Energy Dependence of Current Noise in Superconducting/Normal Metal Junctions

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(Received 10 July 2003; published 12 March 2004)

Interference of electronic waves undergoing Andreev reflection in diffusive conductors determines the energy profile of the conductance on the scale of the Thouless energy. A similar dependence exists in the current noise, but its behavior is known only in a few limiting cases. We consider a metallic diffusive wire connected to a superconducting reservoir through an interface characterized by an arbitrary distribution of channel transparencies. Within the quasiclassical theory for current fluctuations we provide a general expression for the energy dependence of the current noise.

DOI: 10.1103/PhysRevLett.92.107004 PACS numbers: 74.45.+c, 72.70.+m, 74.40.+k

Interference of electronic waves in metallic disordered conductors is responsible for weak localization corrections to the conductance [1]. If these are neglected, the probability of transferring an electron through the diffusive medium is given by the sum of the *modulus squared* of the quantum probability amplitudes for crossing the sample along all possible paths. This probability is denoted as semiclassical, since quantum mechanics is necessary only for establishing the probability for following each path independently of the phases of the quantum amplitudes. In superconducting/normal metal hybrid structures, interference contributions are not corrections, they may actually *dominate* the above defined semiclassical result for temperatures and voltages smaller than the superconducting gap. This is seen experimentally as an energy dependence of the conductance on the scale of the Thouless energy. Indeed, the energy dependence comes from the small wave vector mismatch, linear in the energy of the excitations, between the electron and the Andreev reflected hole. This is responsible for the phase difference in the amplitudes for two different paths leading to interference. The effect is well known, and explicit predictions and measurements exist for a number of systems [2–4].

Interference strongly affects the current noise too [5]. The largest effects are expected in the tunneling limit, when the transparency of the barrier is small and its resistance is much larger than the resistance of the diffusive normal region. Then, the conductance has a strong nonlinear dependence at low bias (reflectionless tunneling) [2,3]. This is actually the case, but the zero-temperature noise (or shot noise) does not give any additional information on the system since it is simply proportional to the current, as shown numerically in a specific example in Ref. [6] and quite generally in Ref. [7]. The double tunnel barrier system has been considered in Ref. [8]. In the case of a diffusive metal wire in contact with a superconductor through an interface of conductance G_B much larger than the wire conductance G_D, Belzig and Nazarov [9] found that the differential shot noise, dS/dV, shows a reentrant behavior, as a function of the voltage bias, *similar,* but not *identical,* to the conductance one. (The extension of the Boltzman-Langevin approach to the coherent regime in Ref. [10] neglects this difference.) In order to compare quantitatively with actual experiments [11–13] and to gain more insight in the interference phenomenon, it is necessary to obtain the energy dependence of noise in more general situations. The numerical method used in Ref. [9] is, in principle, suitable to treat more general cases, notably the case when $G_D \gtrsim G_B$, but only if all channel transparencies, $\{\Gamma_n\}$, that characterize the interface are small. When arbitrary transparencies are present, one has to solve numerically an additional self-consistent equation [14]. This appears particularly heavy numerically if one is interested in treating a distribution of transparencies. We are not aware of results in this direction. In this Letter we present an analytical solution for the diffusion-type differential equation for the noise within the theory of current fluctuations [15] in the quasiclassical dirty limit [9]. It allows one to treat the general case of arbitrary values for $\{\Gamma_n\}$ and G_B/G_D. We express the noise in terms of a function that satisfies a linear differential equation to be solved numerically once the channel distribution is given. We can thus isolate the energy-independent (semiclassical) contribution and the interference contribution to both the conductance and the noise. We discuss their relation in the following.

