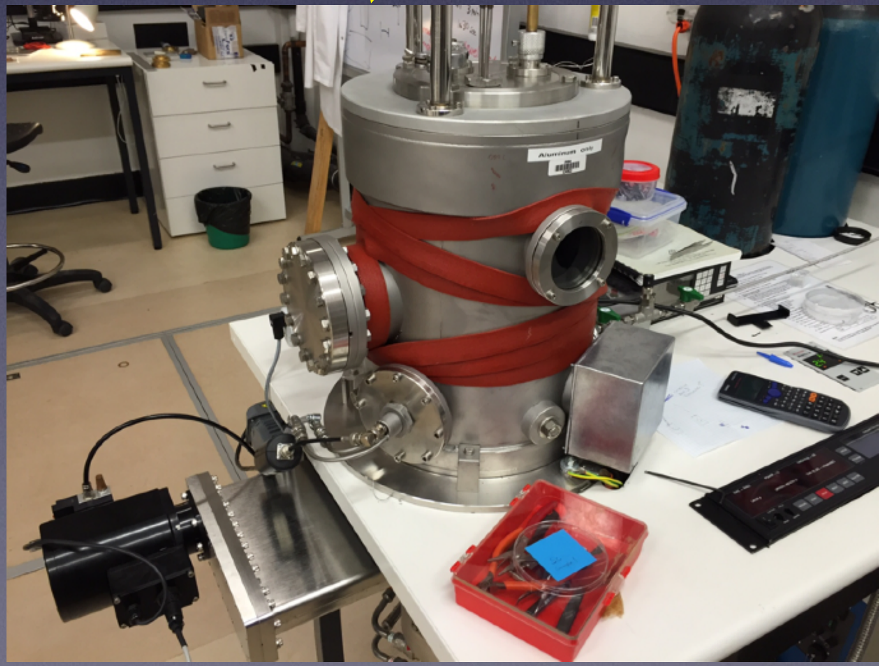
Not so simple, however….noise and finite-size effects complicate interpretation of data

‘Low budget’ fabrication of Josephson-junction devices



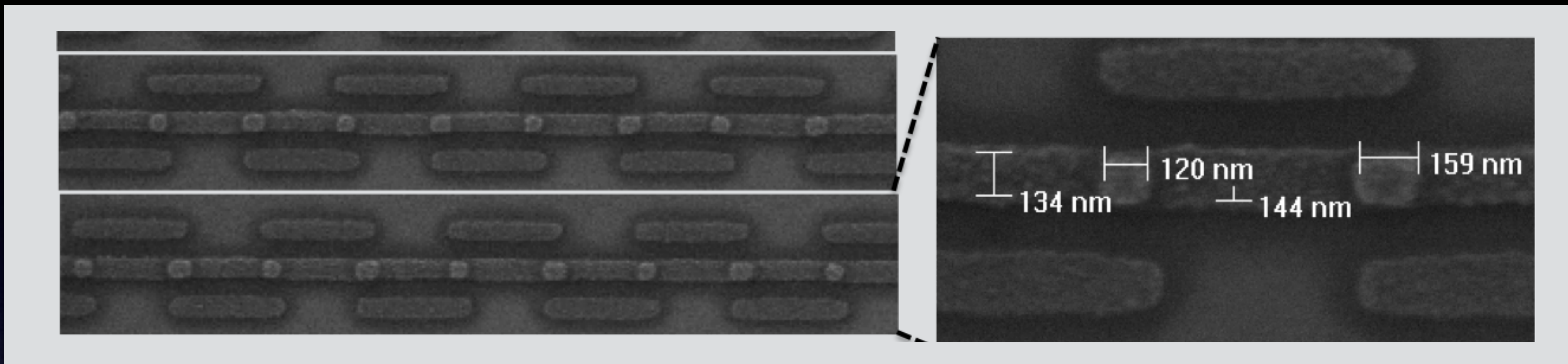
junction and SQUID-chains
up to 20,000 junctions

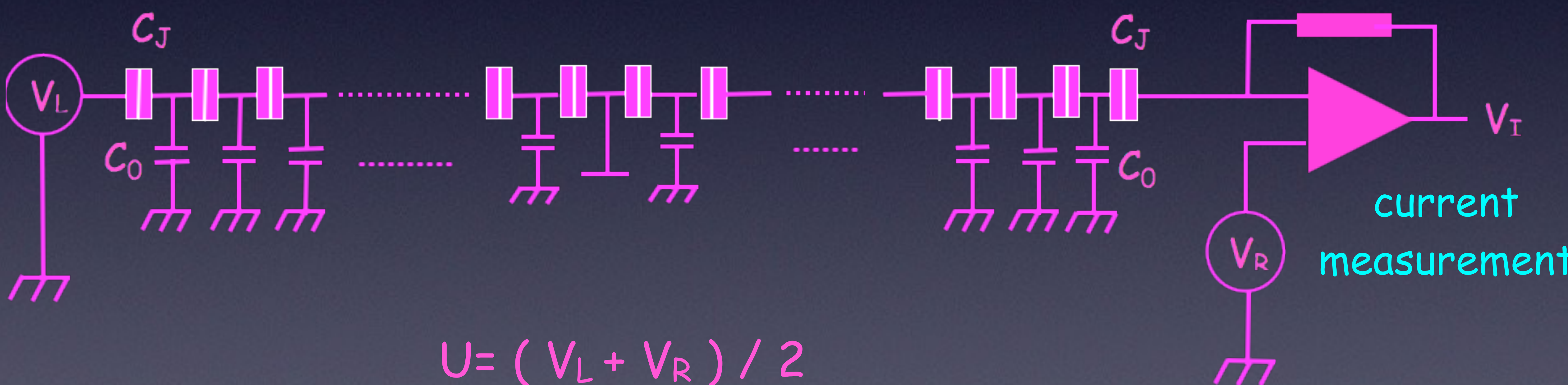




1D junction chain experiments at UNSW

devices with 25-5000 junctions





$$U = (V_L + V_R) / 2$$

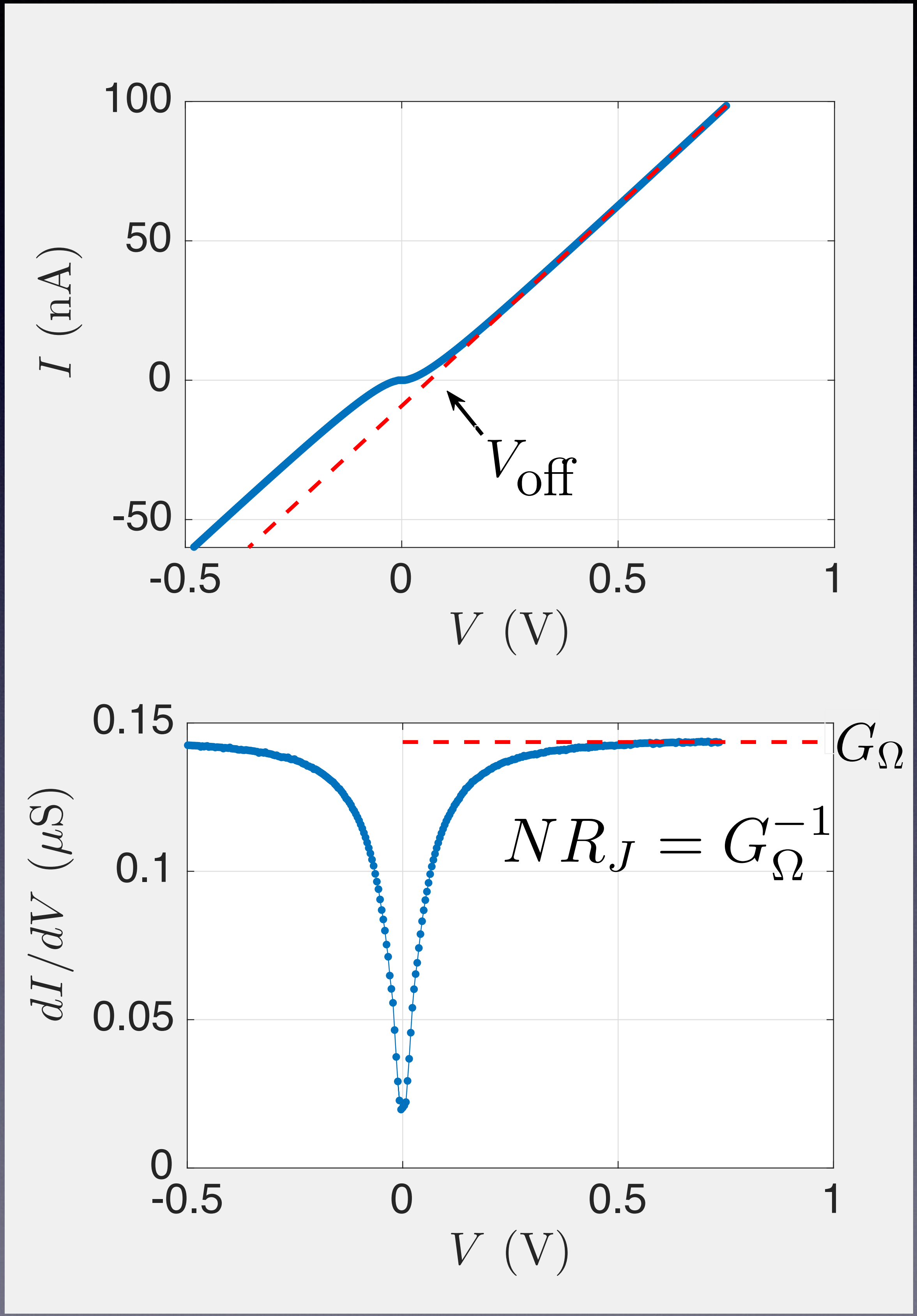
global 'gate' potential U --> 'chemical potenial'

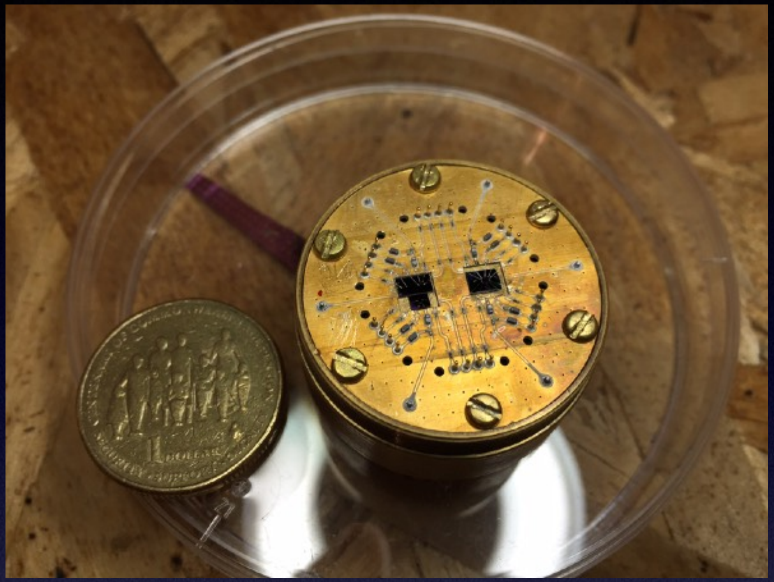
$$V = (V_L - V_R)$$

bias potential V --> chain tilt

Experimental determination of E_Q and E_J from large-scale IVC's

Cedergren, Kafanov, Smirr, Cole and Duty
Phys. Rev. B 92, 104513 (2015)





measurements down to 11mK

charging energy

$$eV_{\mathrm{off}}/N = \frac{e^2}{2C_J}$$
$$= E_C = E_Q/4$$

Josephson energy

$$E_J = \frac{\Delta}{2}\frac{R_Q}{R_J}$$

Experimental determination of E_Q and E_J from large-scale IVC's

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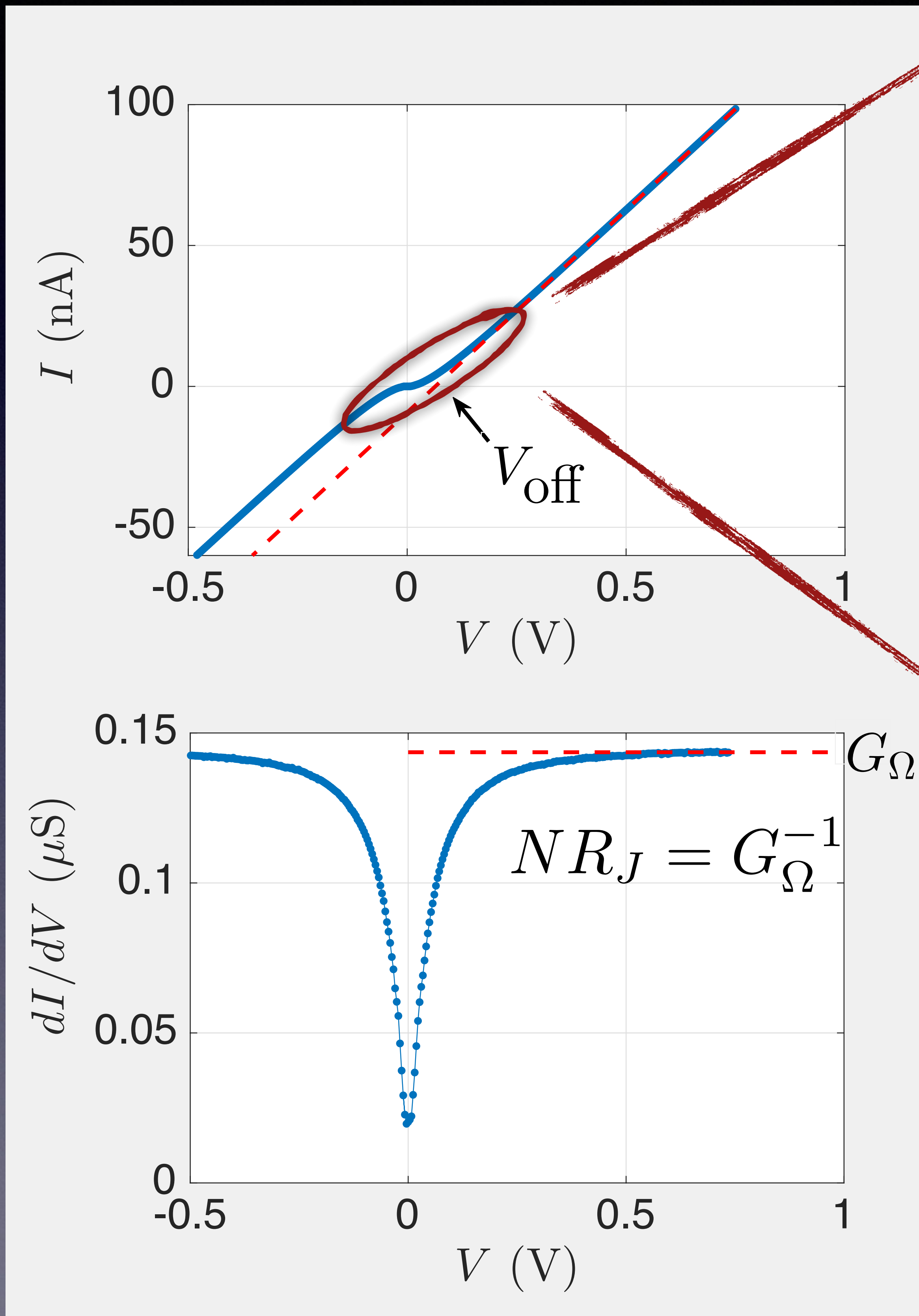
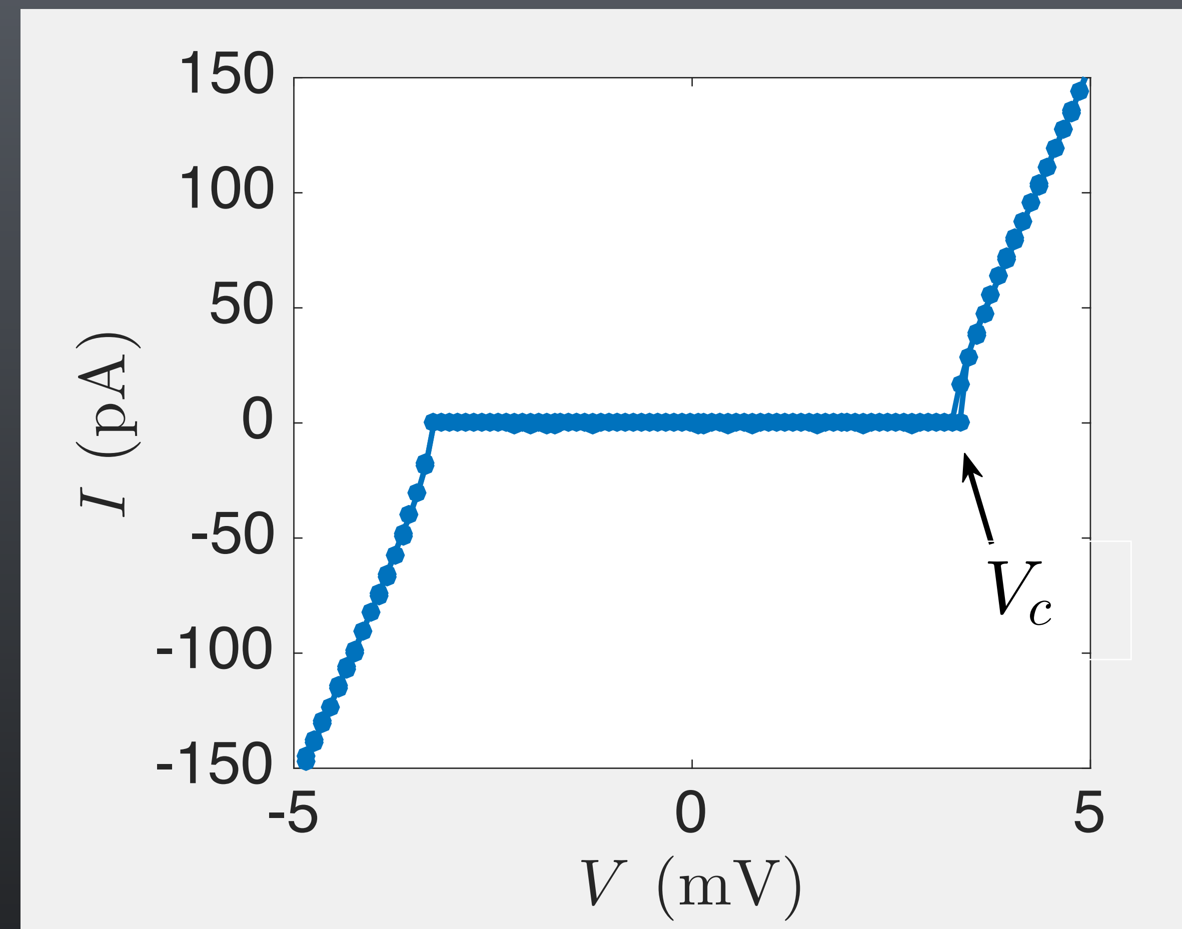
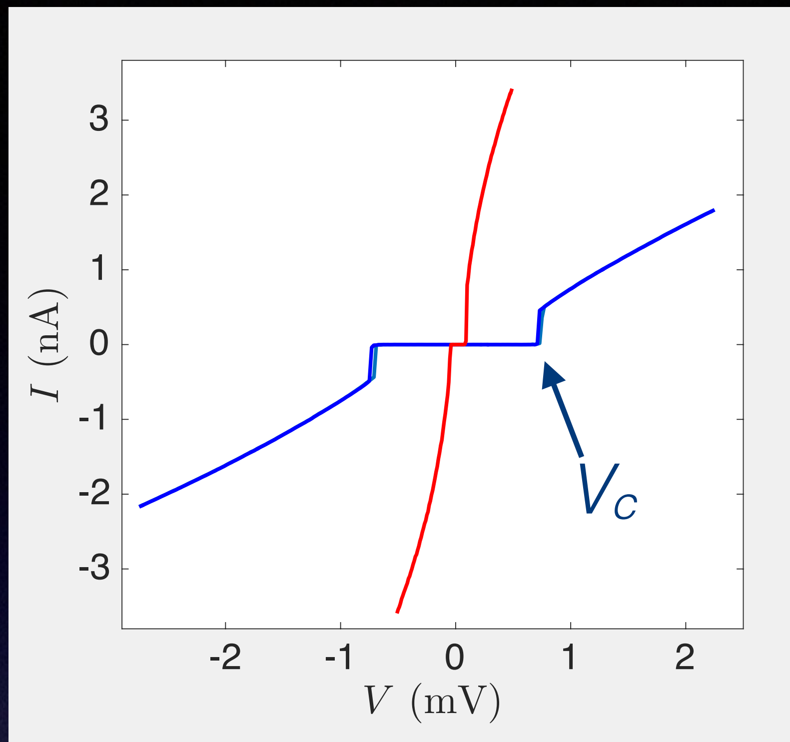




