

Coulomb blockade for an oscillating tunnel junction

N. Pauget and F. Pistolesi

Laboratoire de Physique et Modélisation des Milieux Condensés, CNRS, BP 166, Université Joseph Fourier, 38042 Grenoble, France

M. Houzet

INAC/SPSMS, CEA Grenoble, 17, Rue des Martyrs, 38054 Grenoble Cedex 9, France

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We consider a tunnel junction formed between a fixed electrode and an oscillating one. Accumulation of the charge on the junction capacitor induces a force on the nanomechanical oscillator. The junction is voltage biased and connected in series with an impedance $Z(\omega)$. We discuss how the picture of Coulomb blockade is modified by the presence of the oscillator. Quantum fluctuations of the mechanical oscillator modify the I - V characteristics particularly in the strong Coulomb blockade limit. We show that the oscillator can be taken into account by a simple modification of the effective impedance of the circuit. We discuss in some details the case of a single inductance $Z(\omega)=iL\omega$ and of a constant resistance $Z(\omega)=R$. With little modifications the theory applies also to incoherent transport in Josephson junctions in the tunneling limit.

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I. INTRODUCTION

The improved ability to build electronic devices on the nanometer scale opens the perspective to exploit and study the coupling between electronic transport and mechanical degrees of freedom.^{1,2} The elementary system typically considered in nanoelectromechanics is the harmonic oscillator coupled in some way to the electronic degrees of freedom. A widely studied device is the single-electron transistor, which consists in a small metallic island connected to the leads through two tunnel barriers. The oscillating part of the device can be either a nearby gate electrode or the island itself. The motion of the device modifies mainly two quantities: the capacitances and the (bare) tunneling rates. According to which dependence dominates, several effects have been predicted or observed. When only the capacitance is modified by the oscillation, it has been shown in the weak-coupling limit that the variance of the oscillator position satisfies the equipartition theorem with the voltage bias replacing the temperature.³ Under specific conditions, the system may also undergo dynamical instabilities.⁴⁻⁶ In the strong-coupling limit, a new kind of current blockade (Frank-Condon blockade) has been predicted.⁷⁻¹⁰ This effect also persists in the classical limit.¹¹⁻¹³ When the oscillation modifies the distance between the metallic leads, it has been shown that a dynamical instability can occur.¹⁴⁻²⁰ This instability (called shuttle instability) is characterized by a synchronization of the electronic transport with the mechanical oscillations, as is clearly seen from the full counting statistics of charge transport.^{21,22} Observation of shuttling is difficult since the distance between the metallic leads must be comparable with the tunneling length in order to produce a measurable current.²³⁻²⁶ Very recently indications of shuttling have been found by Kim *et al.*²⁷ probably in an intermediate regime between tunneling and field emission. The wide interest for the single-electron transistor comes from its high sensitivity to a small variation of the gate voltage. Thus, it can be used, for instance, as a sensitive displacement detector²⁸ and it has been shown to reach the quantum back-action limit.²⁹

An even simpler device is the single tunnel junction where the distance between the two metals constitutes the

mechanical degree of freedom. The effect of the position dependence of the tunneling resistance has been already considered in the literature,³⁰⁻³² both for the current and the noise of the device. In this case, the force acting on the oscillator originates from the electron-momentum transfer. Recently, the position fluctuation has been detected by measuring the current fluctuations.³³ This experiment probed the back action of the current crossing the tunnel junction on the oscillator. Apparently, the position dependence of the tunneling matrix elements is not sufficient to explain the intensity of the back action. To our knowledge, the effect of the force coming from the variation of the capacitance C as a function of the distance between the leads has not been considered in this context so far. A particularly interesting case is when the device is voltage biased in series with an impedance $Z(\omega)$ leading, under certain conditions, to the Coulomb blockade physics.³⁴ Then, similarly to what happens for the single-electron transistor, one can expect that a small position dependence of the capacitance can affect the current-voltage characteristics of the device.

In this paper, we consider the effect of the mechanical oscillator on the I - V curve. We will work within the assumptions of the standard Coulomb blockade theory.³⁴ Specifically, we assume that the tunneling resistance is sufficiently large so that the oscillator and the electromagnetic environment have time to relax to equilibrium between two tunneling events. This is the opposite limit with respect to the one considered in Refs. 31 and 32 for the tunneling dependent Hamiltonian. There, the tunneling is so frequent that the system may reach a stationary off-equilibrium state. Restricting to the linear coupling of the oscillator with the electromagnetic modes, we will discuss the effect of the quantum fluctuations of the mechanical oscillator on the current.

The paper is organized as follows. In Sec. II we describe the model and its range of validity. We show that within a linear approximation the mechanical oscillator can be accounted by an effective impedance. In Sec. III we consider two specific cases of $Z(\omega)$ and discuss the resulting conductance at zero temperature. In Sec. IV we consider the case of an oscillating Josephson junction by extending the results of

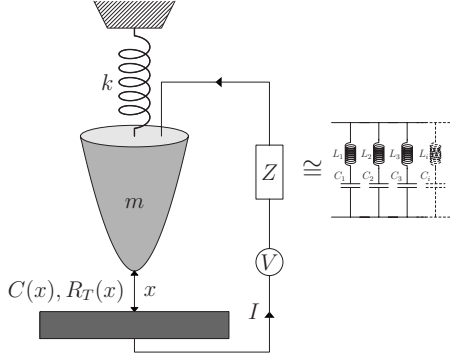


FIG. 1. Schematic representation of the system. A tunneling contact is formed between an oscillating electrode (here depicted as a STM tip) and a fixed surface. The oscillator is modeled by a spring constant k and an effective mass m . The junction between the tip and the plate is characterized by an effective capacitance $C(x)$ and resistance $R_T(x)$, where x denotes the position of the tip. The circuit is closed by an impedance Z and a voltage source V .