We begin by stating the framework for the theory of current fluctuations [9,15]. It relies on the evaluation of the quasiclassical Green's functions $\check{g}(x, \varepsilon, \chi)$ in the Nambu($\hat{\ }$)-Keldysh($\bar{\ }$) space, at a given energy ε and counting field χ. The counting field appears as a gauge transformation of the Green's function in the normal reservoir

$$\check{g}_N(\chi) = e^{-(i/2)\chi\check{\tau}_K}\, \check{g}_N^0\, e^{(i/2)\chi\check{\tau}_K}, \tag{1}$$

where $\check{g}_N^0 = \hat{\tau}_3 \otimes \bar{\sigma}_3 + (f_{T0} + f_{L0}\hat{\tau}_3) \otimes (\bar{\sigma}_1 + i\bar{\sigma}_2)$ is the standard Green's function for a metallic reservoir,

$\bar{\sigma}_i$, $\hat{\tau}_{j(i,j=1,2,3)}$ are Pauli matrices, $\check{\tau}_K = \hat{\tau}_3 \otimes \bar{\sigma}_1$, $f_{L0} = 1 - f_+ - f_-$, $f_{T0} = f_- - f_+$, $f_\pm(\varepsilon) = f(\varepsilon \pm eV)$, f is the Fermi function at temperature T, and V is the voltage bias. Given $\check{g}(x)$, one can calculate the spectral matrix current in the wire

$$\check{J}(x) = -LG_D\check{g}(x)\partial_x\check{g}(x), \tag{2}$$

with x the coordinate along the wire of length L. The current and the zero-frequency noise are given, respectively, by [9]

$$I = J(\chi = 0) \quad \text{and} \quad S = -2ie\frac{\partial J(\chi)}{\partial \chi}\Big|_{\chi=0}, \tag{3}$$

where $J(\chi) = -1/(8e)\int d\varepsilon\,\mathrm{Tr}[\check{\tau}_K\check{J}(x)]$ and $J(\chi)$ does not depend on the position x where $\check{J}$ is evaluated.

The Green's function $\check{g}(x)$ in the wire is determined by the diffusionlike Usadel equation

$$-\frac{\hbar D}{LG_D}\partial_x\check{J}(x) = -i\varepsilon[\check{\tau}_3, \check{g}(x)] \tag{4}$$

(where $\check{\tau}_3 = \hat{\tau}_3 \otimes \bar{1}$ and D is the diffusion constant in the wire) and by boundary conditions at the two extremities. We assume a good contact on the normal side at $x = L$. This implies $\check{g}(L) = \check{g}_N$ as defined in (1). On the superconducting side, at $x = 0$, the boundary condition expresses the conservation of the spectral matrix current (2) through the interface [16,17]:

$$\check{J}(0) = G_Q\sum_n\frac{2\Gamma_n[\check{g}(0), \check{g}_S]}{4 + \Gamma_n[\{\check{g}(0), \check{g}_S\} - 2]}, \tag{5}$$

where the eigenvalues Γ_n of the transmission matrix through the interface appear explicitly and $G_B = G_Q\sum_n\Gamma_n$, with $G_Q = e^2/(\pi\hbar)$ the quantum of conductance. The Thouless energy, $E_T = \hbar D/L^2$, is the relevant energy scale in Eq. (4). We restrict this to the case where it is much smaller than the superconducting gap. With the same restrictions on the voltage bias and temperature, the Green's function in the superconducting reservoir is $\check{g}_S = \hat{\tau}_2 \otimes \bar{1}$. The quasiclassical Green's functions obey the normalization condition $\check{g}^2 = 1$ and the symmetry property

$$\check{g}(x)^\dagger = -\check{\tau}_L\check{g}(x)\check{\tau}_L, \qquad \text{with } \check{\tau}_L = \hat{\tau}_3 \otimes \bar{\sigma}_2. \tag{6}$$

Finding $J(\chi)$ for all values of χ is equivalent to calculating the full counting statistics of charge transfer [15], and it may be a formidable task. One of the main difficulties comes from the normalization condition $\check{g}^2 = 1$. As a matter of fact, for $\chi = 0$ the Green's functions have a triangular structure in Keldysh space:

$$\check{g}_0 = \check{g}(\chi = 0) = \begin{pmatrix} \hat{g}^R & \hat{g}^K \\ 0 & \hat{g}^A \end{pmatrix}. \tag{7}$$