Now look at dependence of V_c on E_J and E_Q

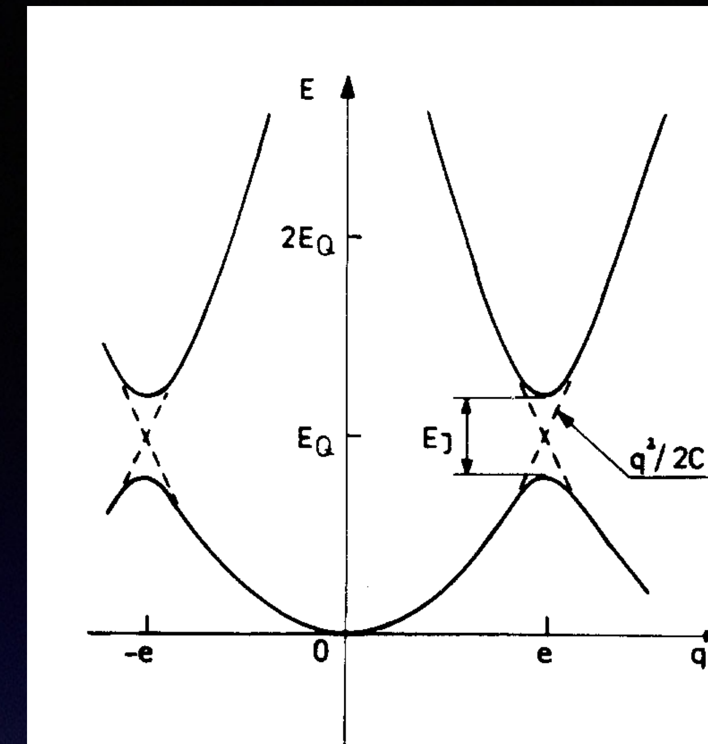
(or equivalently g and ω_p)

Dual Josephson effect in a chain of junctions



Bloch bandwidth W

energy →



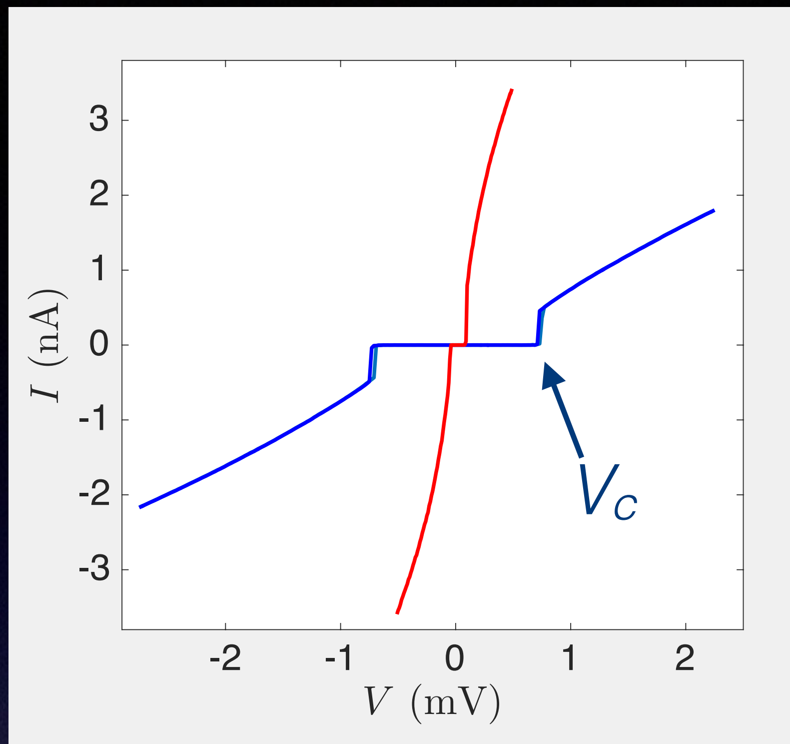
quasicharge q →

V_c

- Need a quantitative and experimental understanding of V_c
- 'rigid' quasicharge model $eV_c/N = \max\left|\dfrac{d\epsilon_0(q)}{dq}\right|$
- For large g ($E_J \gtrsim E_Q$), band is sinusoidal, so $eV_c/N = \pi W$

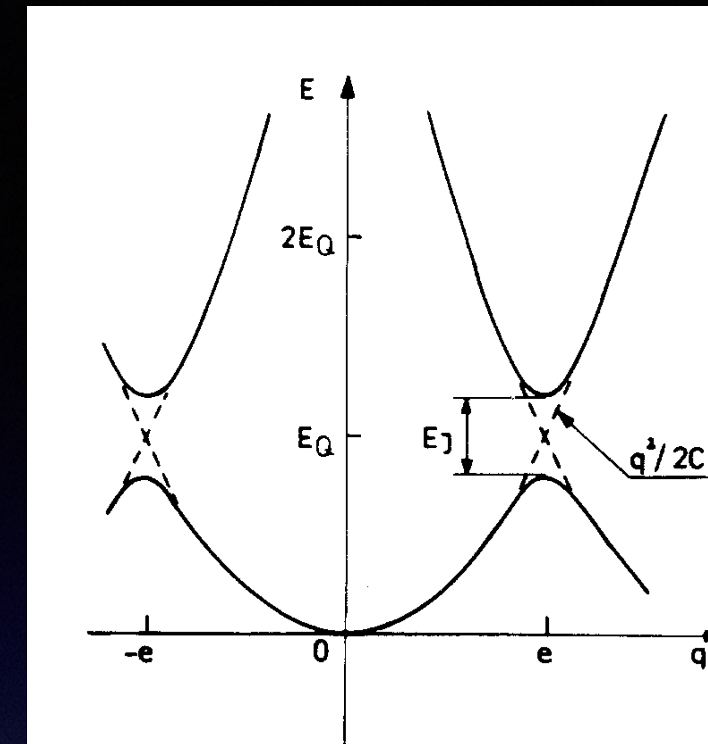
$$W \simeq 16\sqrt{\frac{E_J E_Q}{4\pi}}\,(2g)^{1/4}\,e^{-\sqrt{32g}}, \quad g = E_J/E_Q$$

Dual Josephson effect in a chain of junctions



Bloch
bandwidth W

energy →



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$$W \simeq 16\sqrt{\frac{E_J E_Q}{4\pi}}\,(2g)^{1/4}\,e^{-\sqrt{32g}}, \quad g = E_J/E_Q$$

- define scaled (dimensionless) v and w

plasma frequency
$\omega_p = \sqrt{2E_Q E_J}$

$$\boxed{\begin{aligned} v &= eV_c/N\hbar\omega_p \\ w &= W/\hbar\omega_p \end{aligned}} \rightarrow \boxed{v = Aw^{\alpha}}$$

with $A = \pi/2$
$\alpha = 1$

Disorder and pinning of quasi charge

$$H = \int dx \left\{ \frac{(2e)^2}{2C_0} \left[\partial_x q(x)\right]^2 + \epsilon_0 \left[q(x) + f(x)\right] \right\}$$

elastic energy ***E₀***

Bloch band

random offset charges

Vogt *et al.* Phys. Rev. B, 2015

- Maximal offset charge disorder ➞ equivalent to (classical) pinning of elastic system by random impurities, e.g. pinning of vortices in Type II s.c., charge density waves, …
- Larkin (Fukuyama-Lee, Imry-Ma) length

length scale over which object is rigid

$$N_L = 3^{-\frac{2}{3}} \Lambda^{\frac{4}{3}} W^{-\frac{2}{3}}$$

depinning "force" (voltage…critical current for vortex pinning)

$$eV_c \sim \frac{E_0}{N_L^2}$$

E_0 - *elastic energy scale*

$$v = Aw^{\alpha}$$

with $\alpha = 4/3$

$$A = \frac{1}{2} 3^{4/3} \Lambda^{-2/3}$$

- Classical limit of 1D Luttinger system with $K \to 0$

Tuning *w* (*via* $g = E_J/E_Q$) without using a SQUID configuration

Josephson energy, $E_J \propto \text{jcn area}$

charging energy, $E_Q \propto 1/\text{jcn area}$

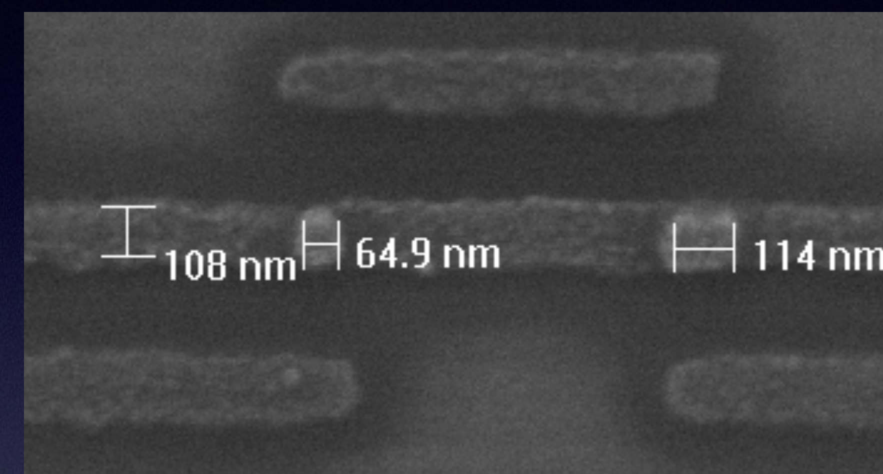
$$g = E_J/E_Q \sim (\text{jcn area})^2$$

$$\hbar\omega_p = \sqrt{2E_J E_Q}$$

independent of area

Dose: [μC/cm²]

1000
1100
1200
1300
1400
1500
1600

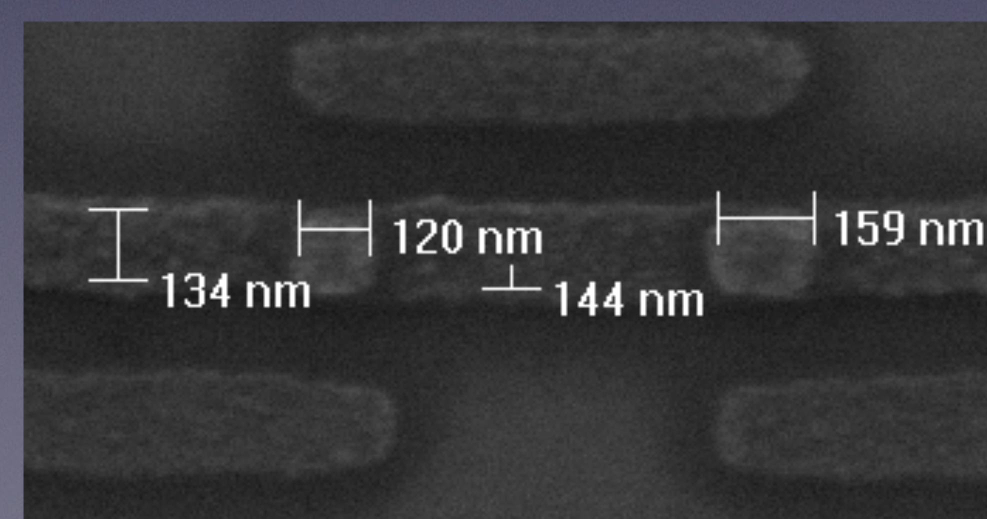


Nominal junction area: 100x100nm

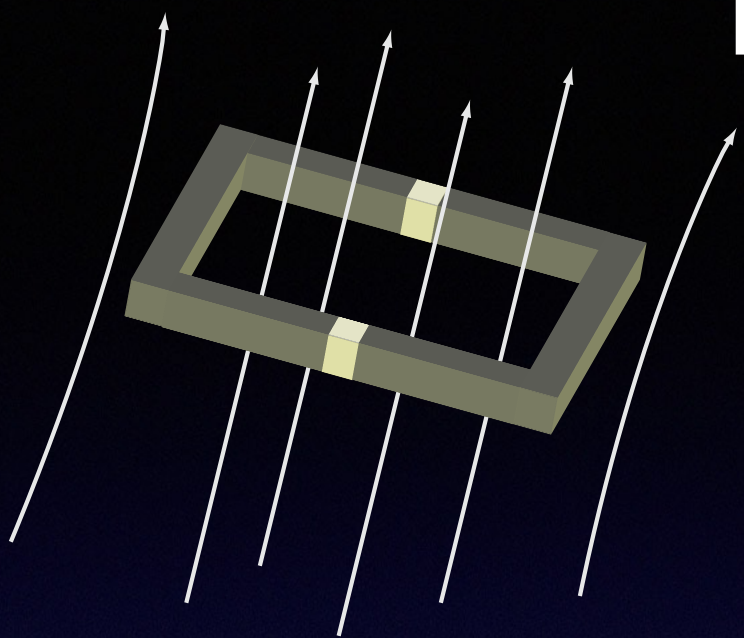
Oxidation:
30s, 0.044 mbar O_2

Vary E_J/E_Q by changing jcn area *via* dose

Vary plasma frequency *via* oxidation



The SQUID

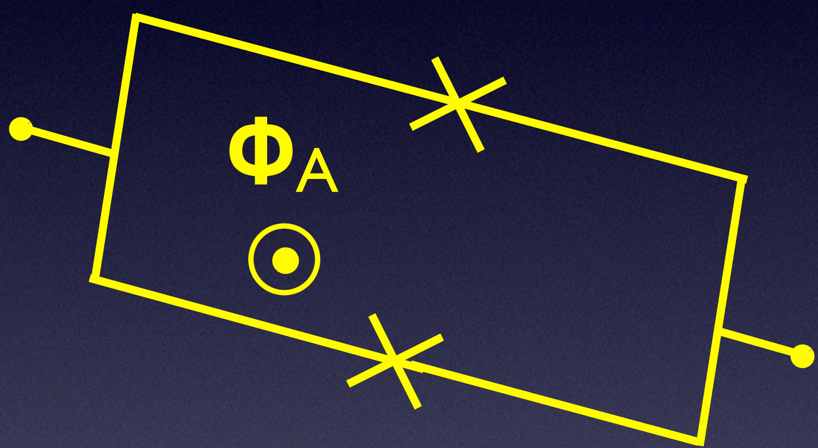


A low (geometrical) inductance SQUID acts like a flux-tunable single Josephson junction

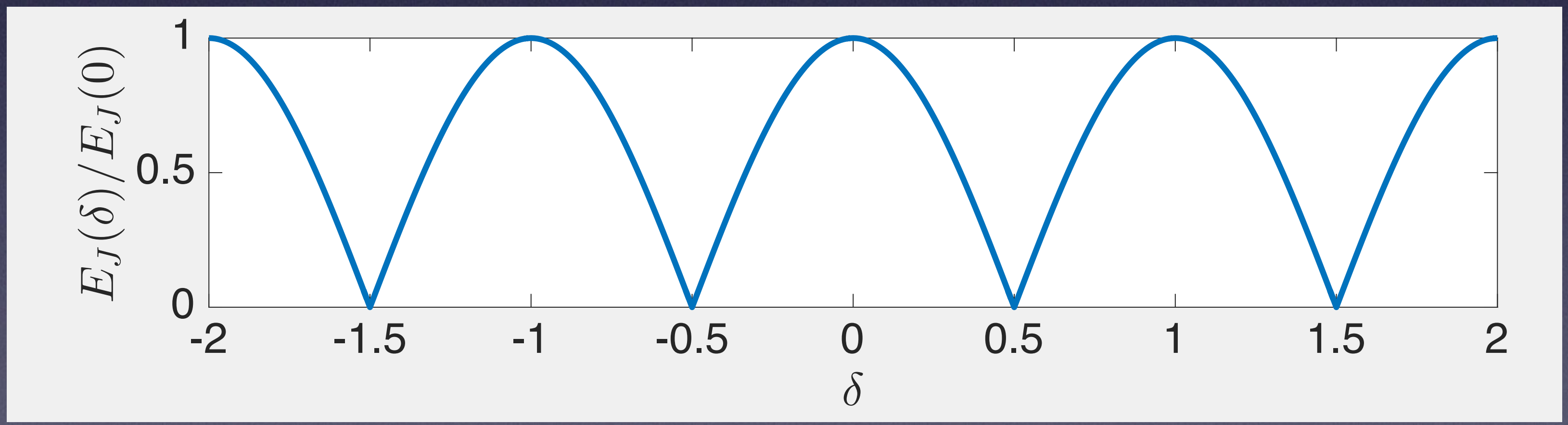
i.e. for $L_{geom} << L_J$

$$H = E_Q n^2 - E_J(\delta)\cos\theta$$

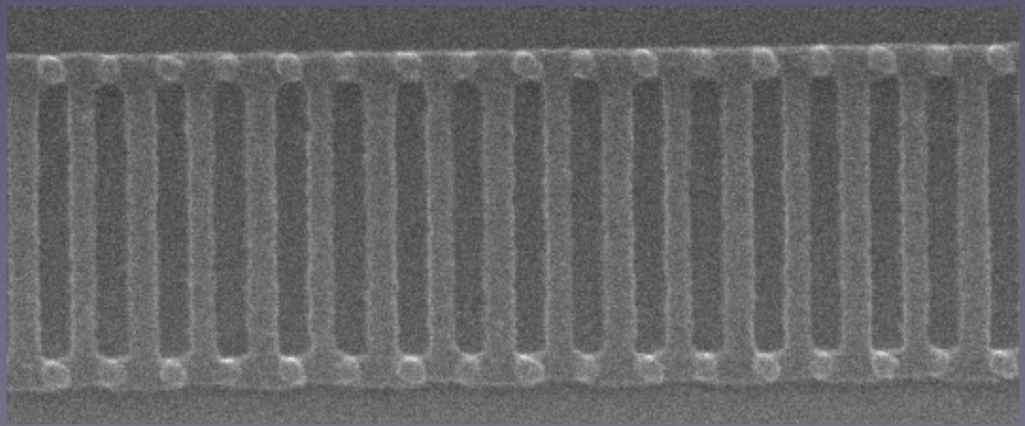
$$\delta = \delta_2 - \delta_1 = \phi_A$$



$$E_J(\delta) = E_J(0)|\cos\delta|$$



SQUID chain



also tunes plasma frequency

$$\hbar\omega_p = \sqrt{2E_J(\delta)E_Q}$$

Tuning *w* (*via* $g = E_J/E_Q$) without using a SQUID configuration

Josephson energy, $E_J \propto \text{jcn area}$

charging energy, $E_Q \propto 1/\text{jcn area}$

$$g = E_J/E_Q \sim (\text{jcn area})^2$$

$$\hbar\omega_p = \sqrt{2E_J E_Q}$$

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Dose: [μC/cm²]

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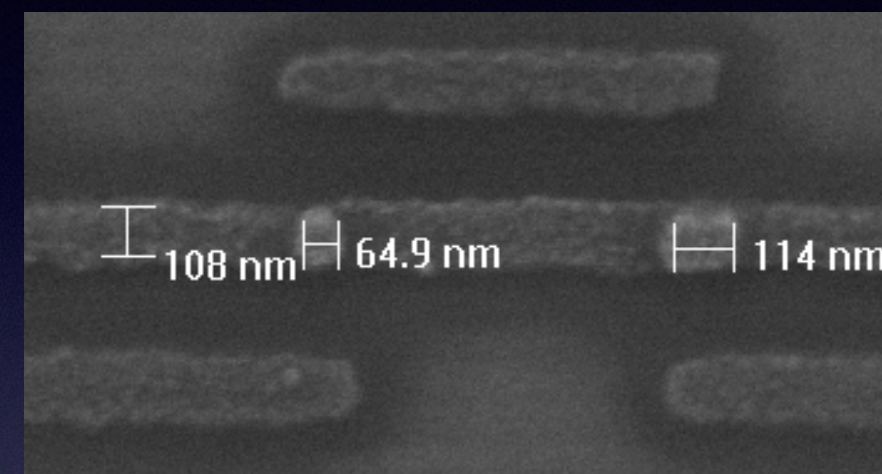
1600