Sec. III to the superconducting case. Section V gives our conclusions.

II. MODEL

We consider the electronic transport through a circuit composed of a tunnel junction in series with an arbitrary impedance $Z(\omega)$ and biased with an ideal voltage source V . We assume that one of the two metallic leads forming the tunnel junction is free to oscillate and can thus modify its distance with respect to the other one. The single degree of freedom describing the oscillation is given by x , the oscillator position. Its effective mass and spring constant are m and k , respectively. The tunneling surface is very small. Thus, it is important to account for the effective capacitor forming at the tunnel barrier. Its capacitance $C(x)$ depends on the position of the oscillator. The tunneling resistance $R_T(x)$ may also depend on x . In the following we will neglect this dependence by showing that its effect is negligible with respect to that of $C(x)$ in the regime of rare tunneling events that we are considering. The device is depicted schematically in Fig. 1.

A. Time scales

As anticipated in Sec. I we will work within the same hypothesis that are commonly used to describe Coulomb blockade. Specifically, we assume that the relaxation times for the electromagnetic environment, $1/\gamma_{RC} = RC$, and for the oscillator, $1/\gamma$, are shorter than the average time between two tunneling events, $\tau_T \sim eR_T/V$. Here, $R \sim \text{Re}[Z]$ is the typical resistance associated with the impedance Z and γ is the damping coefficient of the mechanical oscillator with frequency $\omega_m = \sqrt{k/m}$. If $1/\gamma_{RC}$, $1/\gamma \ll \tau_T$, the system has time to come back to the equilibrium state after an electron has crossed the tunnel barrier. This allows us to calculate the average current on the equilibrium quantum state of the environment. It gives the following limitations on the voltage: $eV \ll E_C(R_T/R)$ and $eV \ll e^2\gamma R_T$, where $E_C = e^2/2C$ is the Coulomb energy. Since we are interested in the range of

voltages around E_C/e , we assume that $R_T \gg R$ and $R_T \gg 1/C\gamma$. The first condition is the standard tunneling condition; the second involves the mechanical damping. One should remember that the damping of the oscillator also depends on the impedance of the circuit. Thus, even if the oscillator is not damped by a mechanical source, its quality factor would not be infinite. In the limit of large R_T the above conditions can thus always be satisfied.

Let us now discuss the effect of the position dependence of the tunneling resistance. To evaluate the importance of this effect for the dynamics of the junction, we consider the effective friction coefficient generated by the position dependence of R_T .³¹

$$\gamma_T \approx \frac{R_Q}{R_T} \left(\frac{x_o}{\lambda_T} \right)^2 \omega_m, \quad (1)$$

where $R_Q = 2\pi\hbar/e^2$ is the quantum of resistance, $x_o^2 = \hbar/2m\omega_m$ is the (square of the) zero-point motion amplitude of the oscillator and λ_T is the tunneling length defined by $\lambda_T^{-1} = (dR_T/dx)/R_T$. The explicit expression of the friction coefficient induced by the electromagnetic dissipation is given below for a specific case [cf. Eq. (28)]. Independently on its explicit form, what matters is that it does not depend on the tunneling resistance since the damping is due to the environment and the capacitive coupling. Thus the ratio γ_T/γ can always be made small by choosing a large enough tunneling resistance. Then we may safely neglect the position dependence of the tunneling matrix elements. We will come back to this point at the end of Sec. III.

B. Derivation of the general expression for the current

The Hamiltonian describing the system is the following:

$$H = H_j + H_{osc} + H_{EM}, \quad (2)$$

where $H_j = \sum_k \xi_k c_k^\dagger c_k + \sum_q \xi_q c_q^\dagger c_q + H_T$ with $H_T = A + A^\dagger$ and $A = \sum_{k,q} t_{kq} e^{i\phi(t)} c_k^\dagger c_q$. Here, k and q label the fermionic eigenstates in the fixed and oscillating electrode, respectively, c_k is a destruction operator in state k , with energy ξ_k , and t_{kq} is the tunneling matrix element. The phase $\phi(t)$ is obtained by integrating the voltage difference at the junction: $\phi(t) = \int_{-\infty}^t (eV_j(t')/\hbar) dt'$. For a constant voltage-biased junction $\phi(t) = eVt/\hbar$. However, due to the impedance in series with the junction, the voltage fluctuates and, in general, V_j is different from V . The Coulomb blockade physics stems from the behavior of the phase ϕ and, specifically, from its correlation function. The current through the junction can be obtained by perturbation theory in H_T . The operator for the current from the mobile electrode to the fixed one reads $I = -ie(A - A^\dagger)/\hbar$. Linear response theory gives

$$I = \frac{e}{\hbar^2} \sum_{k,q} |t_{kq}|^2 \int_{-\infty}^{+\infty} dt e^{-i(\xi_k - \xi_q)t/\hbar} [\langle e^{-i\phi(t)} e^{i\phi(0)} \rangle f_k(1 - f_q) - \langle e^{i\phi(0)} e^{-i\phi(t)} \rangle f_q(1 - f_k)], \quad (3)$$

where $f_{k,q} = 1/(e^{\beta\xi_{k,q}} + 1)$ are the Fermi distributions in the leads and β is the inverse temperature. In order to include the quantum fluctuation of the electromagnetic fields, the phases

in this expression must be regarded as operators whose dynamics is determined by the rest of Hamiltonian (2). We will restrict to the quadratic part of the Hamiltonian. Within this approximation, the fields ϕ satisfy Wick theorem and expression (3) can be simplified by making use of the exact relation,

$$\langle e^{i\tilde{\phi}(t)} e^{-i\tilde{\phi}(0)} \rangle = e^{J(t)-J(0)}, \quad (4)$$

where $\tilde{\phi} = \phi(t) - eVt/\hbar$ and $J(t) = \langle \tilde{\phi}(t) \tilde{\phi}(0) \rangle$.