In the absence of supercurrent it exists a simple parametrization fulfilling the normalization condition and the symmetry property (6): $\hat{g}^R = \hat{\tau}_3\cosh\theta + i\hat{\tau}_2\sinh\theta$, $\hat{g}^K = \hat{g}^R\hat{f} - \hat{f}\hat{g}^A$, and $\hat{f} = f_L + \hat{\tau}_3 f_T$, with $\theta = \theta_1 + i\theta_2$ and f_L, f_T, θ_1, and θ_2 real. When $\chi \neq 0$ the triangular structure is lost and we are not aware of a simple parametrization for $\check{g}$ in this case.

A less ambitious program is to calculate only the noise. In this case it is clear from Eq. (3) that the full dependence of $\check{g}(\chi)$ is not needed: $\partial\check{g}(\chi)/\partial\chi|_{\chi=0}$ is enough. As suggested in Ref. [9], one can thus develop $\check{g}(\chi)$ in powers of χ,

$$\check{g}(x, \chi) = \check{g}_0(x) - i(\chi/2)\check{g}_1(x) + \mathcal{O}(\chi^2), \tag{8}$$

and solve the problem order by order in χ. For the leading order Green's function, $\check{g}_0$, the above parametrization is appropriate. Equation (4) for $\hat{g}^R$ gives the nonlinear equation

$$\hbar D\theta''(x) + 2i\varepsilon\sinh\theta(x) = 0, \tag{9}$$

with boundary conditions $\theta(L) = 0$ and

$$L\theta'(0) = \frac{i}{r}\left\langle\frac{\cosh\theta(0)}{1 + \frac{\Gamma}{2}[i\sinh\theta(0) - 1]}\right\rangle. \tag{10}$$

We defined $\langle\psi(\Gamma)\rangle \equiv \sum_n\Gamma_n\psi(\Gamma_n)/\sum_n\Gamma_n$ and $r = G_D/G_B$. Equation (4) for $\hat{g}^K$ gives $f_L(x) = f_{L0}$ and the differential equation for f_T: $[\cosh^2\theta_1(x)f_T'(x)]' = 0$ with boundary conditions $f_T(L) = f_{T0}$ and

$$f_T'(0) = \frac{f_T(0)\theta_1'(0)}{\cosh\theta_1(0)\sinh\theta_1(0)}. \tag{11}$$

Then the current is $I = 1/(2e)\int d\varepsilon\,\mathcal{G}(\varepsilon)f_{T0}(\varepsilon)$ with [2]

$$\mathcal{G}(\varepsilon) = G_D\left[\mathcal{D}^{-1}(\varepsilon) + \frac{\tanh\theta_1(0)}{L\theta_1'(0)}\right]^{-1} \tag{12}$$

and $\mathcal{D}^{-1}(\varepsilon) = 1/L\int_0^L ds/\cosh^2\theta_1(s)$. At low temperatures, $k_BT \ll eV$, $G(V) = \mathcal{G}(eV)$ is the differential conductance. These equations have been recently exploited in Ref. [18] to discuss the conductance.

At the next order in χ, we get a linear matrix differential equation for $\check{g}_1(x)$:

$$\hbar D\partial_x\check{J}_1(x) = i\varepsilon LG_D[\check{\tau}_3, \check{g}_1(x)] \tag{13}$$

with $\check{J}_1(x) = -LG_D[\check{g}_0(x)\partial_x\check{g}_1(x) + \check{g}_1(x)\partial_x\check{g}_0(x)]$. The boundary conditions read $\check{g}_1(L) = [\check{\tau}_K, \check{g}_N^0]$ on the normal side and $\check{J}_1(0) = 2G_B\langle\check{A}\check{B}\check{A}\rangle$ where $\check{A} = [4 + \Gamma(\{\check{g}(0), \check{g}_S\} - 2)]^{-1}$ and $\check{B} = (4 + 2\Gamma\{\check{g}_S\check{g}_0(0)\check{g}_1(0)\check{g}_S - \check{g}_0(0)\check{g}_1(0) - [\check{g}_1(0), \check{g}_S]\})$ on the superconducting side. Finally, the normalization of $\check{g}$ gives the condition $\{\check{g}_0(x), \check{g}_1(x)\} = 0$ that can be fulfilled with the change of variable $\check{g}_1(x) = [\check{g}_0(x), \check{\phi}(x)]$. The matrix $\check{\phi}(x)$ is now constrained by the symmetry properties (6) only. We find that Eq. (13) can be conveniently solved with the following parametrization for $\check{\phi}$:

$$\check{\phi} = \begin{pmatrix} af_{T0}\hat{\tau}_1 - c\hat{f}\hat{\tau}_3 & b\hat{\tau}_3 + d \\ c\hat{\tau}_3 & a^*f_{T0}\hat{\tau}_1 + c\hat{f}\hat{\tau}_3 \end{pmatrix} \tag{14}$$

with $a = a_1 + ia_2$ and a_1, a_2, b, c, d real functions of x. Substituting $\check{g}_1$ in terms of $\check{\phi}$ into (13) after straightforward but lengthy calculations we obtain the set of equations and boundary conditions for the four parameters

a, b, c, and d. The parameter a plays the role of θ in the lowest order Eq. (9):

$$\hbar D a''(x) + 2i\varepsilon a(x)\cosh\theta(x) = -2E_T \frac{\sinh\theta_1(x)}{\cosh^3\theta_1(x)} \frac{G(\varepsilon)^2}{G_D^2}. \quad (15)$$

The boundary conditions are $a(L) = 0$ and $La'(0) = \alpha a(0)/r + \beta/r$ with

$$\alpha = \left\langle \frac{i\sinh\theta - \Gamma(i\sinh\theta - 1)/2}{[1 + \Gamma(i\sinh\theta - 1)/2]^2} \right\rangle, \qquad \beta = \frac{ic^2}{8}\left\langle \frac{2\Gamma^2\cosh\theta^* + 8(\Gamma - 1)\cosh\theta - 2i\Gamma(\Gamma - 2)\sinh\theta\cosh\theta^*}{|1 + \Gamma(i\sinh\theta - 1)/2|^2[1 + \Gamma(i\sinh\theta - 1)/2]} \right\rangle, \quad (16)$$

both evaluated at $x = 0$. The equation for b resembles that for f_T: it is solved analytically in terms of an integral that enters the final expression for the noise. The parameter c turns out to be simply proportional to $f_T(x)$: $c(x) = -f_T(x)/f_{T0}$ with $c(0) = 1 - \mathcal{G}(\varepsilon)/[G_D\mathcal{D}(\varepsilon)]$. Finally, we find $d(x) = 2f_{L0}f_{T0}[1 + c^3(x) - 2\tan\theta_2(x)]$. Its knowledge is not necessary to obtain the noise.

The expression for the noise is obtained by evaluating Eq. (3):

$$S = \int d\varepsilon\, \mathcal{G}(\varepsilon)\{1 - f_{L0}^2(\varepsilon) - [1 - \mathcal{F}(\varepsilon)]f_{T0}^2(\varepsilon)\}, \quad (17)$$

where

$$\mathcal{F}(\varepsilon) = \frac{2}{3}[1 + c(0)^3] + \frac{2\mathcal{G}(\varepsilon)}{G_D}\int_0^L \frac{\sinh\theta_1 a_1}{\cosh^3\theta_1} ds - c(0)\left(\frac{G_D a_1'(0)c(0)}{\mathcal{G}(\varepsilon)\tanh\theta_1(0)} + \frac{2a_1(0)}{\sinh 2\theta_1(0)}\right). \quad (18)$$

In equilibrium, Eq. (17) yields the fluctuation dissipation relation $S = 4k_BTG$, as expected. Out of equilibrium, in the shot-noise regime $k_BT \ll eV$, Eq. (18) defines the experimentally accessible differential Fano factor $F(V) \equiv [dS(V)/dV]/[2eG(V)] = \mathcal{F}(eV)$.