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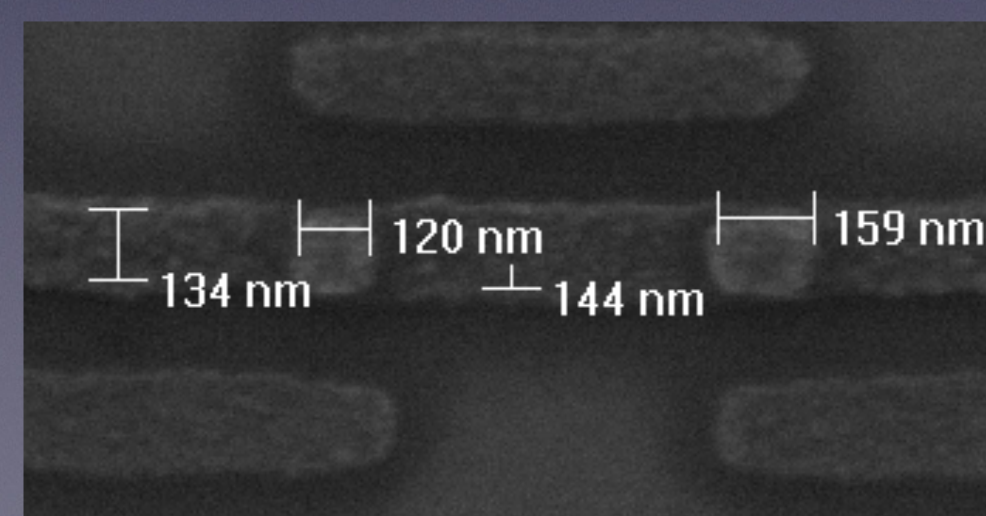
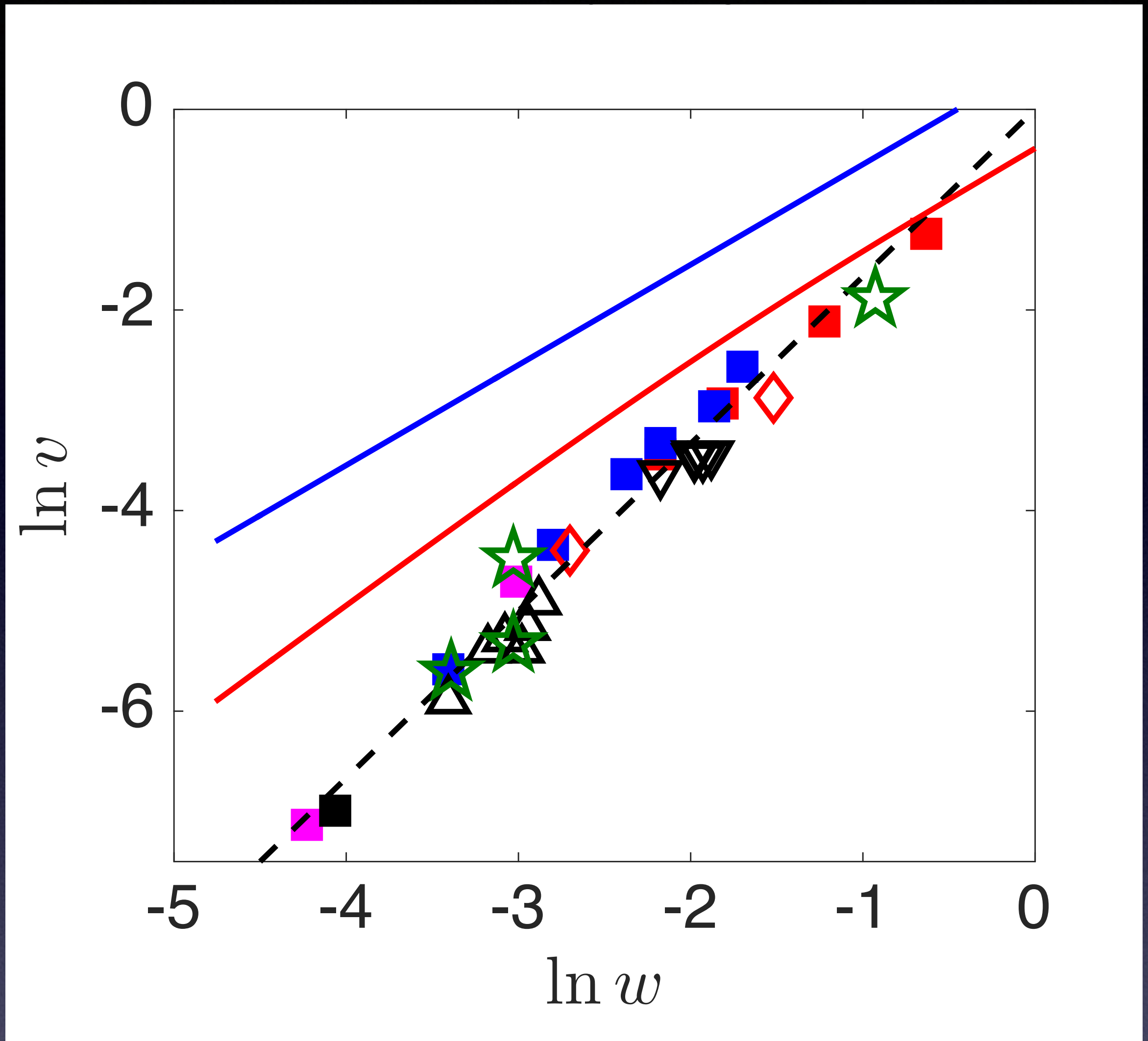
Vary E_J/E_Q by changing jcn area *via* dose

Vary plasma frequency *via* oxidation



Universal scaling of critical voltage



'rigid' quasicharge $\alpha = 1, A = \pi/2$

compressible quasicharge

$\alpha = 4/3, A = B\Lambda^{-2/3}$

(Vogt *et al.* Phys. Rev. B, 2015)

$$\boxed{\begin{aligned} v &= eV_c/N\hbar\omega_p \\ w &= W/\hbar\omega_p \end{aligned}}$$

$$\boxed{v = Aw^{\alpha}}$$

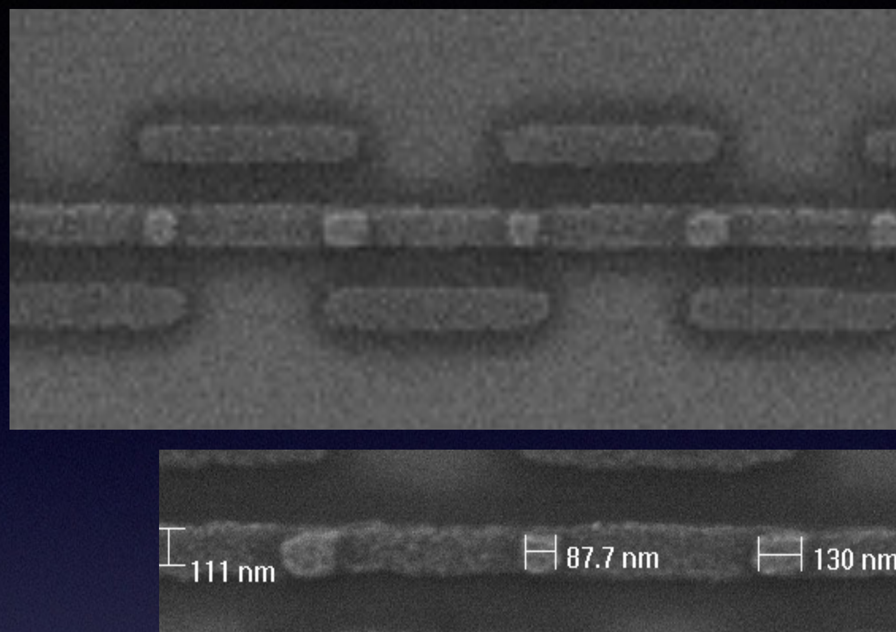
Data departs significantly from 4/3 exponent

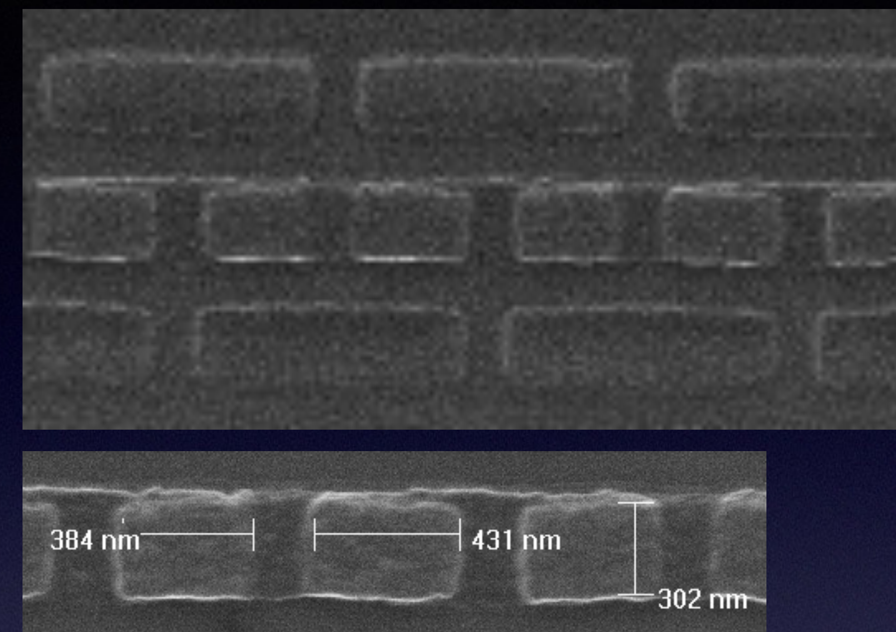
note: quantitative comparison to

since we experimentally obtain

screening length $\Lambda = \sqrt{C_J/C_0}$

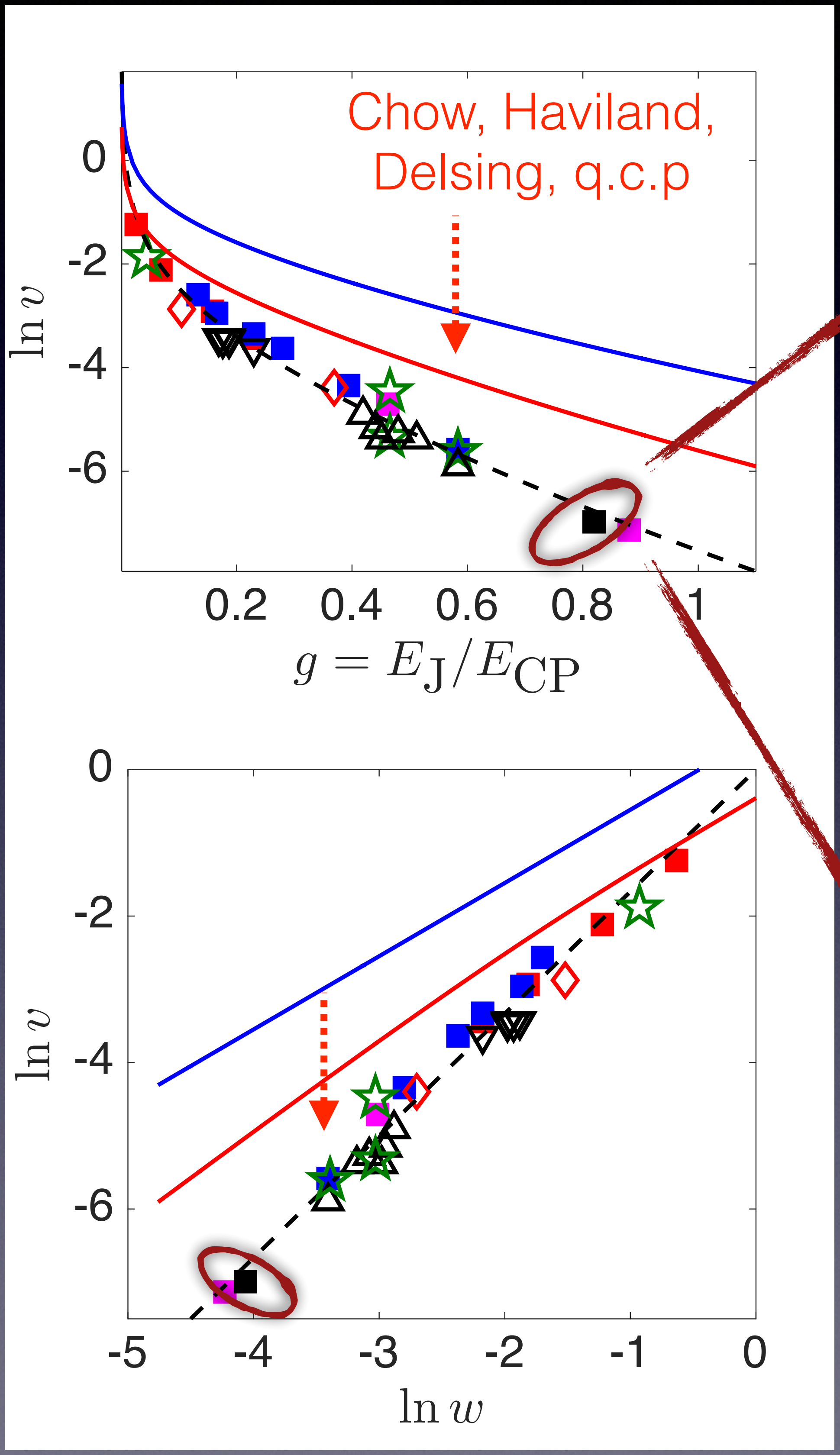
Fabrication control of the plasma frequency

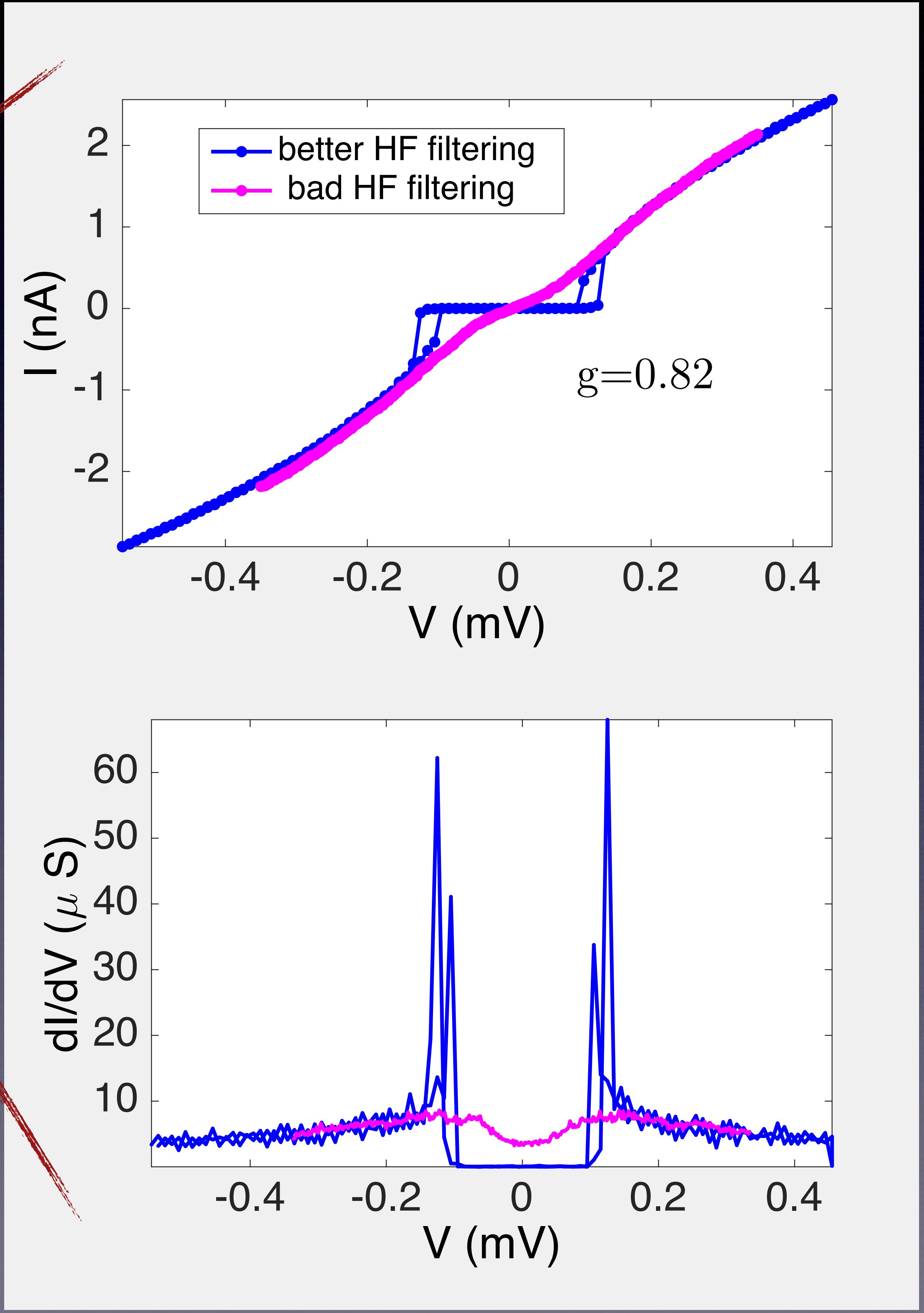




	A_{junc} [μm^2]	Oxidation	R_{junc} [kΩ]	E_J [μeV]	E_Q [μeV]	E_J/E_Q	V_{th} [mV]	ω_p
□	*0.015*	*30 s 0.044 mbar*	*4.056*	*167*	*280*	*0.60*	*0.29*	*306*
○	*0.120*	*10 min 200 mbar*	*22.5*	*30*	*49*	*0.61*	*0.06*	*54*

Where is that pesky SIT ?!

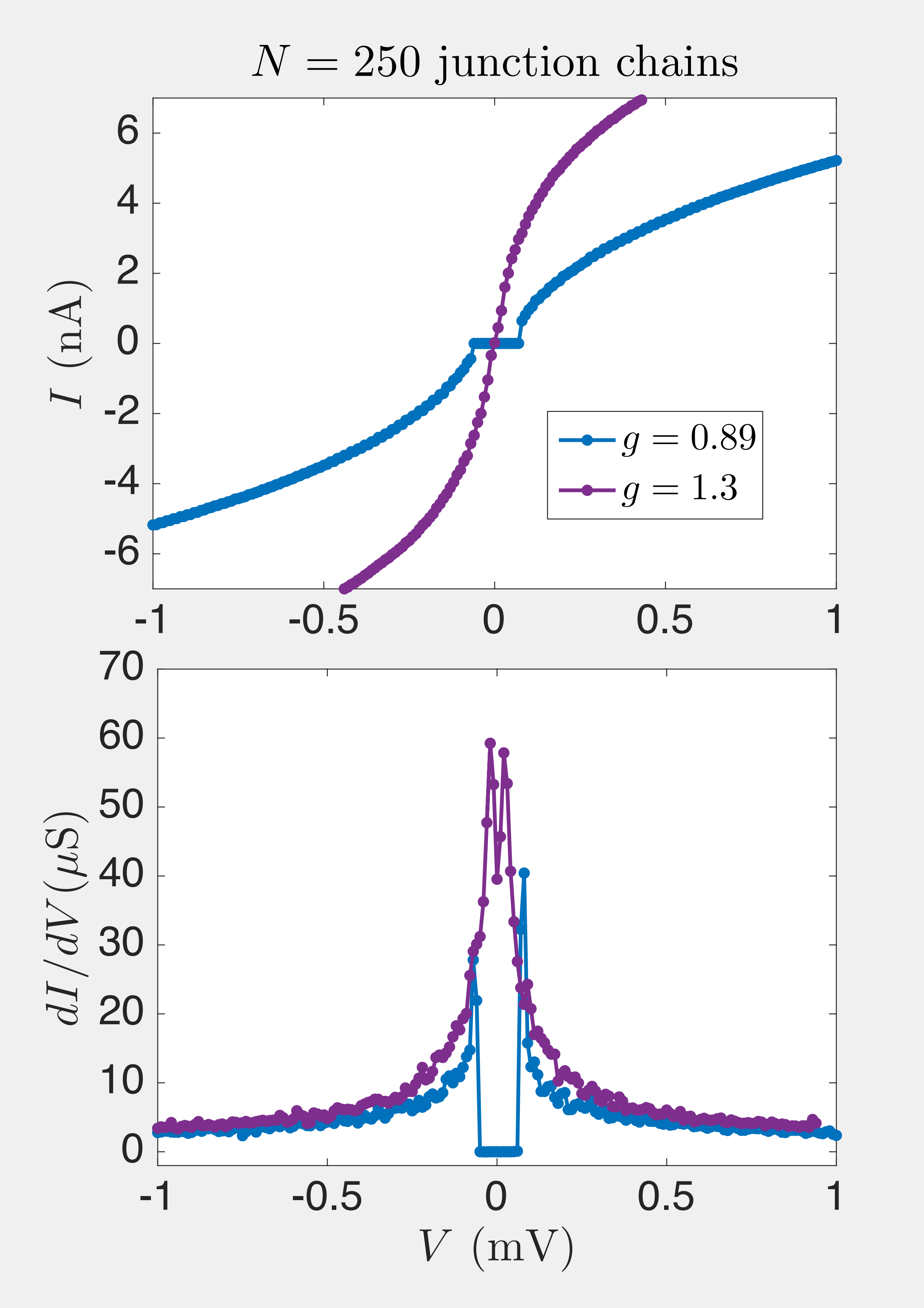


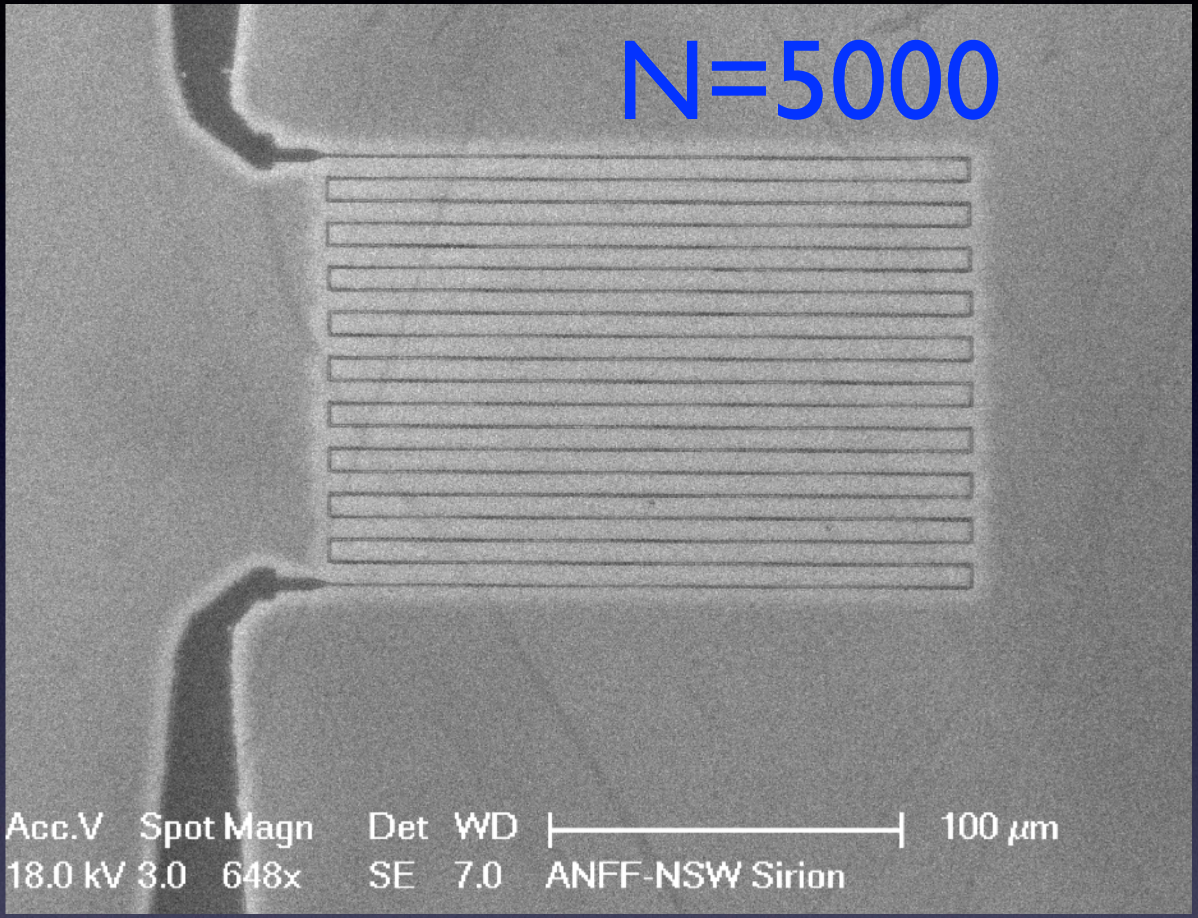


Where is that pesky SIT ?!

$$V_c \propto N$$

Need longer chains!