The current can then be expressed in terms of the Fourier transform of $e^{J(t)-J(0)}$,

$$P(E) = \int_{-\infty}^{+\infty} \frac{dt}{2\pi\hbar} e^{J(t)-J(0)+iEt/\hbar}. \quad (5)$$

The function $P(E)$ gives the probability that the environment absorbs a quantity of energy equal to E during the tunneling process. Finally the current in terms of $P(E)$ reads³⁴

$$I = \frac{1}{eR_T} \int_{-\infty}^{+\infty} EdEP(eV - E) \frac{(1 - e^{-\beta eV})}{1 - e^{-\beta E}}, \quad (6)$$

where R_T is the tunneling resistance in the absence of the environment [i.e., when $P(E) = \delta(E)$].

In order to find the behavior of the device we need to determine the function $P(E)$ in the presence of the oscillator. Since we will restrict to the quadratic part of the Hamiltonian, we can obtain the quantum phase correlation function $J(t)$ from the classical response function of the phase to a current source at the junction. Specifically we add to the total Hamiltonian (2) the term $-I_{\text{ext}}(t) \tilde{\phi}_{\text{int}}(t) \hbar/e$, where $\tilde{\phi}_{\text{int}}(t)$ is the phase in the interacting picture. The linear response to the external field I_{ext} gives

$$\langle \tilde{\phi}(t) \rangle = - \int_{-\infty}^{+\infty} dt' \chi(t-t') I_{\text{ext}}(t') \frac{\hbar}{e}, \quad (7)$$

with $\chi(t) = \frac{i}{\hbar} \langle [\tilde{\phi}_{\text{int}}(0), \tilde{\phi}_{\text{int}}(t)] \theta(t) \rangle$. Using the fluctuation-dissipation theorem, one can show that

$$J(\omega) = -2\hbar \frac{\text{Im}[\chi(\omega)]}{1 - e^{\beta\hbar\omega}}. \quad (8)$$

Since the Hamiltonian is quadratic, $\chi(\omega)$ can be calculated from the classical response function (see, for instance, Ref. 35). The problem is now reduced to the determination of the classical retarded response function. This can be readily obtained by solving the classical equations of motion by Laplace transform.

We determine now the equations of motion for the phase, the position of the oscillator, and the degrees of freedom of the environment. To describe the dissipation we introduce a bath of harmonic oscillators coupled to the mechanical oscillator and a bath of electromagnetic oscillators coupled to the current through the tunnel junction. The coupling of the mechanical and electrical part comes from the charge energy $Q^2/2C(x)$ of the capacitor. The equations of motion read

$$m\ddot{x} = -kx - \sum_p k_p(x - x_p) + \frac{1}{2} C'(x) \dot{\phi}^2 \left(\frac{\hbar}{e} \right)^2, \quad (9)$$

$$C(x) \ddot{\phi} = -C'(x) \dot{x} \dot{\phi} + \sum_n \frac{1}{L_n} (\phi_e - \phi_n - \phi) + I_{\text{ext}}(t) \frac{e}{\hbar}, \quad (10)$$

$$m_p \ddot{x}_p = k_p(x - x_p), \quad (11)$$

$$C_n \ddot{\phi}_n = \frac{1}{L_n} (\phi_e - \phi_n - \phi). \quad (12)$$

Here x_p and ϕ_n are the degrees of freedom of the environment, $\phi_e(t) = eVt/\hbar$ is due to the constant external voltage source, and we choose the zero of x as the equilibrium position of the oscillator when $V=0$. We also define $C'(x) = dC/dx$.

For $I_{\text{ext}}=0$, this system of equations has a stationary solution given by $x=x_{\text{eq}}=C'(x_{\text{eq}})V^2/2k$, $\dot{\phi}=eV/\hbar$, $x_p=x_{\text{eq}}$, and $\phi_n=0$. This solution is unique for a given gauge. We thus expand the equations around this solution and consider the quadratic fluctuations of the fields only: $x(t)=x_{\text{eq}}+\tilde{x}(t)$, $x_p(t)=x_{\text{eq}}+\tilde{x}_p(t)$, and $\phi(t)=eVt/\hbar+\tilde{\phi}(t)$. Let us introduce the Laplace transform for the fluctuating fields. For example, for x it reads $\tilde{x}(s) = \int_0^{+\infty} dt e^{-st} \tilde{x}(t)$ with $\text{Re}[s] > 0$ and $\tilde{x}(t) = \int_{-i\infty+a}^{+i\infty+a} (ds/2\pi i) e^{st} \tilde{x}(s)$ with $a > 0$. The system of differential equations then becomes an algebraic system of equations. We are interested in the response function. Thus, we can neglect transient terms. Keeping only linear terms in the fluctuating fields and solving first for the degrees of freedom of the environment, one finds

$$[s^2 + \gamma(s)s + \omega_m^2] \tilde{x} = \frac{C'V\hbar}{em} s \tilde{\phi}, \quad (13)$$

$$[Cs + Y(s)] s \tilde{\phi} = -C' s \tilde{x} \frac{eV}{\hbar} + I_{\text{ext}}(t) \frac{e}{\hbar}, \quad (14)$$

where we have introduced the memory function of the oscillator and the impedance of the circuit,

$$\gamma(s) = \frac{1}{m} \sum_p k_p \frac{s}{s^2 + \omega_p^2}, \quad Z(s/i)^{-1} = \sum_n \frac{1}{L_n} \frac{s}{s^2 + \omega_n^2}.$$