Equation (18) is the central result of this Letter. Once the problem for the conductance has been solved and $\theta(x)$ and $f_T(x)$ are known [2,18], we provide a simple and efficient way to calculate the noise. It suffices to solve the linear differential Eq. (15) with the given boundary conditions, and substitute the result into Eq. (18). This program has to be followed numerically in most situations, but its implementation is straightforward and allows one to obtain quantitative predictions for a wide range of realizable experiments. The previous approach to the same problem has been to discretize Eq. (4) from the outset and solve it numerically for $\Gamma_n \ll 1$ [9,17].

We can now discuss the crossover from the coherent ($\varepsilon \ll E_T$) to the incoherent semiclassical ($\varepsilon \gg E_T$) regime quite generally. In the fully coherent regime we recover known expressions obtained with random-matrix theory [4,19] or quasiclassical Green's functions [20]. In the opposite limit, propagation is incoherent: because of the large value of ε the phases accumulated along two different paths are always uncorrelated. In order to verify this point, we first repeated the calculation when both reservoirs are normal. It suffices to substitute $\check{g}_S$ with $\check{g}_N$ into (5) with $eV = 0$. As expected, we find no energy dependence for G and F. For the conductance the usual Ohm's law holds, $G^{-1} = G_D^{-1} + G_B^{-1}$, while for the Fano factor we find

$$F = \frac{1}{3}\left[1 + \left(2 - 3\frac{\sum_n \Gamma_n^2}{\sum_n \Gamma_n}\right)\frac{G_D^3}{(G_D + G_B)^3}\right]. \quad (19)$$

Equation (19) coincides with the semiclassical result obtained in Ref. [21]. In the superconducting case, Eqs. (12) and (18) can be evaluated analytically for $\varepsilon \gg E_T$ defining $G_{\rm inc}$ and $F_{\rm inc}$, respectively. It turns out that the conductance and the Fano factor are given by the same expressions for the normal case multiplied by a factor 2 and where the substitutions $G_D \to G_D/2$ and $\Gamma_n \to \Gamma_n^A = \Gamma_n^2/(2 - \Gamma_n)^2$ have been operated [the last one implies $G_B \to G_B\langle\Gamma/(2 - \Gamma)^2\rangle$] as expected from semiclassical arguments [22–24].

Quantum corrections to both $G(\varepsilon)$ and $F(\varepsilon)$ can be evaluated as an expansion for large ε. We give the result for all $\Gamma_n = \Gamma$ [25]: $G = G_{\rm inc} + G_1(\varepsilon)$, with $G_1 = 2\sqrt{E_T/\varepsilon}\,(4 - 4\Gamma - \Gamma^2)/[r(\Gamma - 2)^2 + 2\Gamma]^2$, and

$$F(\varepsilon) = F_{\rm inc} - r\gamma(\Gamma, r)[G(\varepsilon) - G_{\rm inc}] \quad (20)$$

with $\gamma = [r(2 - \Gamma)^2\{64 + (2 - \Gamma)\Gamma[64 + r(2 - \Gamma) \times (12 - 12\Gamma - \Gamma^2)]\}]/\{[r(2 - \Gamma)^2 + 2\Gamma]^2(4 - 4\Gamma - \Gamma^2)\}$. γ is positive for most values of Γ. Equation (20) relates the interference contributions to G with those to F for large ε. When the barrier dominates ($r \gg 1$), the relation (20) holds for any value of ε up to first order in $1/r$ with $\gamma(\Gamma) = \gamma(\Gamma, r \to \infty)$ and $G_1(\varepsilon) = 2\phi(2\sqrt{\varepsilon/E_T}) \times (4 - 4\Gamma - \Gamma^2)/[r^2(\Gamma - 2)^4]$ and $\phi(x) = (\sinh x + \sin x)/[x(\cosh x + \cos x)]$. This suggests that the simple relation (20) stands, though approximately, beyond the range of validity for which it has been proved. A possible interpretation is that interference modifies the effective transparency of the whole system. The linear relation between the quantum corrections to G and F then corresponds to the single channel quantum result: $G \propto \Gamma$ and $F \propto 1 - \Gamma$ [5].