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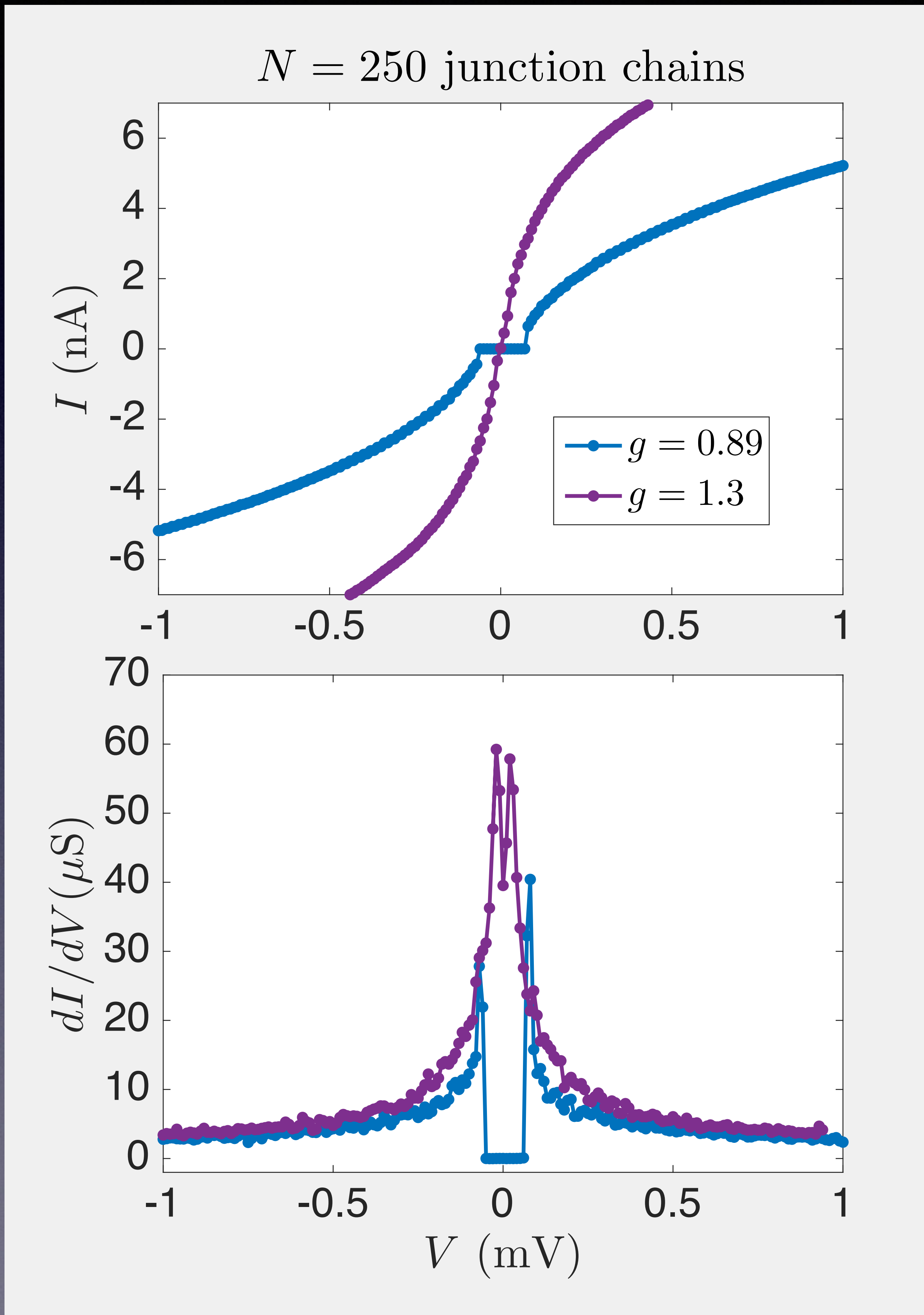
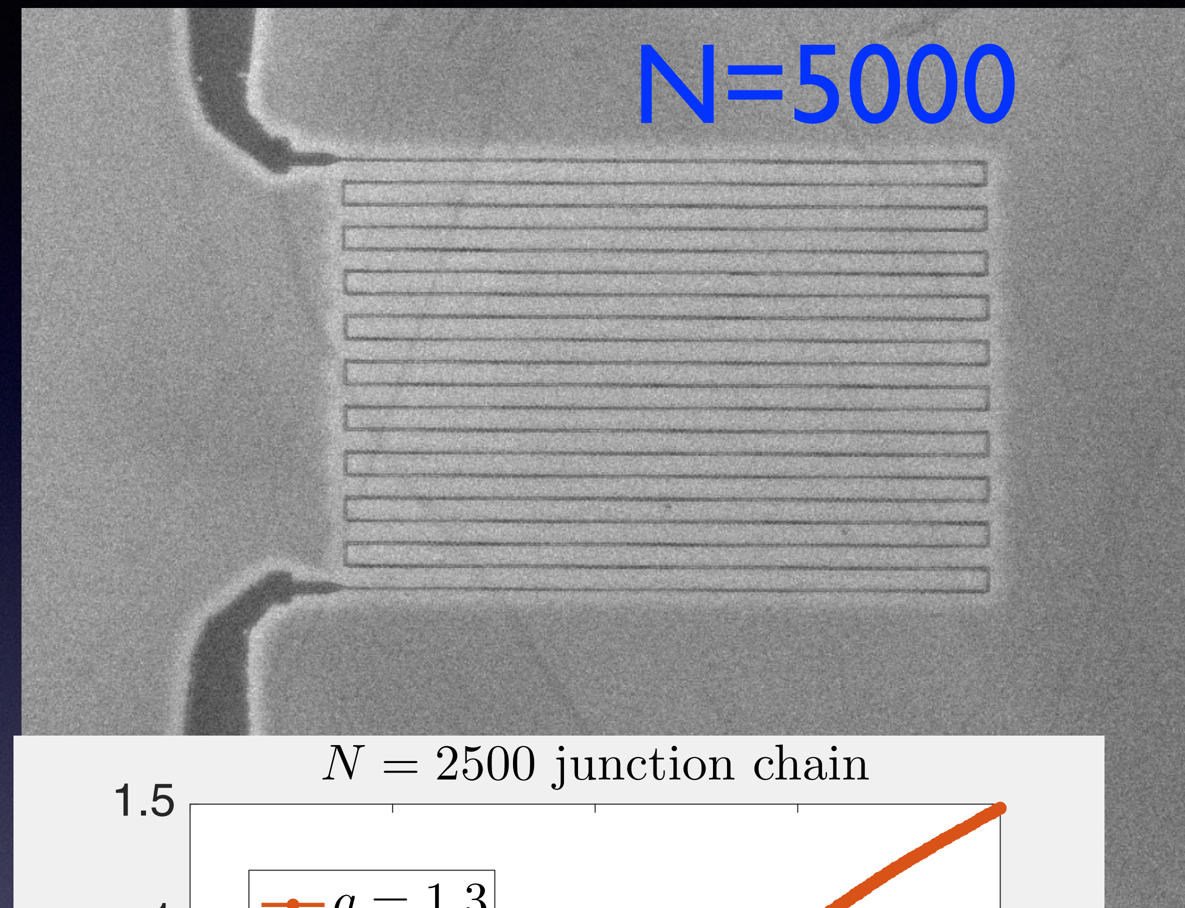
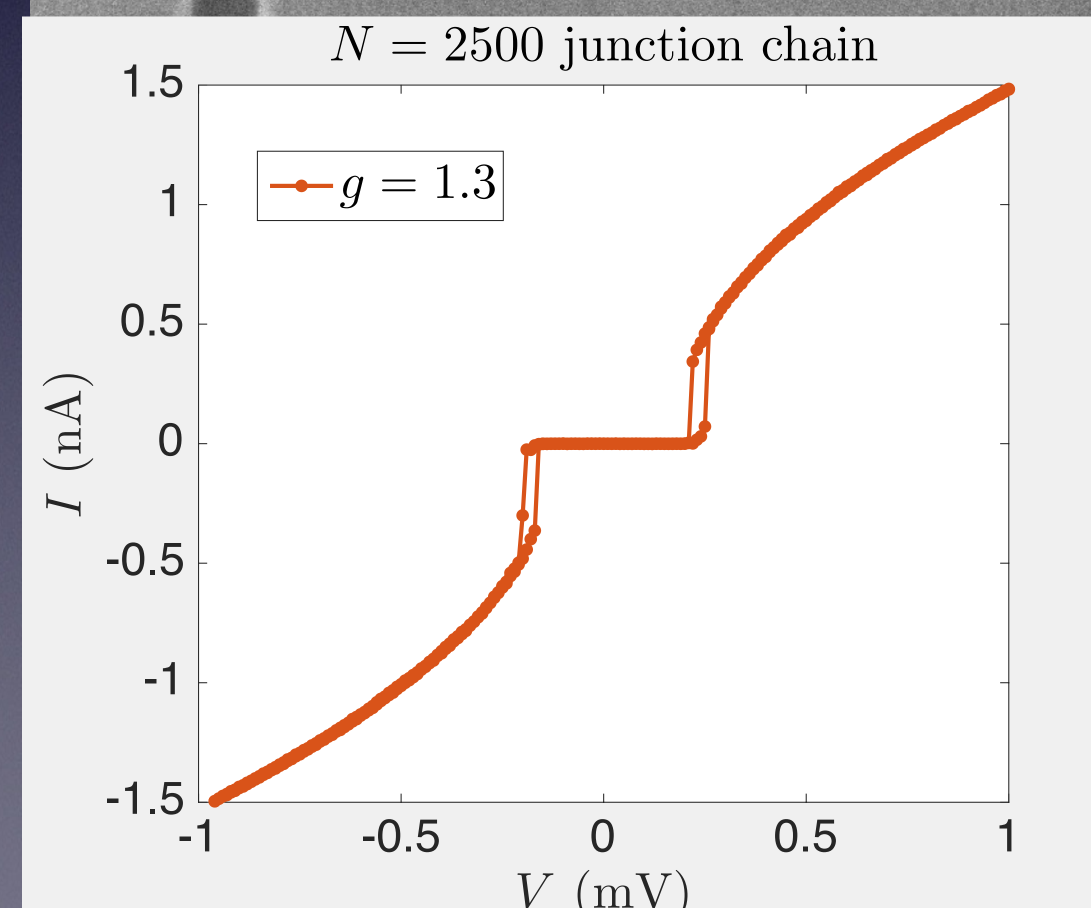
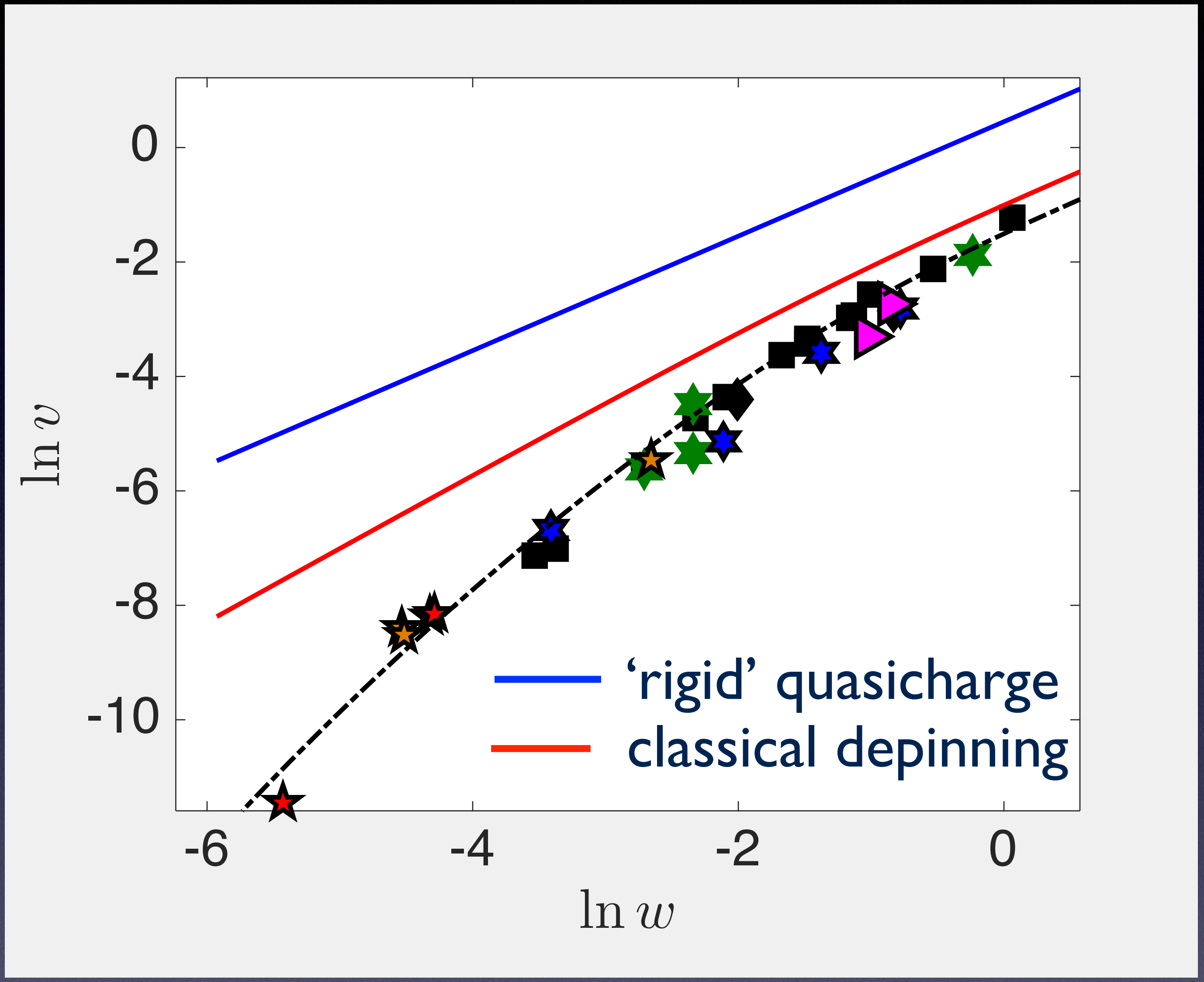






quantum enhancement of the pinning length



Luttinger liquid pinned by random disorder, no fitting!

$$N_L \propto W^{-\frac{2}{3-2K}}$$

Suzumura and Fukuyama (1983)

Giamarchi and Schulz, PRB 1988

see also Giamarchi's book:

Quantum Physics in One Dimension

$$\boxed{v \sim E_0/N_L^2 = Aw^\alpha}$$

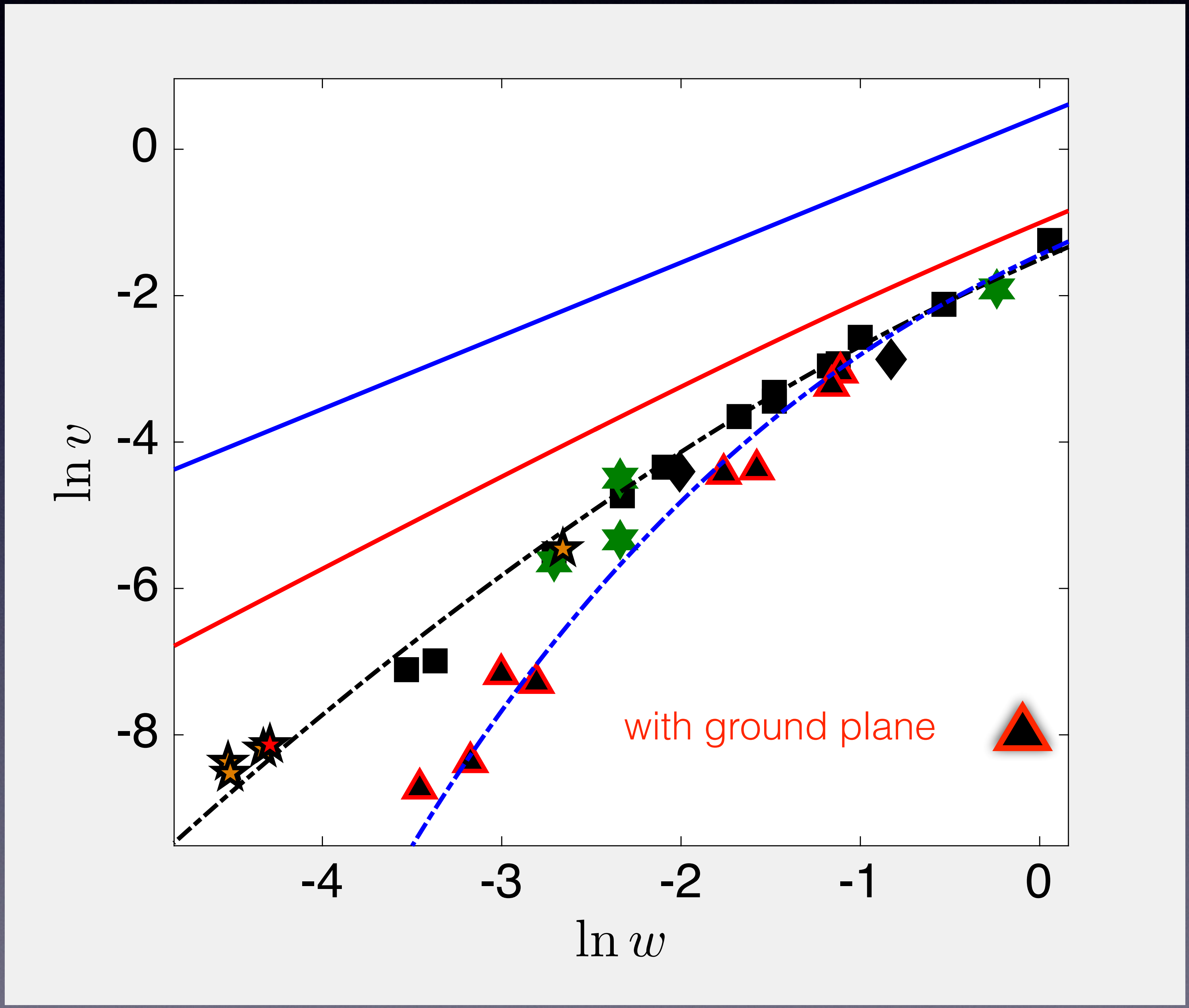
$$K \propto \Lambda^{-1} \ln w$$

$$\alpha = \frac{4}{3-2K}$$

… a precursor of the SF-BG QPT

OK, so prove it!