The two coupled Eqs. (13) and (14) describe the response of the phase $\tilde{\phi}$ to the external current I_{ext} flowing through the tunnel junction. Solving for $\tilde{\phi}$ and letting $s \rightarrow i\omega + 0^+$, we obtain the expression for $\chi(\omega)$. Using Eq. (8), the expression for $J(\omega)$ reads

$$J(\omega) = \frac{4\pi \text{Re}[Z_i(\omega)]}{\omega R_Q} \frac{1}{1 - e^{\beta\hbar\omega}}, \quad (15)$$

with

$$Z_i(\omega) = \left[i\omega C + Z(\omega)^{-1} + \frac{C'^2 V^2 i\omega/m}{\omega_m^2 + \gamma(i\omega)i\omega - \omega^2} \right]^{-1}. \quad (16)$$

The usual result of the Coulomb blockade can be obtained by setting $C'=0$ in this expression. Then, only the electromagnetic environment would contribute to the response function. The presence of the oscillator simply modifies the response

of the system; for γ constant, it would be equivalent to a RLC circuit in parallel with the tunnel junction.

The current-voltage characteristics can now be obtained by combining Eqs. (5), (6), (15), and (16). In the following section, we consider two representative cases for the circuit impedance and discuss the expected conductances.

III. RESULTS IN SPECIFIC CASES

We consider now two simple cases for the impedance of the circuit: $Z=i\omega L$ and $Z=R$. In the first case (Sec. III A), the electromagnetic environment is an LC oscillator. Thus, the system reduces to two coupled oscillators: one mechanical and the other electromagnetic. This example is interesting for his simplicity. It clearly shows how the mechanical and electric part becomes coupled and how this coupling appears in the current-voltage characteristics. By contrast, its experimental realization is nontrivial since it is in general not easy to get rid of parasitic capacitances and resistances. An inductance and a capacitance are present in the environment for the oscillating tunneling junction in the experiment of Ref. 33. The environment forms in this way a radio frequency resonator that is used to improve the bandwidth of the detection scheme. However, one should note that experiment of Ref. 33 is not in the regime of parameters considered in the present paper.

In the second case (Sec. III B), the environment provides the dissipation and the conditions for the standard Coulomb blockade with the suppression of the current at low voltages. This case is more relevant from the experimental point of view since high impedance environment can be more easily devised. For electronic transport without mechanical motion an accurate experimental study of the effect of a purely resistive environment has been carried out recently by tuning the environment resistance for a given tunnel junction.³⁶

A. Inductive case $Z(\omega)=iL\omega$

We consider the case of undamped harmonic oscillator. Strictly speaking, the hypothesis of thermal equilibrium for the oscillator and for the electromagnetic modes is not valid. We will assume that an infinitesimal dissipation is present and that the tunneling resistance is sufficiently large to let the time for the system to relax between two tunneling events. The electromagnetic part of the circuit is characterized by the oscillator frequency $\omega_{LC}=1/\sqrt{LC}$. The pure electromagnetic case is discussed in the review paper Ref. 34. Due to the coupling with the mechanical mode two resonant frequencies appear, as can be seen by finding the poles of $Z_I(\omega)$ [cf. Eq. (16)],

$$\omega_{\pm}^2 = \frac{[\omega_m^2 + \omega_{LC}^2 + \omega_I^2 \pm \sqrt{(\omega_{LC}^2 + \omega_m^2 + \omega_I^2)^2 - 4\omega_{LC}^2\omega_m^2}]}{2}. \quad (17)$$

Here we introduced the ‘‘coupling frequency’’ $\omega_I^2 = C'^2 V^2 / Cm$. The real part of Z_I reduces to a sum of four delta functions,

$$\frac{\text{Re}[Z_I(\omega)]}{R_Q} = \sum_{\sigma=\pm} \frac{\rho_{\sigma}\omega_{\sigma}}{2} [\delta(\omega - \omega_{\sigma}) + \delta(\omega + \omega_{\sigma})], \quad (18)$$

with $\rho_{\pm} = (E_C / \hbar \omega_{\pm}) |\omega_{\pm}^2 - \omega_m^2| / |\omega_{\pm}^2 - \omega_{\pm}^2|$.