For intermediate values of the parameters r, ε, and Γ_n, quantum interference contribution can be studied numerically. We report the results in Fig. 1 for $G_D = G_B$ and different values of $\Gamma = \Gamma_n$ for all n. The transparency Γ drives a smooth crossover from a reflectionless tunneling to a reentrant behavior that can be seen both in the conductance and in the Fano factor. This proves that the energy dependence of both may be strongly affected by the precise set $\{\Gamma_n\}$ of transparencies of

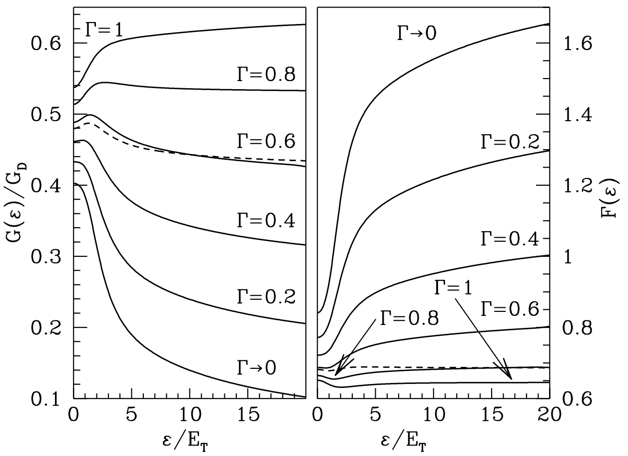

FIG. 1. Energy dependence of the conductance and of the differential Fano factor when the wire and the interface have the same conductance ($G_D = G_B$) for different values of $\Gamma = \Gamma_n$ [25]. The dashed lines correspond to the universal distribution $\{\Gamma_n\}$ for disordered interfaces.

the interface. As a relevant example we plot the result for the universal distribution for disordered interfaces $\sum_n \delta(\Gamma - \Gamma_n) \propto 1/(\Gamma^{3/2}\sqrt{1-\Gamma})$ [26]. Figure 1 also confirms that Eq. (20) is actually qualitatively satisfied in the whole range of parameters.

In the limit $G_D \ll G_B$ ($r \to 0$) the actual values of $\{\Gamma_n\}$ drop from the boundary conditions. Indeed, Eqs. (10), (11), and (16) force $\theta(0) = -i\pi/2$, $f_T(0) = 0$ [and thus $c(0) = 0$], and $a(0) = 0$. The conductance becomes $G(V) = \mathcal{D}(eV)G_D$ and the Fano factor is given by Eq. (18) where only the 2/3 and the integral terms survive. The first term is the semiclassical incoherent value [27], which also coincides with the fully coherent one [4]. The integral term singles out the interference contribution to the Fano factor. The energy dependence coincides with that obtained previously in a different way [9], which, in turn, agrees qualitatively with the experimental result [12]. However, a broader voltage range for the reentrance in $G(V)$ and $S(V)$ is predicted [28]. A possible explanation is that the barrier resistance is not negligible. Assuming a disordered interface, we find that the reasonably small value of $r = 0.3$ allows one to fit the conductance. It also improves the fit for the noise, but the agreement is not perfect. This may be either due to a different distribution of transparencies at the barrier or, as suggested in Ref. [9], due to heating or interaction effects.

In conclusion, we provide a framework to calculate both conductance and noise in a normal metallic wire connected to a superconducting lead through an arbitrary interface. We predict a strong energy dependence of the coherent contribution to the Fano factor. We suggest to exploit this dependence to experimentally characterize the transparency of interfaces.

We thank V. P. Mineev, M. Sanquer, and F. W. J. Hekking for useful discussion. F. P. acknowledges financial support from CNRS through Contract No. ATIP-JC 2002.

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