New family of chains 50 nm over a buried gold ground plane to increase C_0 , *i.e.* decrease Λ, increase *K*, move closer to transition



$$\Lambda = \sqrt{C_J/C_0}$$

$$K \propto -\Lambda^{-1} \ln w$$

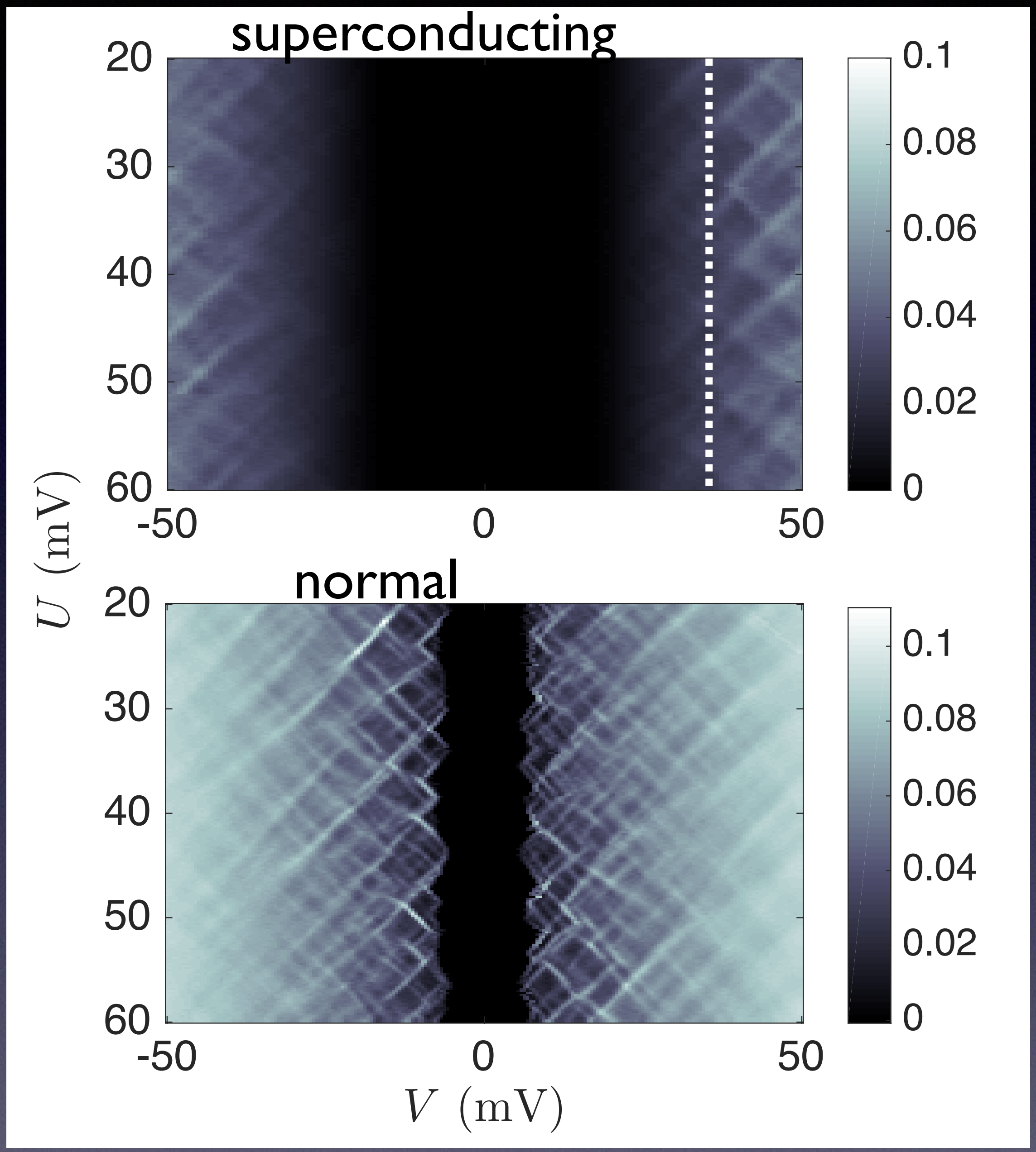
$$\propto \Lambda^{-1}\sqrt{g}$$

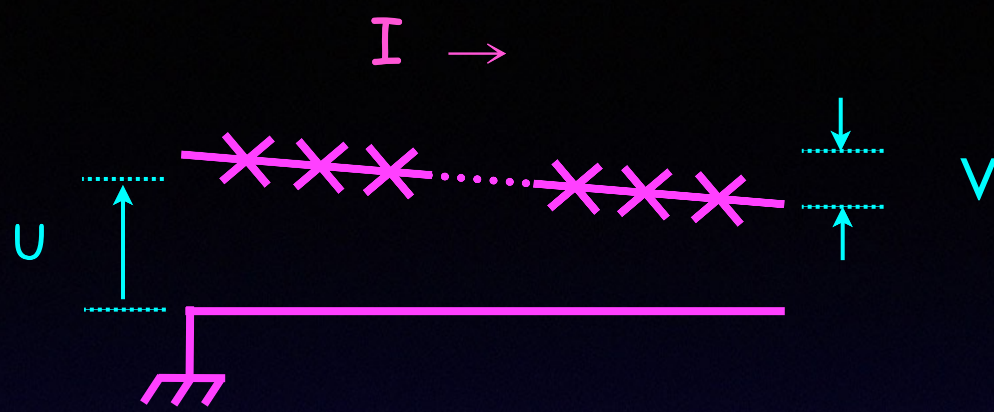
$$\boxed{v = Aw^{\alpha}}$$

$$\alpha = \frac{4}{3-2K}$$

cond-mat > arXiv:1702.04386

Gate dependence of conductance *dI/dV*



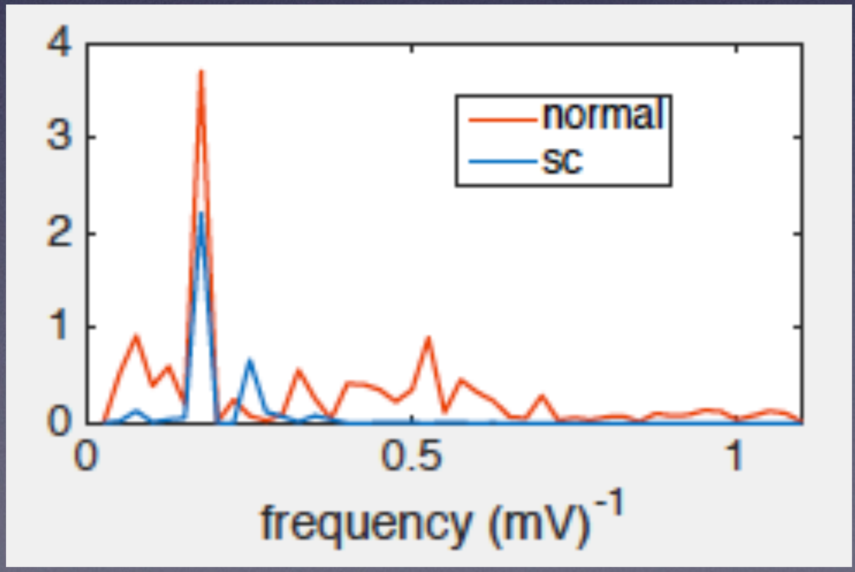


U= (V_L + V_R) / 2

global 'gate' potential U --> 'chemical potenial'

periodicity $C_0 \Delta U = e, 2e$

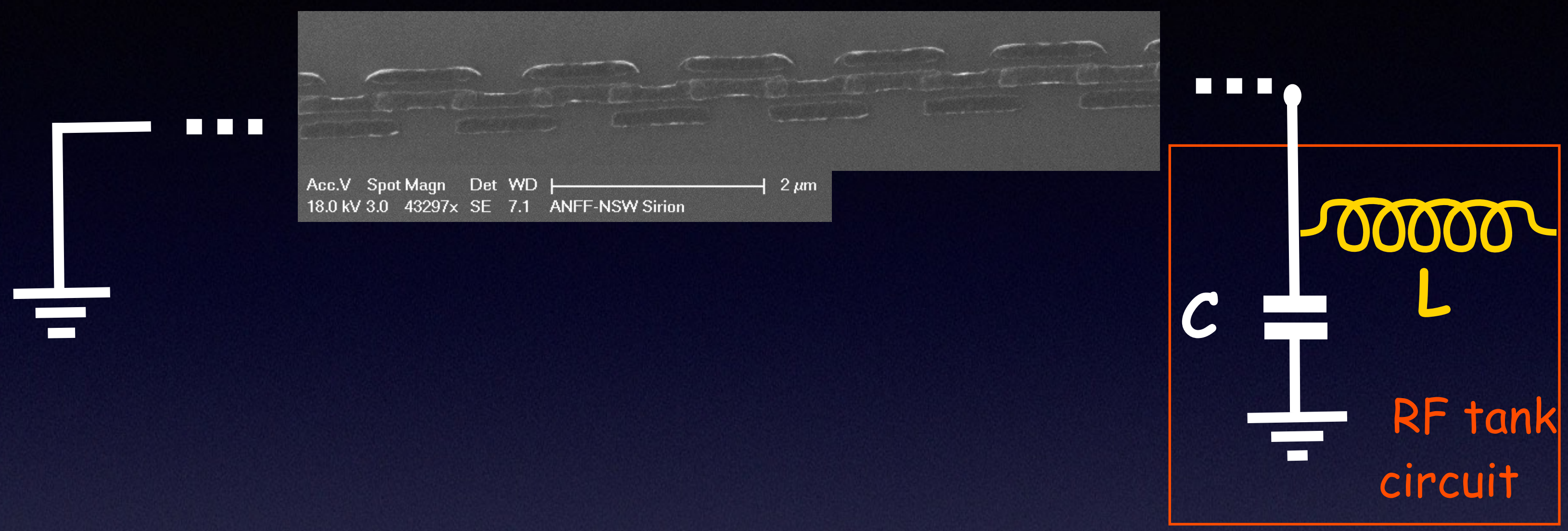
single electrons, Cooper pairs



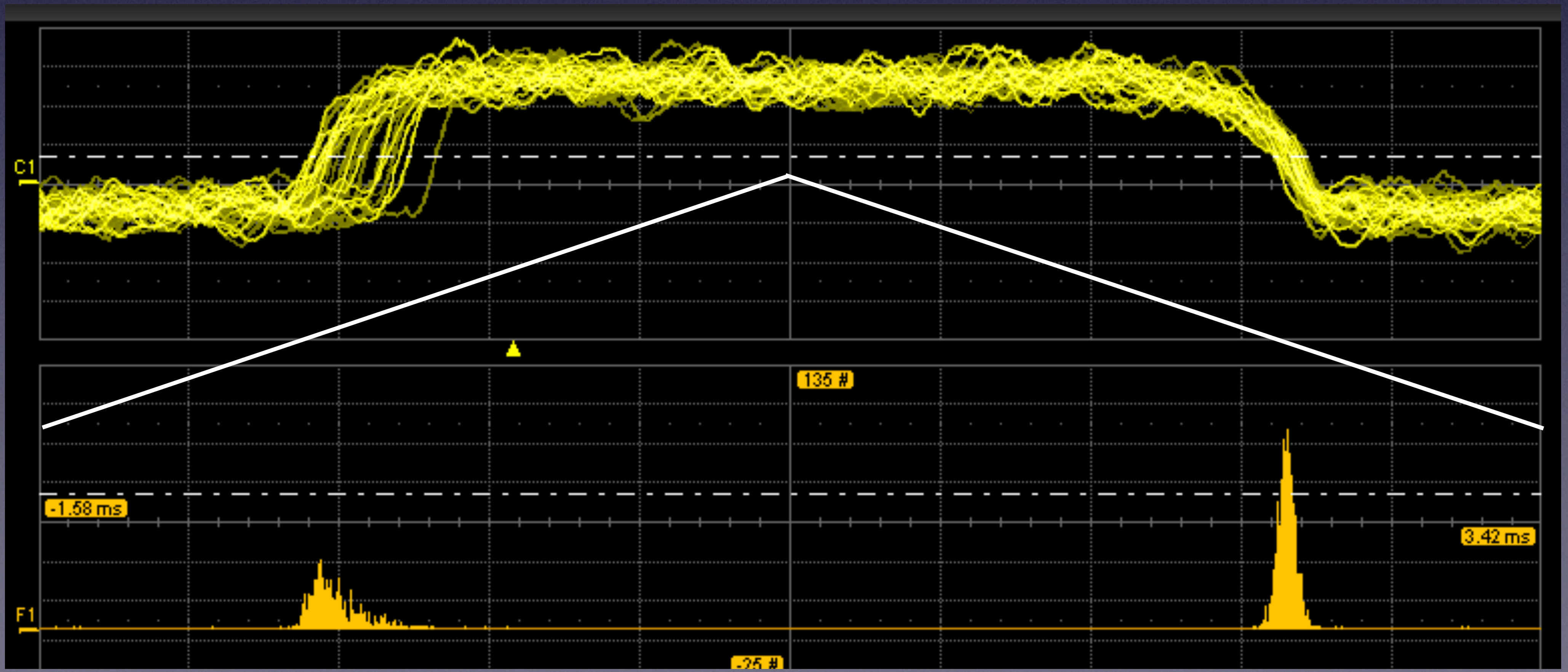
We find $\Lambda = 8(3)$

without (with) buried ground plane

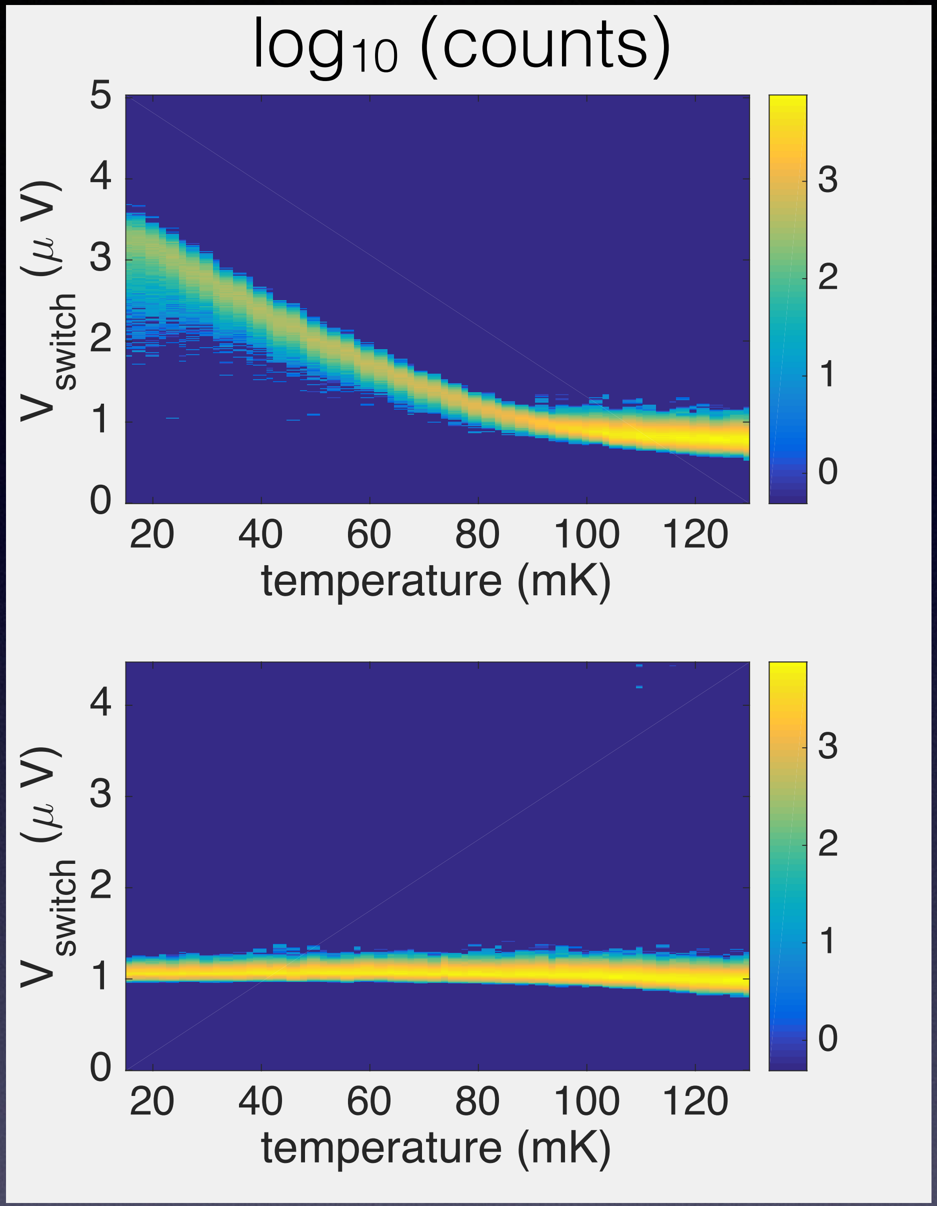
Measuring switching from the zero-current state in a chain of small Josephson junctions

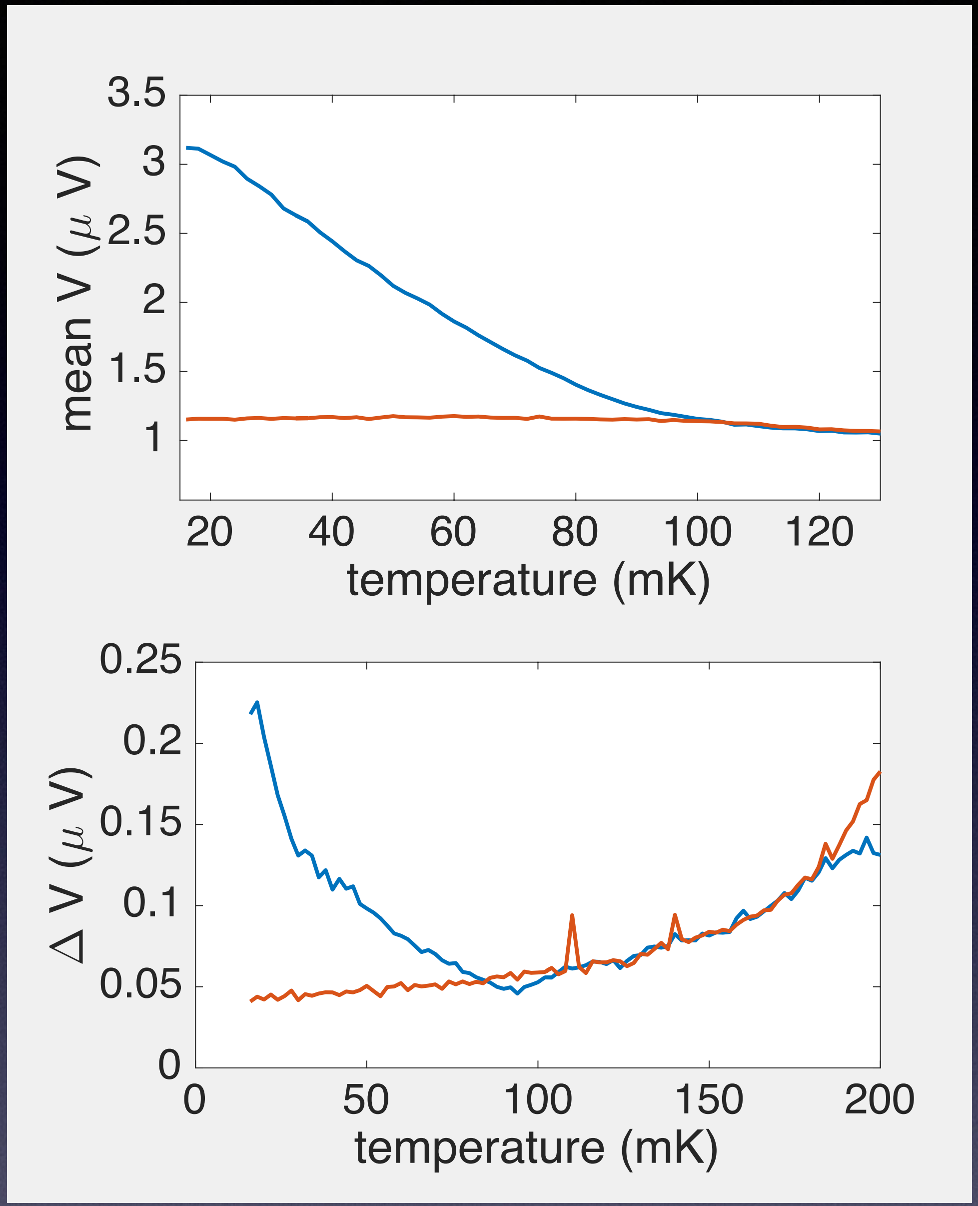


- Measured using RF-reflectometry
- f ~350 MHz with a few MHz bandwidth



switching histograms *vs* temperature





- Histograms narrow sharply as temperature increases!
- Remarkably similar to phase diffusion seen in large junctions (usual Josephson effect) when there is <u>frequency-dependent dissipation,</u> *i.e.* relatively large noise at the plasma frequency

Kautz and Martinis (1990), Krasnov *et al.* (2005), Kivioja *et al.* (2005)

JJ chains in the charge regime

- Insulating JJ chains are ‘pinned’ Luttinger liquids
  - Origin of critical voltage, related to localisation length
- Solved some outstanding mysteries and confirmed an important theoretical result

  1. Superfluid-insulator transition in chains is to a Bose glass, Why does the theory work so well?

     note: do not require disorder in E_J, E_Q

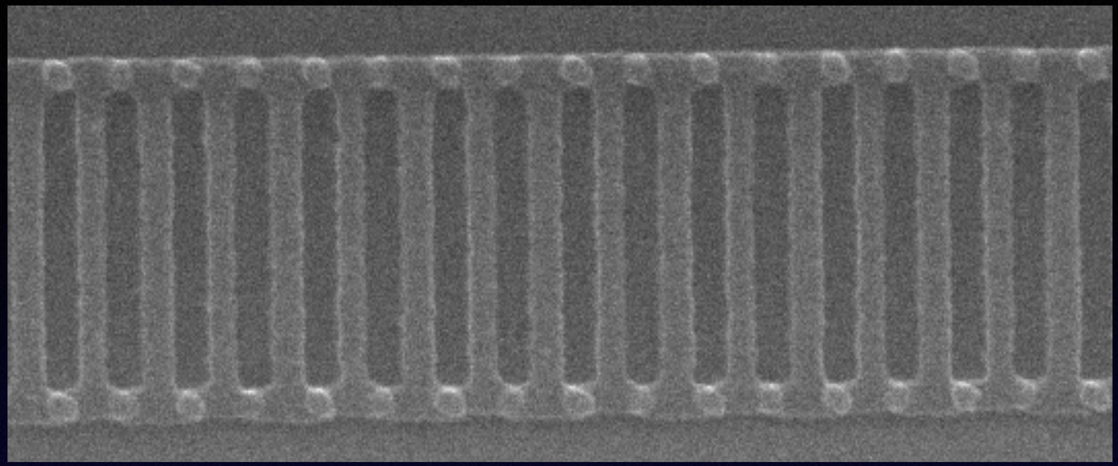
  2. Unlike Mott insulator, the Bose glass is ***compressible.*** Localised Cooper pairs rearrange randomly and easily under applied voltages — explains lack of success in obtaining synchronised current steps

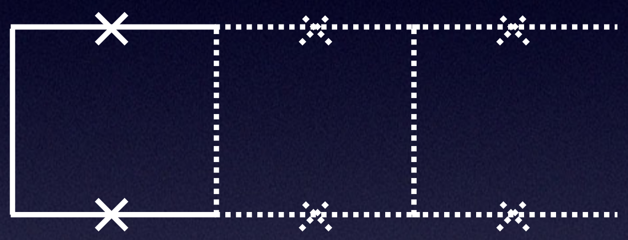
- Universal scaling of the critical voltage: orders of magnitude in *w* and *N*, one in plasma freq: a precursor of SF-BG QPT