It is convenient to express the coupling of mechanical and electronic degrees of freedom in terms of a parameter that depends only on the device and does not depend on the voltage bias. The reason is that it is easy to vary experimentally the voltage for a given device. A physically relevant parameter is the variation of the elastic energy E_e when the oscillator is displaced by a distance $\Delta x = E_C(C'/C)/k$ in response to the Coulomb force generated by a single electron on the capacitor. With this definition, we have

$$E_e = E_C^2 (C'/C)^2 / 2k \quad \text{and} \quad \omega_I^2 = \omega_m^2 E_e (eV)^2 / E_C^3. \quad (19)$$

For a junction made of a scanning tunneling microscopy (STM) tip on top of a suspended carbon nanotube, an electromechanical coupling with the radial breathing mode of the tube was reported.^{37,38} In this context, ω_m and E_C were of the same order of magnitude (around 10 meV). Thus, we estimate the coupling parameter $E_e/E_C \sim (x_0/L_c)^2$, where x_0 is the zero-point motion of the mode (a fraction of angstroms) and $L_c = C/C'$ (a fraction of nm). In the following, we assume a (rather optimistic) ratio of $E_e/E_C \lesssim 0.1$

For small value of E_e/E_C and $\omega_m \neq \omega_{LC}$, the new frequencies ω'_m and ω'_{LC} are only weakly modified by the interaction with respect to their bare values,

$$\omega_m'^2 / \omega_m^2 = 1 + \omega_I^2 / (\omega_m^2 - \omega_{LC}^2), \quad (20)$$

$$\omega_{LC}'^2 / \omega_{LC}^2 = 1 - \omega_I^2 / (\omega_m^2 - \omega_{LC}^2) \quad (21)$$

(note that $\omega'_m = \omega_+$ if $\omega_m > \omega_{LC}$ and $\omega'_m = \omega_-$ if $\omega_m < \omega_{LC}$) and the weights of the poles simplify

$$\rho_m \approx \frac{E_C}{\hbar \omega_m} \frac{\omega_I^2 \omega_m^2}{(\omega_{LC}^2 - \omega_m^2)^2} \ll \rho_{LC} \approx \frac{E_C}{\hbar \omega_{LC}}. \quad (22)$$

The inequality holds if the bare frequencies ω_m and ω_{LC} are of the same order of magnitude. On the other hand, when $\omega_m = \omega_{LC} \gg \omega_I$, we find $\omega_{\pm} = \omega_m \pm \omega_I/2$ and $\rho_+ = \rho_- = \rho_{LC}/2$.

From Eq. (15), one can obtain $J(t) - J(0) = \sum_{\sigma} \rho_{\sigma} j_{\sigma}(t)$ with³⁴

$$j_{\sigma}(t) = \coth \frac{\beta \hbar \omega_{\sigma}}{2} (\cos(\omega_{\sigma} t) - 1) - i \sin(\omega_{\sigma} t). \quad (23)$$

We consider now the zero-temperature limit. In this case, the function $P(E)$ obtained from Eq. (5) has a simple expression in terms of an infinite series,

$$P(E) = \sum_{n, n'=0}^{\infty} p_n(\rho_m) p_{n'}(\rho_{LC}) \delta(E - n \hbar \omega'_m - n' \hbar \omega'_{LC}), \quad (24)$$

where $p_n(\rho) = e^{-\rho} \rho^n / n!$.

The differential conductance $G = dI/dV$ can be obtained from the current (6). Neglecting the weak V dependence of ρ_{σ} and ω_{σ} on the derivative, we obtain

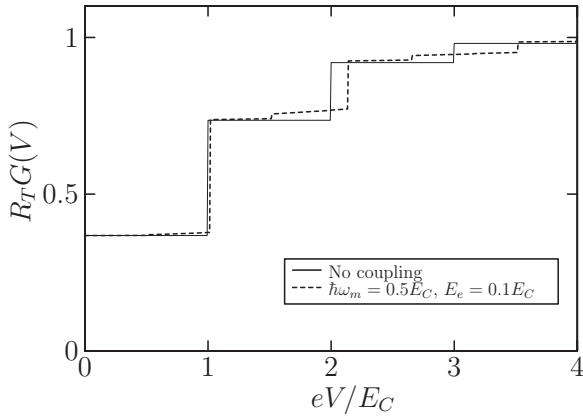


FIG. 2. Differential conductance of the device as a function of the bias voltage for the case $Z=i\omega L$. The bare oscillating frequencies are $\hbar\omega_m=0.5E_C$ and $\hbar\omega_{LC}=E_C$. The electromechanical coupling is $E_e=0$ (full line) and $E_e=0.1E_C$ (dashed line) (in this case, signatures of mechanical coupling are not visible for $\hbar\omega_m$ below $0.5E_C$).

$$R_T G(V) = \sum_{\hbar n\omega'_m + \hbar n'\omega'_{LC} < eV} p_n(\rho_m) p_{n'}(\rho_{LC}). \quad (25)$$

It leads to a double series of steps spaced by $\hbar\omega_m$ and $\hbar\omega_{LC}$. If $\rho_m=0$, one recovers the usual sequence of steps with a suppression of the conductance at low voltage and $R_T G=1$ for $eV \gg E_C$. The interesting situation is when ρ_{LC} is of the order of 1. In this case steps are clearly visible for $eV \sim E_C$ (cf. Fig. 2). If $\rho_m \neq 0$ and $\omega_m < \omega_{LC}$, the mechanical oscillator manifests itself as a series of additional steps of width $\hbar\omega'_m$ particularly visible at the beginning of each step of the LC circuit. In practice the picture is slightly more complex due to the dependence on V of the four parameters ρ_σ and ω_σ . The renormalization of the frequency and the change in ρ introduces a shift of the steps and a weak smooth V dependence of the conductance plateaux, as shown in Fig. 2. The presence of the mechanical oscillator thus gives a similar result to what has been predicted for a single-electron transistor⁷ with Frank-Condon steps and a reduction of the current at low voltage. This result is found here in a much simpler structure: a single tunnel junction coupled in series with an external inductance.