Cedergren, Ackroyd, Kafanov, Vogt, Shnirman and Duty

cond-mat > arXiv:1702.04386

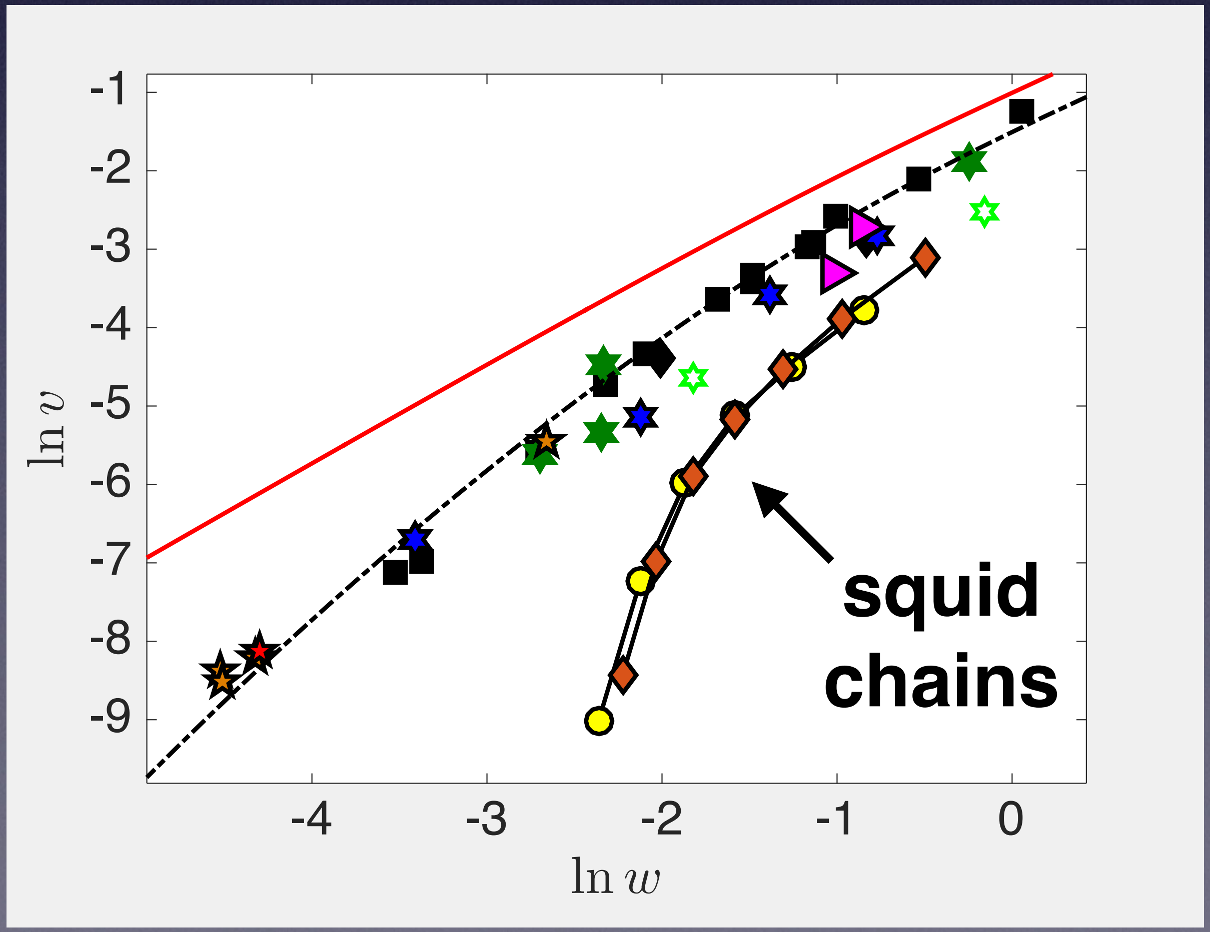
Critical voltage in SQUID chains





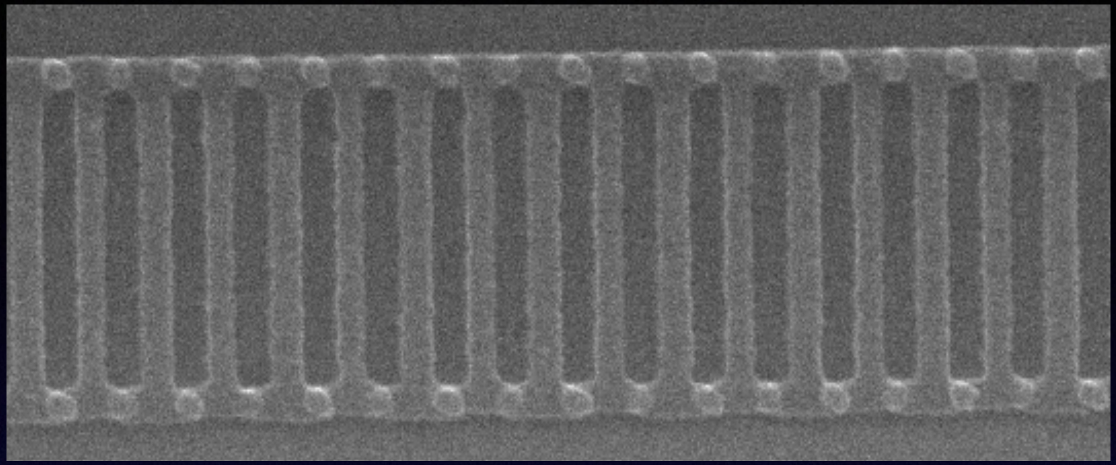
Huge surprise!

Not on "the line".



- Parity (even-odd) effect from non-trivial interplay of flux and charge in SQUID chains?
- Chain of N SQUID's not equiv. to N SQUID's in series?
- Could explain anomalous SIT result by Chan, Delsing and Haviland '98

Critical voltage in SQUID chains



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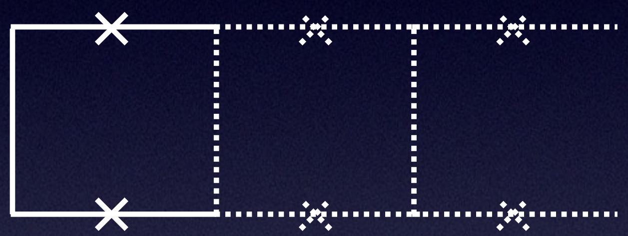

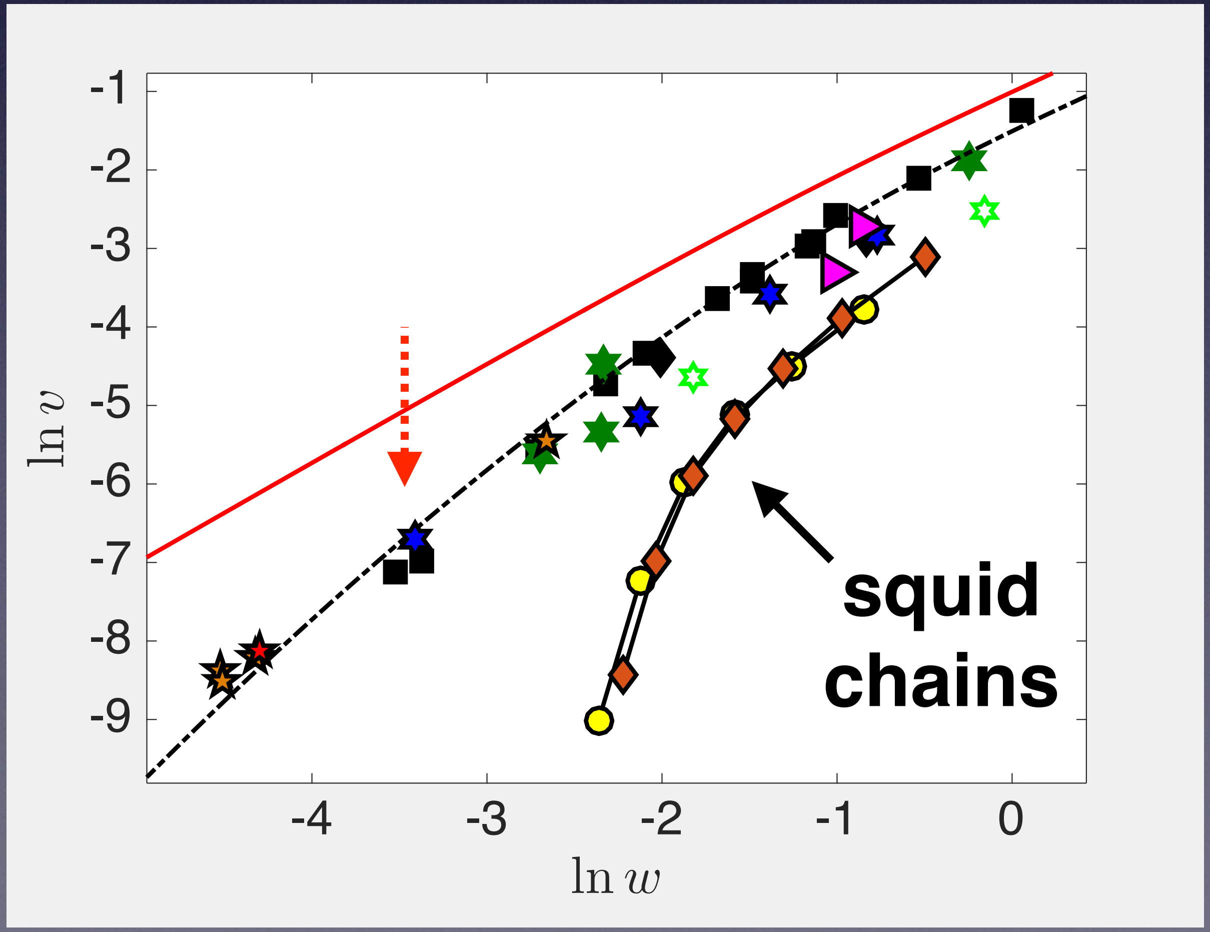

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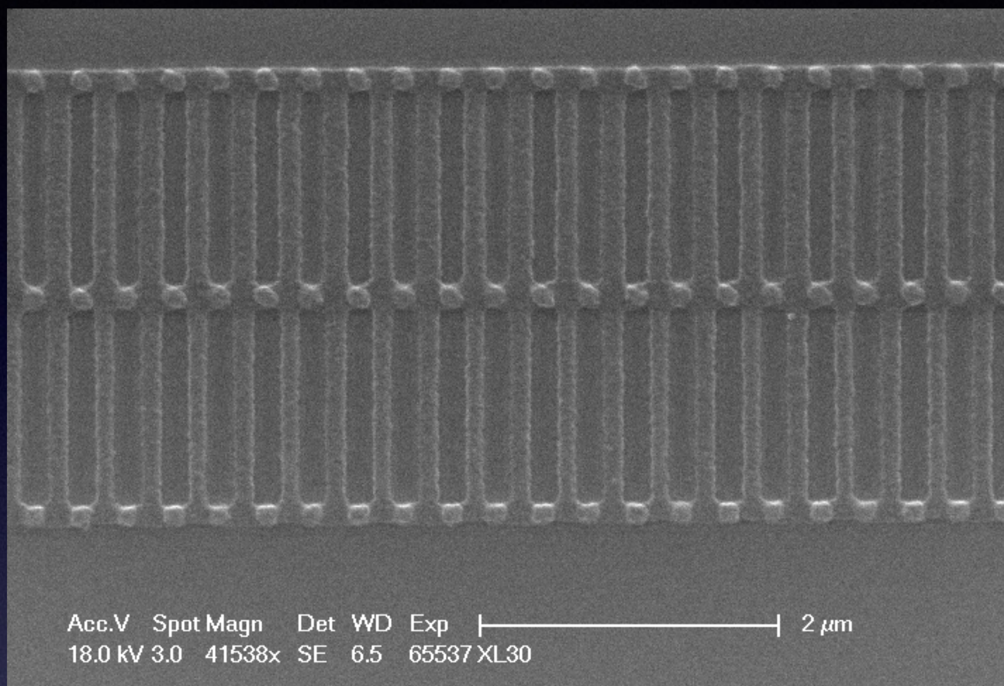
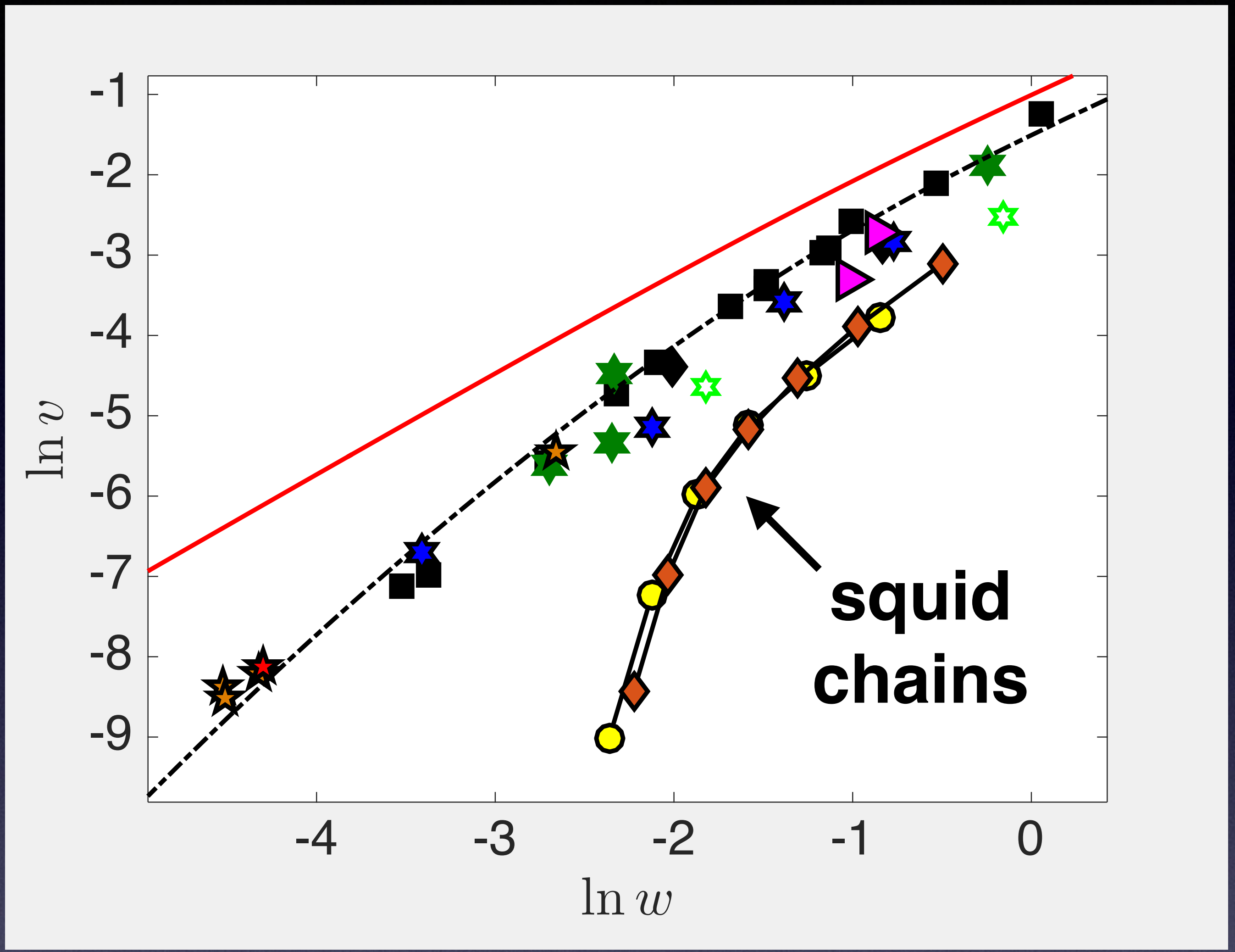
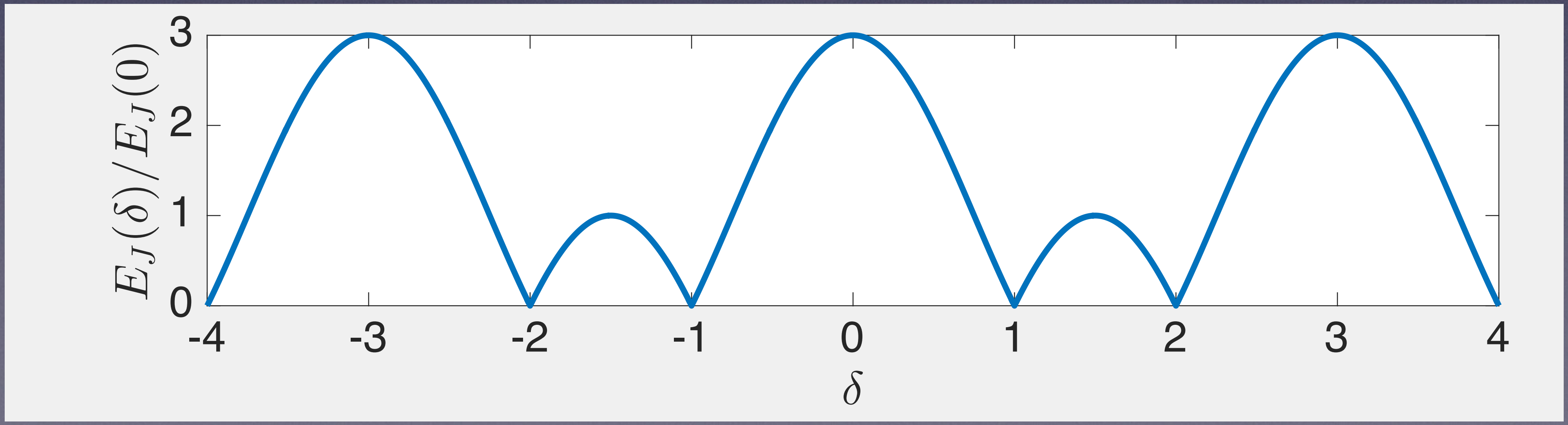
Critical voltage in ‘double’ SQUID chains

i.e. symmetric 3-slit interferometer





$$E_J(\phi) = E_J^0 \left| 1 + 2\cos\frac{2\phi}{3} \right|$$



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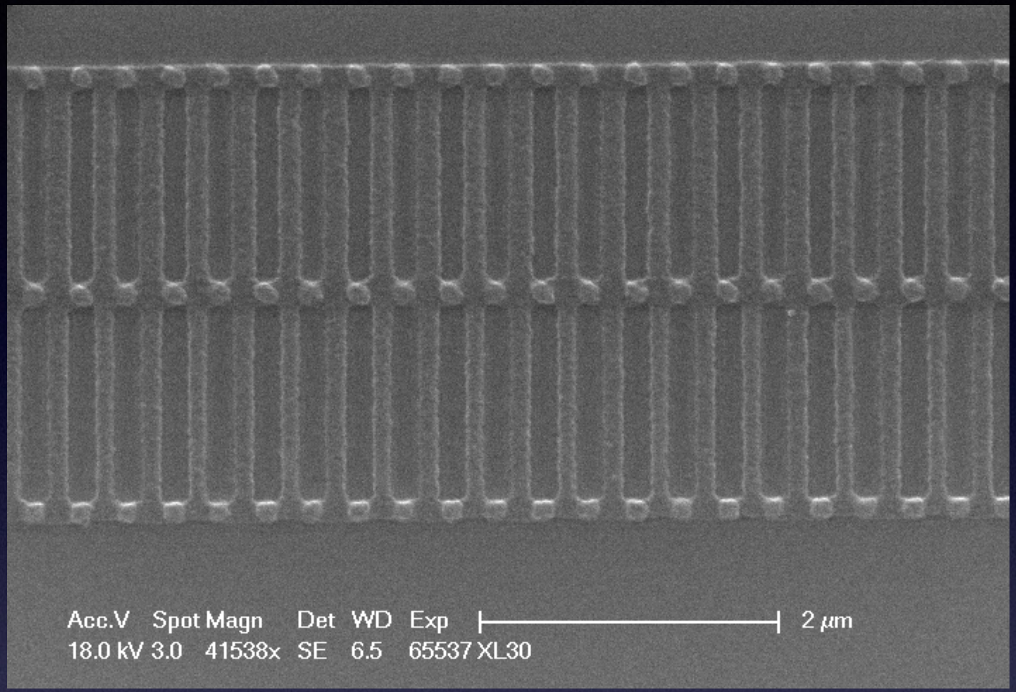

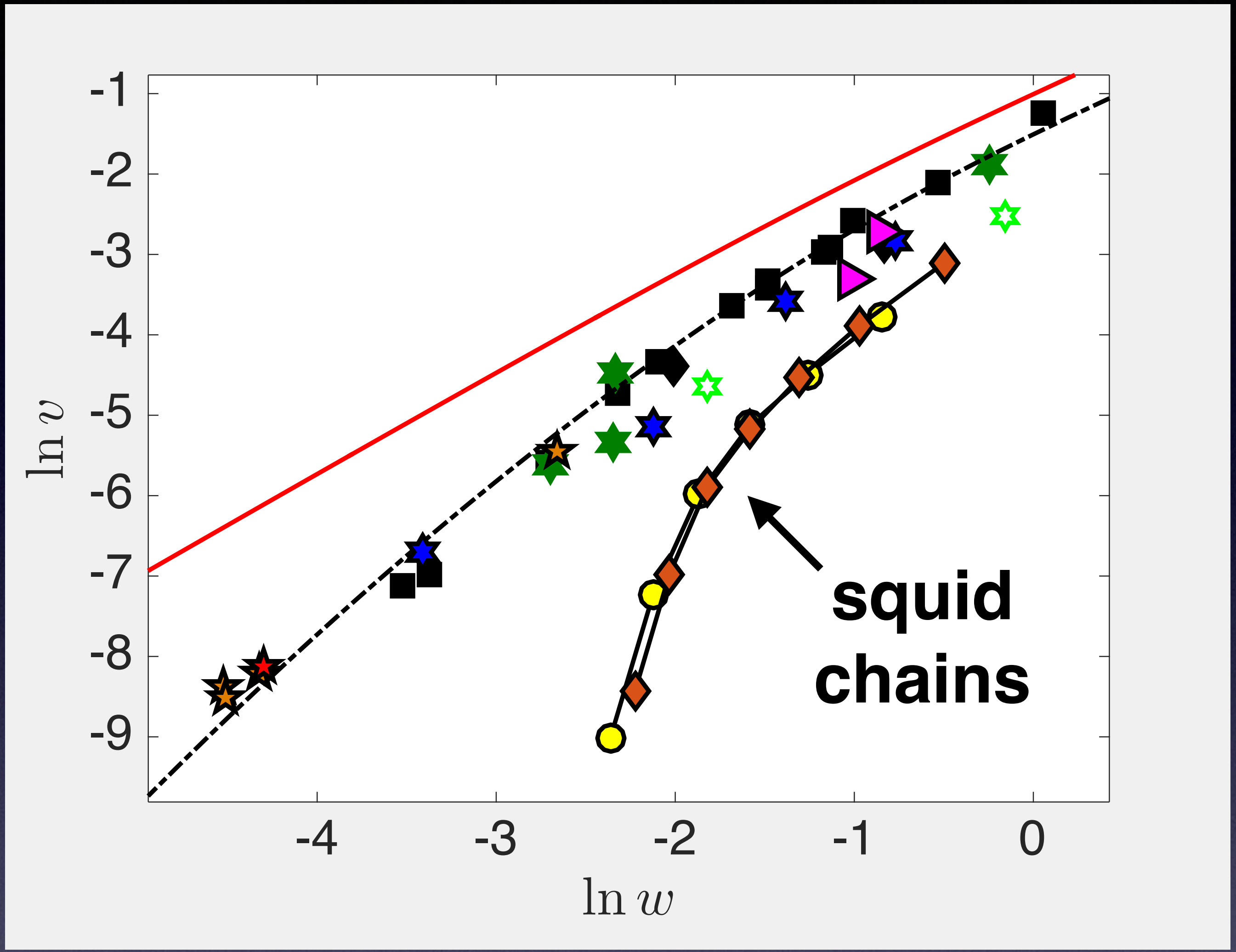

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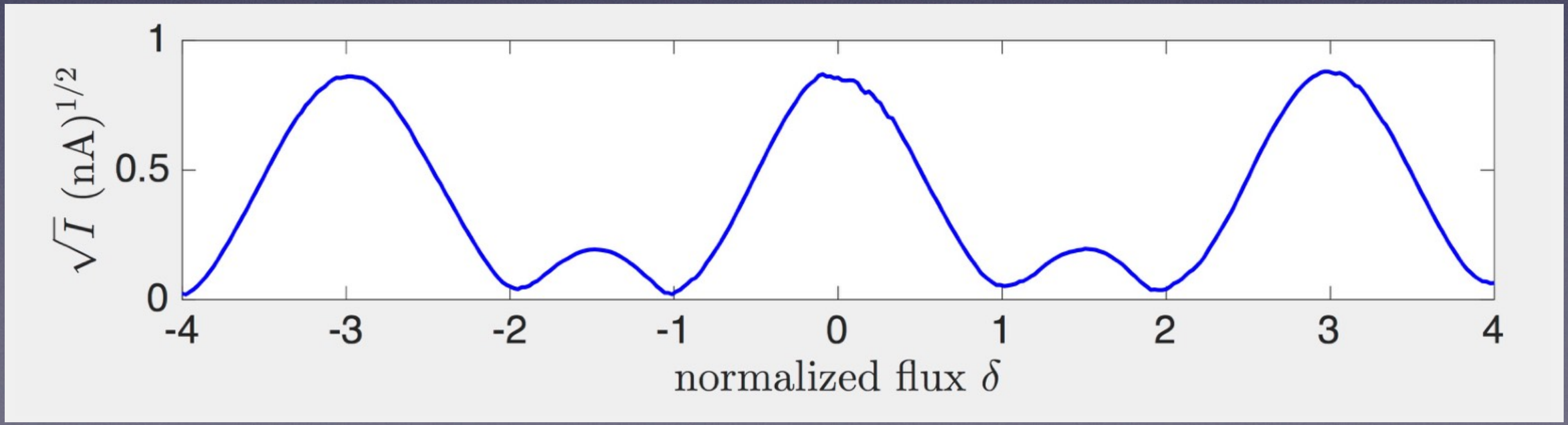

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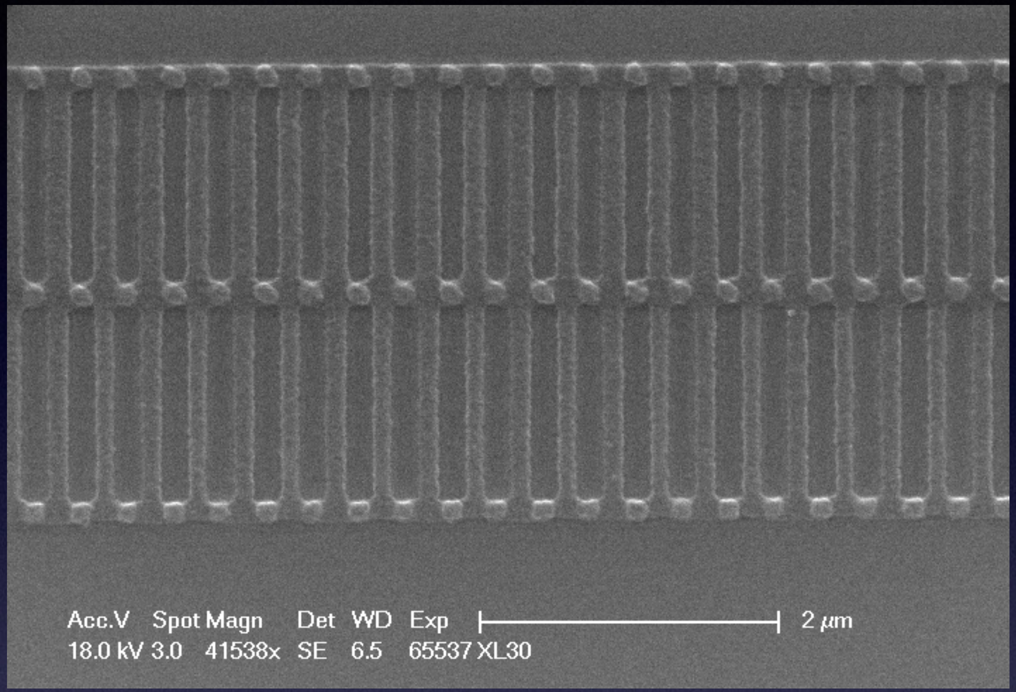

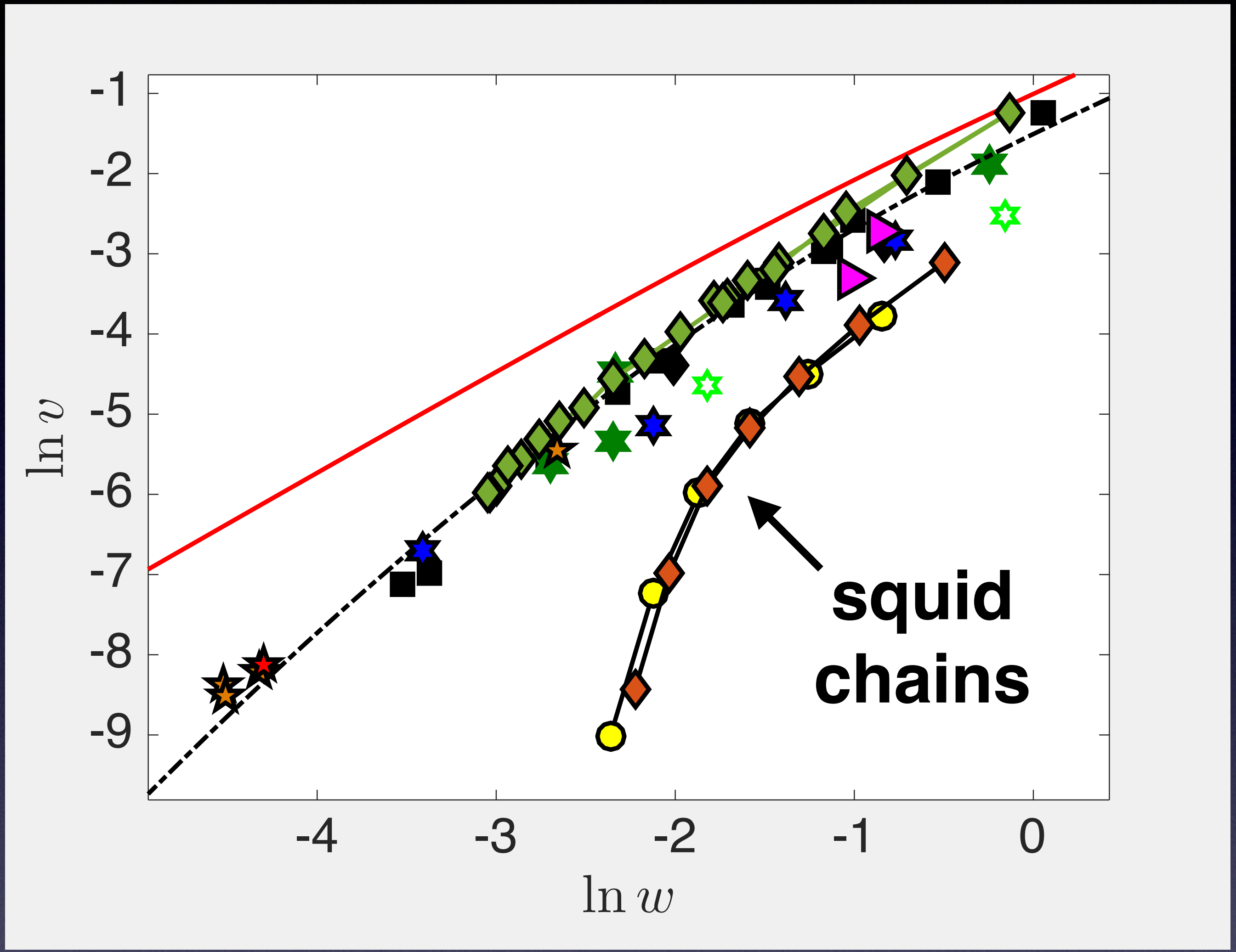

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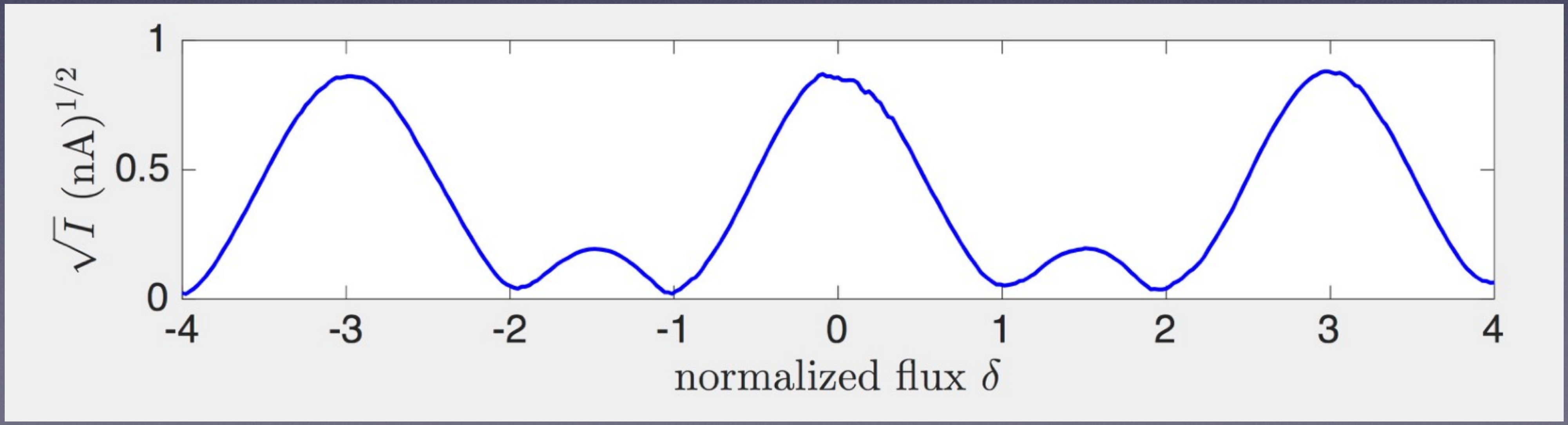
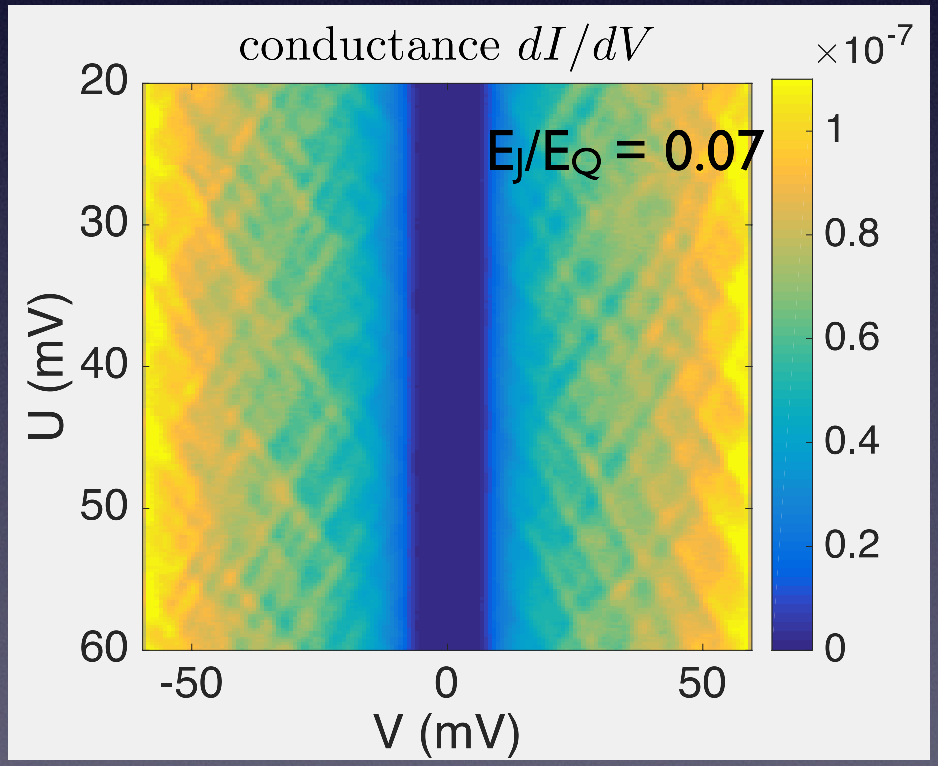
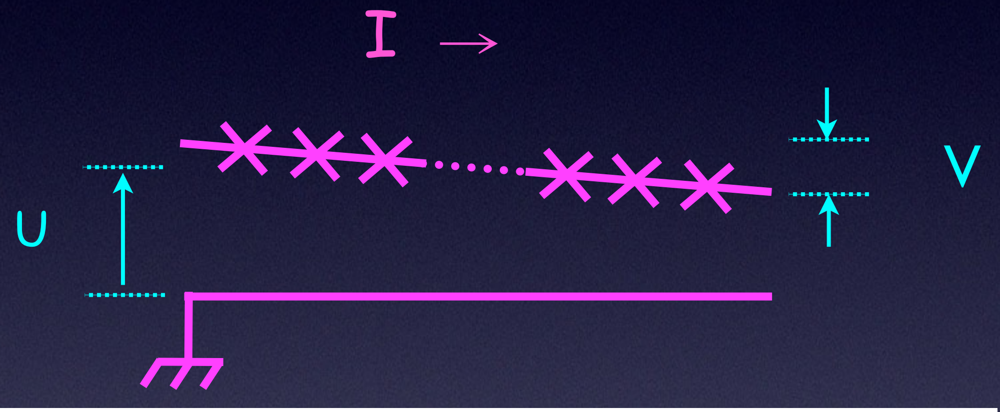


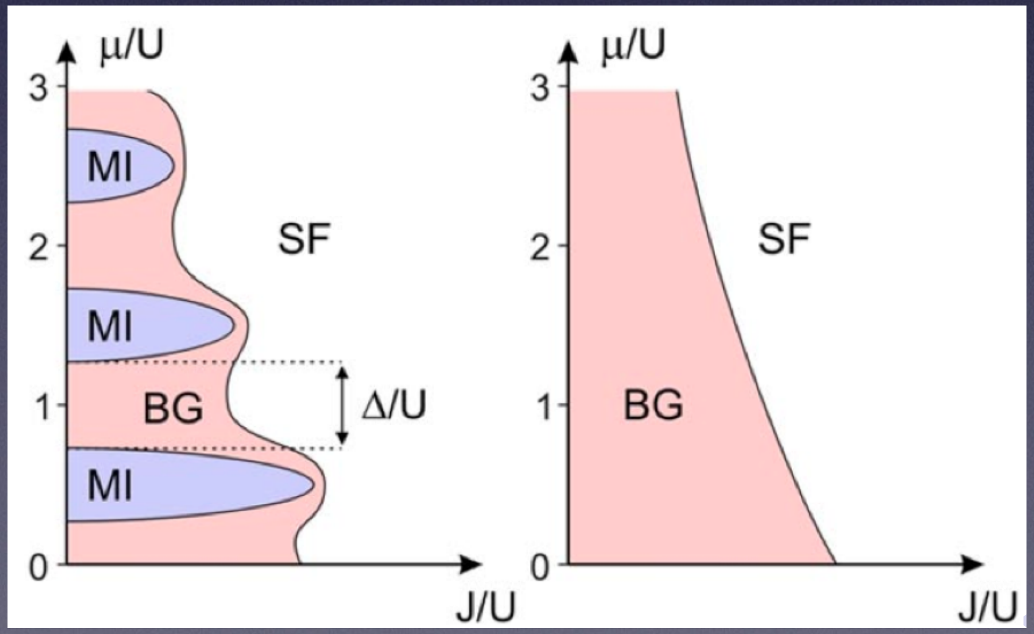
Open and very important question:

Can we get ever get away from Bose glass to reach the Mott insulator state?

theory: ubiquity of Bose glass tied to ubiquity of "maximal" charge disorder







experiment hints that it is not quite maximal. How can we reduce offset-charge distribution?

Mott insulator *versus* Bose Glass

- Maximal offset charge $$N_L = \left(\frac{4\sqrt{3}\pi^2 E_0}{W}\right)^{2/3}$$
- In the absence of disorder, sine-Gordon-like model: soliton length $$N_s = 2\sqrt{E_0/W}$$
- With disorder $N_s = 2\sqrt{E_0/aW}$

  a $\propto$ to drift of random walk

  maximal charge disorder: $a \to 0$

  … leads to non-commensurate LL with weak disorder!
- Mott insulator becomes ground state when $N_s \lesssim N_L$

  Fukuyama J. Phys. Soc. Japan (1978)
- Also holds when quantum fluctuations are included

  Giamarchi, Le Doussal and Orignac, PRB 2001

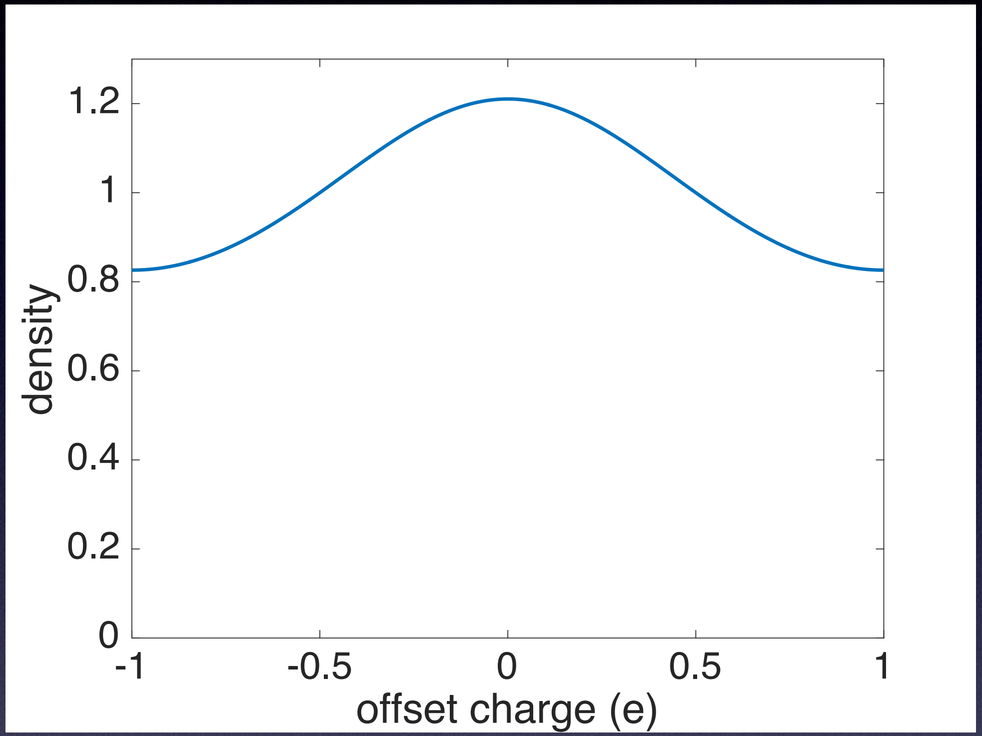
Mott insulator *versus* Bose Glass

Estimate we only need

$$a \gtrsim 0.1$$

a small deviation from maximal offset charge distribution

for example $a = 0.5$



- AlOx is known to prefer large numbers of fixed negative charges, e.g. used to passivate solar cells
- Deposition of NiOx to dope in holes….
- Use ScOx as barrier material….
- Novel circuit designs?