B. Ohmic case $Z(\omega)=R$

Let us now consider the pure Ohmic case for the external impedance $Z(\omega)=R$. The first step is again to find the poles of $Z_t(\omega)$. For $\gamma(\omega)$ constant, three poles are present: one is pure imaginary, $i\gamma'_{RC}$, and the two other ones are complex, $\omega_\pm = \pm \omega'_m + i\gamma'/2$. In the relevant limit of small coupling $\omega_I \ll \omega_m, \gamma_{RC}$ and low mechanical damping $\gamma \ll \omega_m, \gamma_{RC}$, we get

$$\gamma'_{RC} = \gamma_{RC} - \frac{\omega_I^2 \gamma_{RC}}{\gamma_{RC}^2 + \omega_m^2} + \dots, \quad (26)$$

$$\omega'_m = \omega_m + \frac{1}{2} \frac{\omega_m \omega_I^2}{\gamma_{RC}^2 + \omega_m^2} - \frac{\gamma^2}{8\omega_m} + \dots, \quad (27)$$

$$\gamma' = \gamma + \frac{\gamma_{RC} \omega_I^2}{\gamma_{RC}^2 + \omega_m^2} + \dots. \quad (28)$$

The mechanical oscillator acquires a damping even if $\gamma=0$. This intrinsic damping is proportional to the coupling. From here, we drop the bare mechanical damping since it does not induce new effects compared to those already introduced by the presence of R . An important parameter for this device is the dimensionless conductance of the tunnel junction $g=R_Q/R$. For $g \gg 1$ the Coulomb blockade is no more observable since the charge can vary continuously. In the absence of the oscillator the appearance of the Coulomb blockade is signaled by the change of the low-energy behavior of $P(E)$, which goes like $E^{2/g-1}$.³⁴ Thus for $g < 2$ the current is suppressed at low voltage $V < E_C/e$. If we consider the interesting case of $\hbar\omega_m \sim E_C$, then the condition $g \ll 1$ implies also that $\gamma_{RC} \ll \omega_m$. In this limit the real part of Z_t is simply the sum of three Lorentzian functions,

$$\frac{\text{Re}[Z_t(\omega)]}{R_Q} = \frac{E_C}{\hbar} \left[\epsilon L(\omega, 2\gamma'_{RC}) + \frac{\gamma'}{\gamma_{RC}} [L(\omega - \omega'_m, \gamma') + L(\omega + \omega'_m, \gamma')] \right], \quad (29)$$

with $\epsilon = \gamma'_{RC}/\gamma_{RC}$ and $L(\omega, \gamma) = (\gamma/2)/(\omega^2 + \gamma^2/4)/\pi$. Since the coupling is weak, the damping of the mechanical mode is also very small and the Lorentzian function can be approximated by a Dirac delta function [$L(\omega, \gamma \rightarrow 0) = \delta(\omega)$]. At zero temperature this gives for the phase correlation function,

$$J(t) - J(0) = \frac{E_C}{\pi \hbar \gamma_{RC}} j_o(\gamma'_{RC} t) + \rho_m [e^{-i\omega'_m t} - 1], \quad (30)$$

where $\rho_m = \frac{2E_C \gamma'}{\hbar \omega_m \gamma_{RC}}$ and $j_o(\tau) = 2 \int_0^\infty \frac{e^{ix\tau} - 1}{x(x^2+1)} dx$ is the correlation function of the circuit in the absence of the mechanical oscillator. $P(E)$ thus reads

$$P(E) = \sum_{n=0}^{\infty} \frac{p_n(\rho_m)}{\epsilon} P_o[(E - n\hbar\omega'_m)/\epsilon], \quad (31)$$

where

$$P_o(E) = \frac{1}{2\pi\hbar} \int_{-\infty}^{+\infty} dt e^{iEt/\hbar + (E_C/\pi\hbar\gamma_{RC}) j_o(t\gamma_{RC})} \quad (32)$$

is the function $P(E)$ in the absence of the oscillator. For the case at hand of small g the function $P_o(E)$ is peaked at $E \approx E_C$ with a width $\sim (E_C/\pi) \sqrt{2g|\log g|}$ that vanishes for $g \rightarrow 0$. Thus, the resulting $P(E)$ is a sequence of peaks shifted by $\hbar\omega_m$. Figure 3 shows the dependence of $P(E)$ on E and V as obtained by numerical integration. For $eV/E_C \ll \sqrt{E_C/E_e}$ [Eqs. (21) and (29)], the mechanical coupling is negligible and one recovers the usual picture of Coulomb blockade with a single peak at $E \approx E_C$. For larger values of eV/E_C new peaks appear and their positions slowly drift as a function of V toward higher energies. The first peak instead drifts toward smaller energies due to the ϵ factor in Eq. (31). Since γ'_{RC}

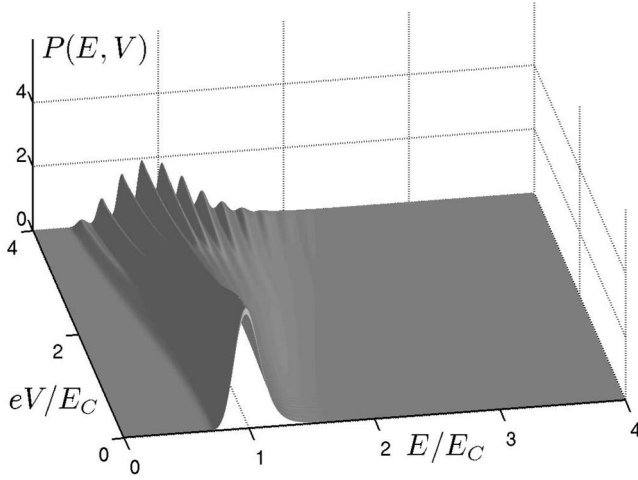


FIG. 3. $P(E)$ is displayed for different values of V . We take $g = 0.015$, $\hbar\omega_m = 0.1E_C$, and $E_e = 0.1E_C$.

reduces for larger values of V , the width of the peaks also reduces.