Kitaev's topologically protected qubit based on the "current mirror"

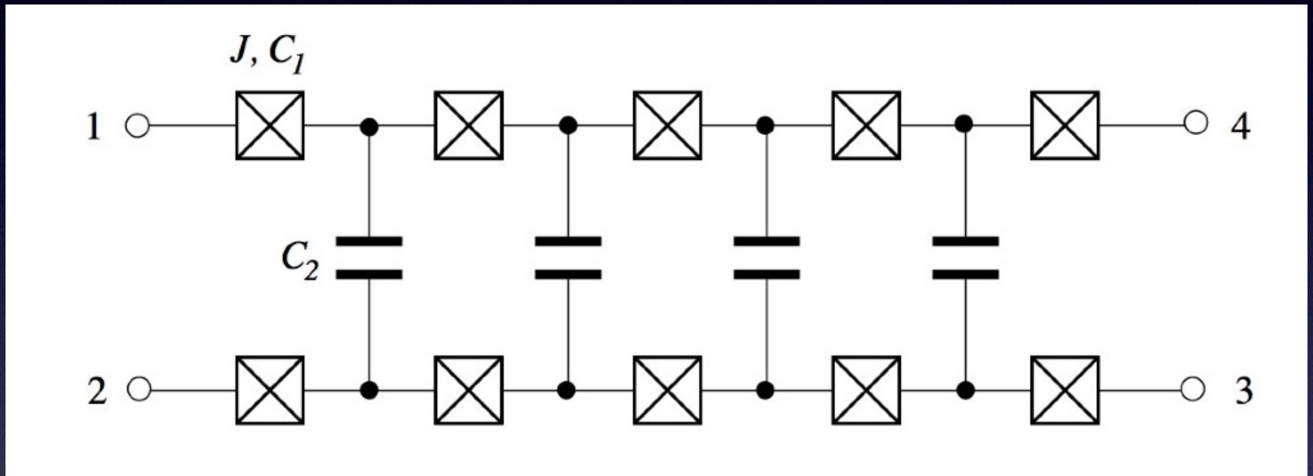
Kitaev, arXiv: cond-mat/0609441
Choi, Choi, Choi, and Lee PRL 1998

Strongly couple two insulating chains such that lowest energy excitations are CP on one chain, CP hole on other

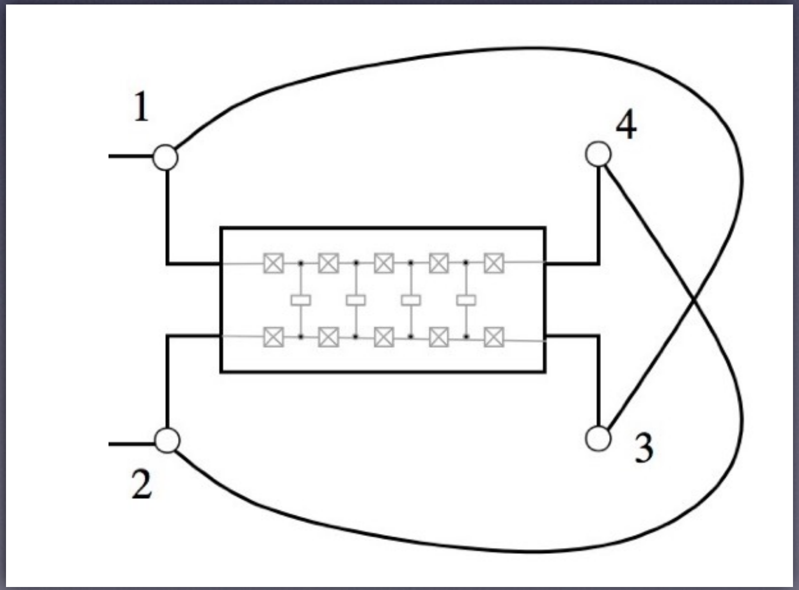
→ Bose condensation of neutral dipoles

2e
-2e

supercurrent I_s ⟶

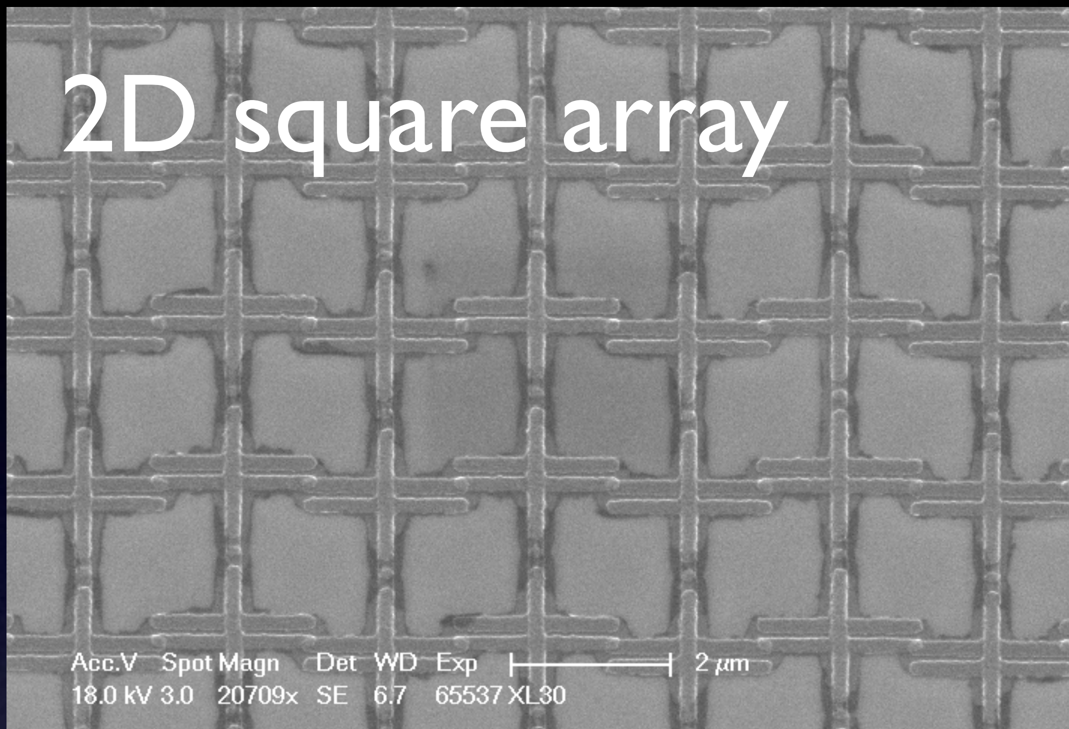


⟵ reflected supercurrent I_s

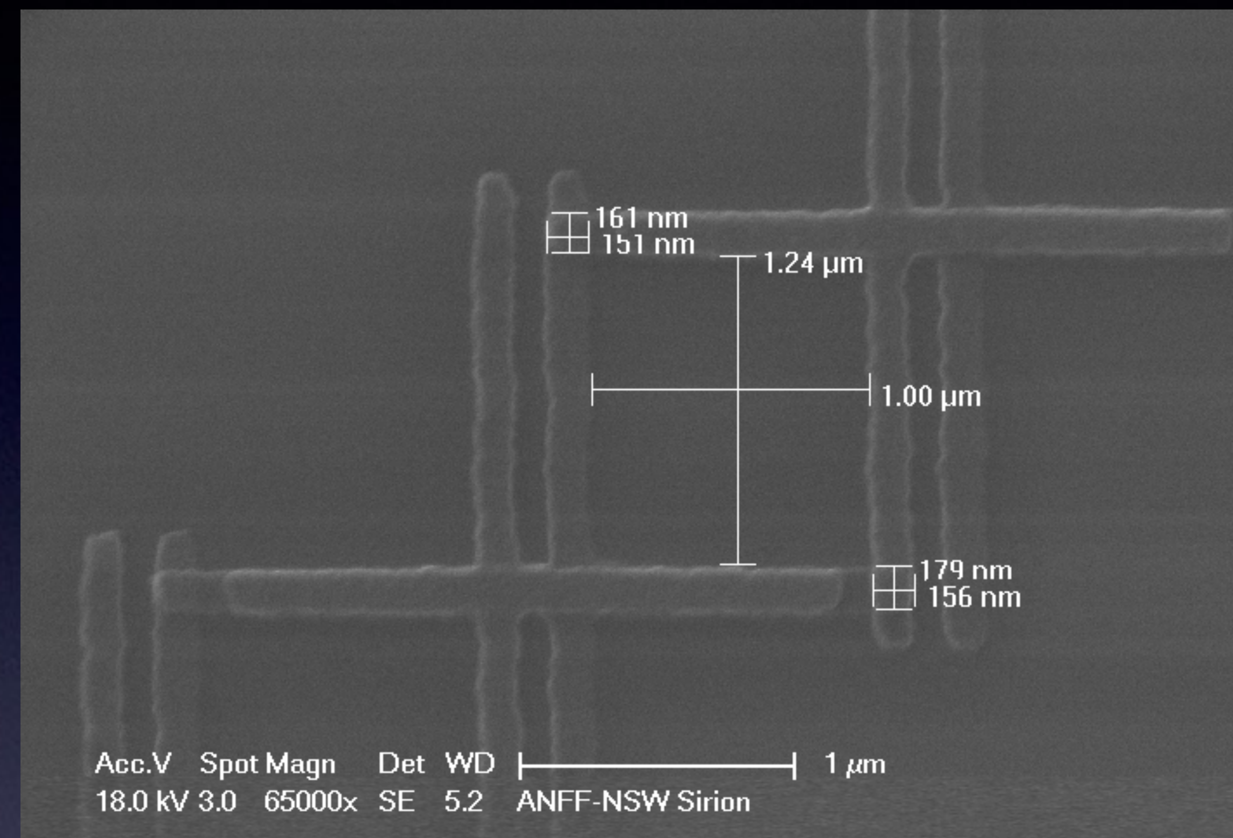


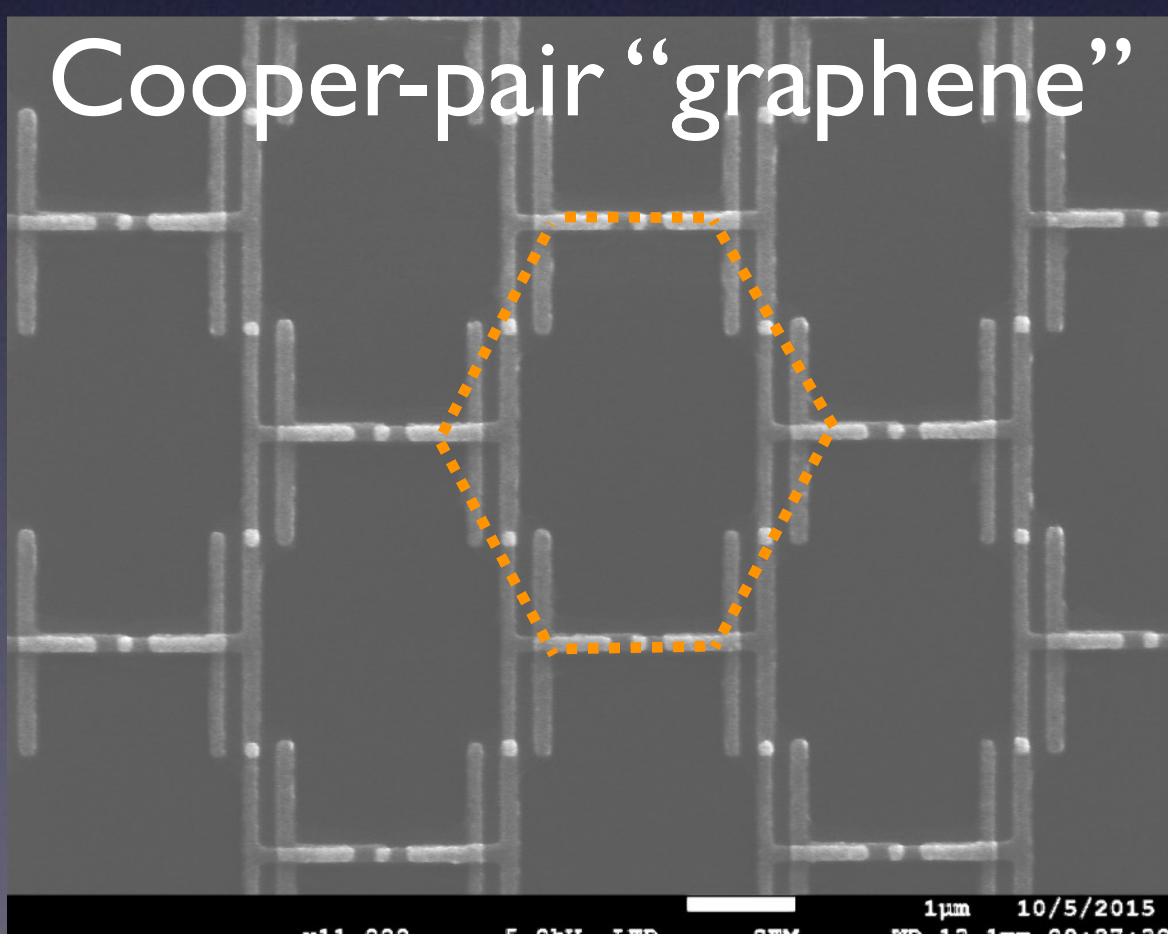
Exponentially protected two-level ground state arises from a novel quantum phase — a neutral superfluid of charge dipoles

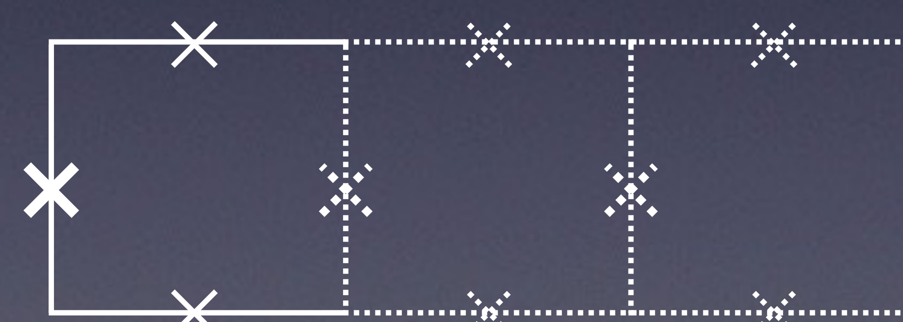
Quasi-1D and 2D Josephson-junction arrays



SQUID “ladder”

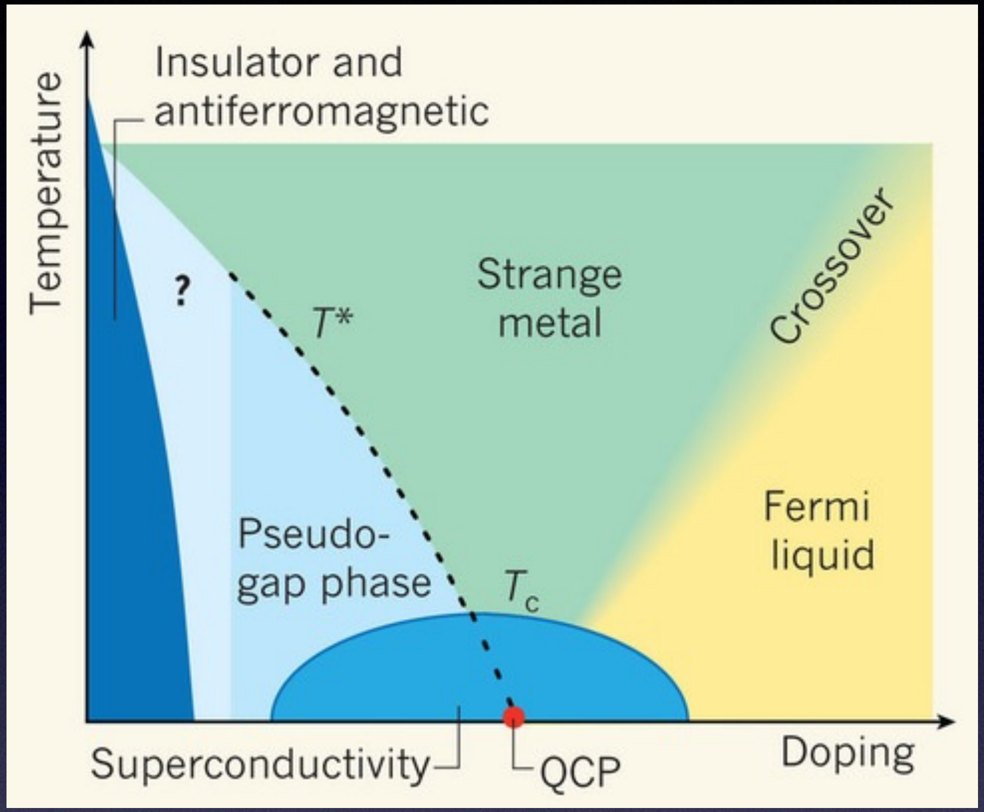






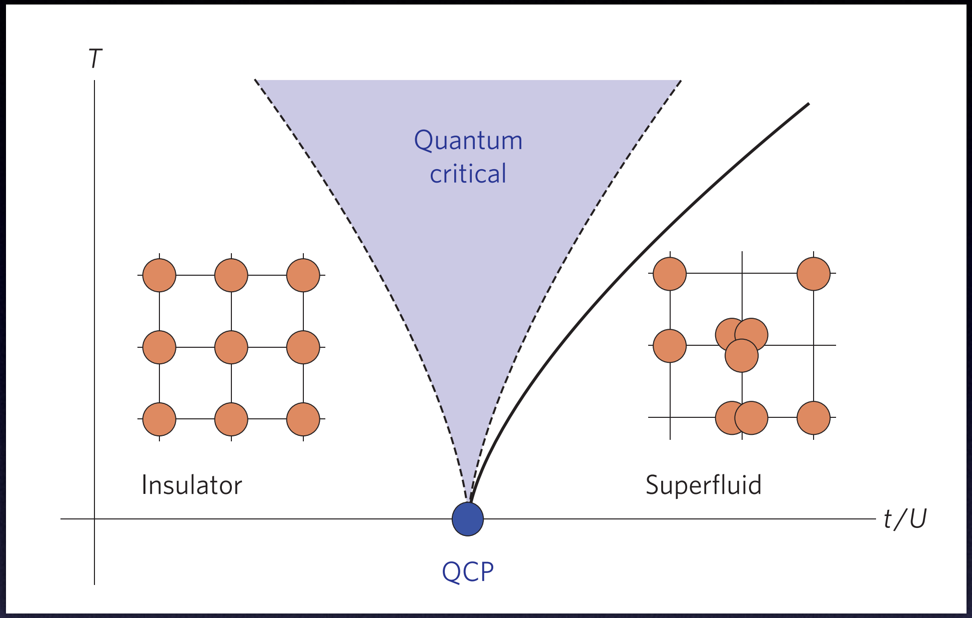
Quantum phase transitions and 2D quantum criticality

high Tc SC phase diagram with QCP?



2D Bose-Hubbard Model

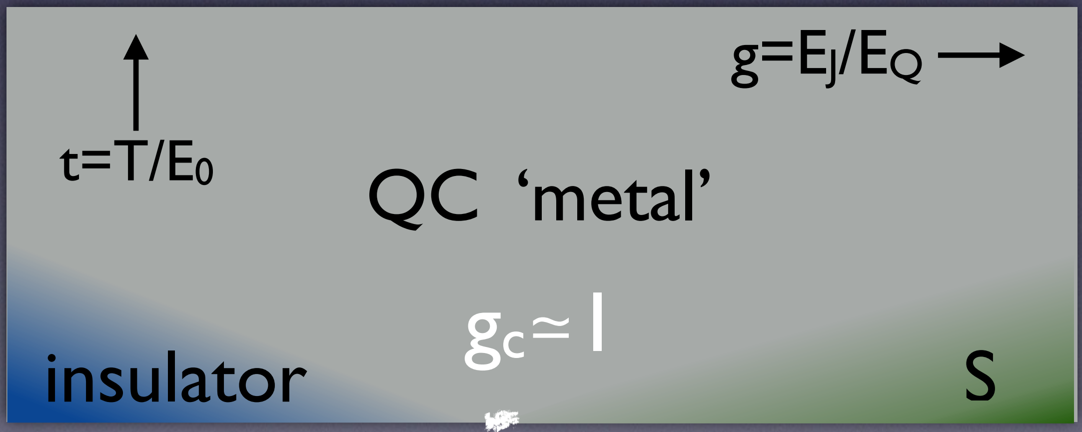
Witczak-Krempa,Sørensen and Sachdev
Nature Physics 10, 361 (2014)



quantum critical region:

- no simple quasiparticle description
- computationally 'hard' (entangled)
- Does AdS / CFT (gauge-gravity) correspondence actually apply to real CM systems?

Josephson-junction arrays



For the future:

- Can we identify and reduce origin of offset charge and reach the Mott insulator?
- Possible new physics in SQUID chains and ladders. Non-trivial and unexpected interplay of flux and charge? Parity?
- Can we surpass what is possible with real atomic systems: create exotic quantum phases, study quantum phase transitions, make novel devices and applications?
- Complex engineered (or synthetic) quantum matter using plain old s.c. aluminum. Precise nanofab is the key!