In Fig. 4 we show the same function, but for a higher mechanical frequency $\hbar\omega_m/E_C = 0.5$. It shows the appearance of the peaks and their shrinking. An important case is $g \rightarrow 0$, for which $P_o(E) = \delta(E - E_C)$. This is clearly the most favorable case to observe the quantum fluctuations of the oscillator directly in the I - V characteristics. Indeed, even in the weak electromechanical coupling the sharpness of the peaks gives rise to steps in $G(V)$ like in the case of a purely electromagnetic resonator. The conductance has thus the form (again neglecting the small voltage dependence of ρ_m and $\hbar\omega_m$ on performing the derivative)

$$R_T G(V) = \sum_{\hbar n \omega'_m < eV - \epsilon E_C} p_n(\rho_m). \quad (33)$$

The result is plotted in the insert of Fig. 5 for $E_e/E_C = 0.1$ and $g = 10^{-4}$.

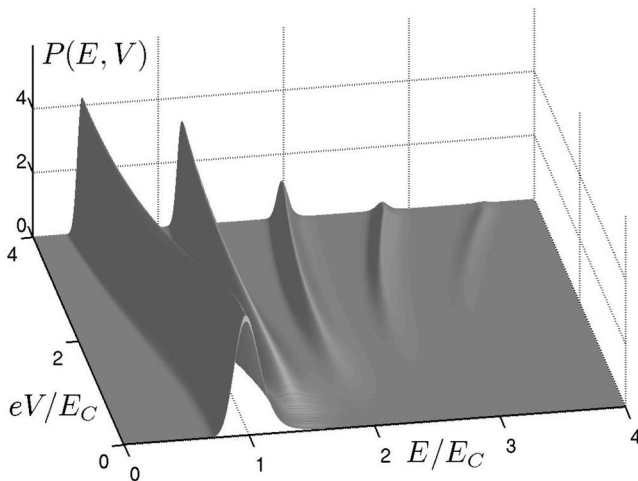


FIG. 4. $P(E)$ is obtained numerically for different values of V for $g = 0.015$, $\hbar\omega_m = 0.5E_C$, and $E_e = 0.1E_C$.

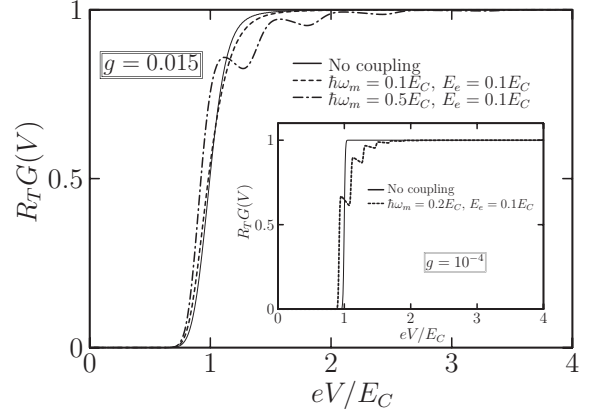


FIG. 5. Main plot: $G(V)$ for the same parameters as in Fig. 3 (dashed line) and Fig. 4 (dot-dashed line) and for vanishing coupling (full line). Inset: $G(V)$ for $g = 10^{-4}$ from the analytical expression (33) for $\hbar\omega_m = 0.1E_C$, $E_e = 0$ (full line), and $E_e = 0.1E_C$ (dashed line). The conductance is much more sensitive to the electromechanical coupling in presence of a high impedance environment.

For the general case the conductance can be obtained numerically by integrating $P(E)$. It is shown for three different values of the ratio E_e/E_C in Fig. 5. The presence of the mechanical coupling introduces oscillations of the conductance on the scale $eV \sim \hbar\omega_m$ that are precursors of the step behavior in the extreme $g = 0$ case. The small increase in the conductance at low voltage is the consequence of the renormalization of E_C due to the electromechanical coupling (the ϵ factor).

A last comment is in order for the resistive case. Even if this system is described by a very similar electric circuit to that used for a single-electron transistor without gate, some differences should not be overlooked. In the single-electron transistor the two resistances are tunnel junctions, while here one is a tunnel junction and one a Ohmic resistor. A tunnel junction is substantially different from an Ohmic resistor since in the first case the charge can be transferred only by a sudden tunneling event, while in the second the charge slowly leaks through the resistor. The charge quantization is thus not enforced by the Ohmic resistor, and this has consequences also on the time dependence of the force acting on the mobile part. For these reasons our results complement those obtained by others authors for suspended single-electron transistors.^{3-5,7-9,11-13}

C. Range of validity

Let us return to the comparison of position dependence of the tunneling and capacitive terms. Using the explicit expression (28) for γ' , we find that the condition to neglect the effect of the tunneling force (case $\gamma_{RC} \ll \omega_m$) is

$$1 \gg \frac{\gamma_T}{\gamma'} = \left(\frac{x_o}{\lambda_T} \right)^2 \frac{\omega_m R_Q E_C^3}{\gamma_{RC} R_T E_e e^2 V^2}. \quad (34)$$

This gives the explicit condition on the voltage in order to neglect the position dependent part of the Hamiltonian. Formally, it is clear that it can be always fulfilled for R_T suffi-

ciently large. A second possible source of deviation from the behavior we found comes from the cubic term of the interaction energy. We neglected the term $(\dot{\phi})^2$ with respect to $eV\dot{\phi}/\hbar$. This is justified when $\langle(\dot{\phi})^2\rangle^{1/2} \ll eV/\hbar$. We thus obtain the condition of validity of the quadratic theory at zero temperature,

$$\frac{eV}{E_C} \gg \sqrt{g|\ln g|}. \quad (35)$$

IV. JOSEPHSON JUNCTION

Observing a Josephson supercurrent through a STM tunnel junction has been shown to be feasible.³⁹ Particularly appealing is the possibility of studying an oscillating Josephson junction. The effect of the position dependence of the electronic tunneling amplitude on the dynamics of the Josephson junction has been considered recently.⁴⁰ However the effect of the position dependence of the capacitance (Coulomb interaction) has not been explored so far. The theory of Coulomb blockade for normal metallic electrodes presented above can be generalized with little change to the superconducting case. The main difference is that the phase difference is now a truly observable quantity and it is conjugated with the number of Cooper pairs that cross the junction.

The Josephson junction is characterized by a critical current I_c that defines the energy scale $E_J = \hbar I_c / 2e$. For $E_J / E_C \ll 1$ a tunneling approach can be used to obtain the dissipative current of the device. We remind the main steps of the procedure.³⁴ The current operator is $I_S = I_c \sin(2\phi)$, while the Josephson part of the Hamiltonian reads $H_J = E_J \cos(2\phi)$ (we keep the same definition of the phase, the factor two intervenes in the usual Josephson relation). Calculating the current in the linear-response limit, we get

$$I_S = I_c \frac{E_J}{4\hbar} \int_{-\infty}^{+\infty} dt' \langle [e^{2i\phi(t)}, e^{-2i\phi(t')}] \rangle e^{2ieVt/\hbar}. \quad (36)$$

One can thus express the current again in terms of the Fourier transform of the phase correlator,

$$I(V) = \frac{\pi E_J^2 e}{\hbar} [P_s(2eV) - P_s(-2eV)], \quad (37)$$

where the definition of $P_s(E)$ differs from the definition of $P(E)$ simply by a factor of 4 in front of $J(t)$,

$$P_s(E) = \int_{-\infty}^{+\infty} dt e^{AJ(t) - 4J(0) + iEt/\hbar}. \quad (38)$$

The function $J(t)$ depends on the mechanical response function as shown in Sec. II. The results of Secs. II and III can thus be applied to the superconducting case to obtain $P_s(E)$ and hence $I(V)$. In the Fig. 6 we show the expected current-voltage characteristics for an oscillating Josephson junction in the pure resistive case of Sec. III. The superconducting device is much more sensitive to the presence of the mechanical oscillator since the voltage dependence is directly related to $P_s(E)$, and not to its integral. One can then observe

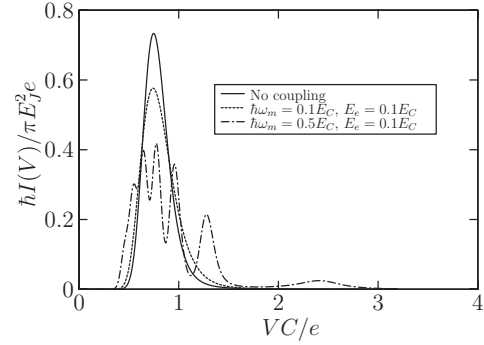


FIG. 6. $I(V)$ is plotted for $g=0.065$. We distinguish the case $E_e=0$ (full line), $E_e=0.1E_C$ with $\hbar\omega_m=0.1E_C$ (dashed line), and $E_e=0.1E_C$ with $\hbar\omega_m=0.5E_C$ (dot-dashed line). As long as $\hbar\omega_m$ remains smaller than the width of the peak, the coupling manifests itself in a broadening of the Coulomb peak. For $\hbar\omega_m/E_C=0.5$, contributions of the mechanical oscillator clearly appear.

directly the Coulomb blockade peaks in the I - V characteristics.

V. CONCLUSION

In this work, we considered the current-voltage characteristics of an oscillating tunnel junction. At low tunnel coupling and moderately low-voltage bias, we found that the coupling between the electronic and mechanic degrees of freedom mainly originates from the position dependence of the junction capacitance. That is, the electrons crossing the tunnel junction couple to an electromagnetic environment formed by the rest of the circuit which is itself coupled to the mechanical oscillator. In our study, we included both the electromagnetic and mechanical environments in the Coulomb blockade theory. We found that the mechanical properties of the junction result in an additional mechanism for the current suppression that is similar to the Frank-Condon blockade predicted in an oscillating single-electron transistor. As in the conventional Coulomb blockade, similar, though more pronounced, effects would also appear in a voltage-biased oscillating Josephson junction.

At larger tunneling rates, two new ingredients should be added to our study. First, the electromagnetic and mechanical degrees of freedom would not have time to relax to equilibrium between two tunneling events. Second, the position dependence of the tunneling matrix element may not give anymore negligible effects compared to those arising from the position dependence of the capacitance. Considering both, it may be interesting to investigate whether a shuttling instability takes place in a single oscillating tunnel junction.

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