PHYSICAL REVIEW B **76**, 060504(R) (2007)

Long range triplet Josephson effect through a ferromagnetic trilayer

M. Houzet[1] and A. I. Buzdin[2]
[1]*DRFMC/SPSMS, CEA Grenoble, 17, rue des Martyrs, 38054 Grenoble Cedex 9, France*
[2]*Institut Universitaire de France and Université Bordeaux I, CPMOH, UMR 5798, 33405 Talence, France*
(Received 16 July 2007; published 15 August 2007)

We study the Josephson current through a ferromagnetic trilayer, in both the diffusive and clean limits. For collinear (parallel or antiparallel) magnetizations in the layers, the Josephson current is small due to the short range proximity effect in superconductor/ferromagnet structures. For noncollinear magnetizations, we determine the conditions for the Josephson current to be dominated by another contribution originating from the long range triplet proximity effect.

DOI: 10.1103/PhysRevB.76.060504 PACS number(s): 74.45.+c, 74.78.Fk, 85.25.Cp

The coexistence of superconductivity and ferromagnetism is very rare in bulk systems. However, it can be easily achieved in artificially fabricated superconductor/ferromagnet (S/F) heterostructures. The S/F proximity effect is characterized by the damped oscillatory behavior of the Cooper pair wave function in the ferromagnet. This phenomenon leads to nonmonotonic dependence of the critical temperature of S/F multilayers on the F layer thickness and the realization of Josephson π junctions (for a review, see Refs. 1 and 2). In the diffusive limit, the proximity effect in a F metal is rather short ranged due to the large value of the ferromagnetic exchange field. This is related to the incompatibility between singlet superconductivity and ferromagnetism.

Interestingly, nonuniform magnetization can induce triplet superconducting correlations which are long ranged [on the same scale as for the superconductor/normal (N) metal proximity effect].[3] Several experimental indications exist for this triplet proximity effect.[4,5] However, the transition from the usual to the long range triplet proximity effect has not been observed in the same system.

In the present work, we investigate the conditions for the observation of the Josephson current due to a long range triplet component under controllable conditions. Noncollinear magnetization may serve as a source of the long range triplet component. However, it is not possible to have a Josephson current due to the interference of the triplet and singlet components. Two sources of triplet components are needed to observe the long range triplet Josephson effect between them. Thus, the simplest experimental realization of such a situation may be a S/F′/F/F″/S system with the magnetic moments of the F′ and F″ layers noncollinear with the F interlayer (see Fig. 1). The optimal condition for triplet Josephson current observation is when the thicknesses d_L and d_R of the layers F′ and F″ are of the order of the coherence length ξ_f in the ferromagnet. Indeed, for large d_L and d_R, the triplet component is exponentially small due to the short range proximity effect in the layers F′ and F″, while for very thin d_L and d_R, it is also small. Thus, we predict that the magnitude of the Josephson current in a structure with F layer thickness much larger than ξ_f will be comparable to that of a S/N/S junction with the same length.

A similar phenomenon could be observed in lateral Josephson junctions made of a nanostructured ferromagnetic film, allowing control of its magnetic domain structure. Then, the described effect would give a much larger critical current than the one predicted in S/F/S junctions with in-plane magnetic domain walls.[6]

In addition, the triplet Josephson effect provides the possibility of 0 and π junction realization due to the different orientations of the magnetic moments in the F′ and F″ layers. This effect was revealed in $S_F/I/S_F$ junctions, where S_F are magnetic superconductors with helical magnetic order separated by a thin insulating (I) layer.[7] It was also obtained in diffusive F/S multilayers with noncollinear magnetizations in successive F layers.[8] In this case, the triplet Josephson effect is mediated by the inverse proximity effect in the thin S layers. This effect would compete with the reduction of critical temperature and gap amplitude, but these were not taken into account. In Ref. 9, an idealized circuit-theory model for the triplet proximity effect in a S/F/I/F/I/F/S junction was proposed. The spatial range of the singlet and triplet proximity effect was not considered. Our work is somewhat complementary to these. The question of the concrete realization and optimization of the triplet Josephson effect was outside the scope of these approaches, while it is of primary importance in the present study.

We also provide an analysis of the triplet Josephson current in the ballistic (clean) limit. In this case, the singlet component reveals nonexponential oscillatory decay but nevertheless the decay of the triplet component is even weaker, and it is again possible to observe the crossover between singlet and triplet Josephson effects.

The needed conditions for the observation of the triplet proximity effect in the Josephson current are rather stringent.

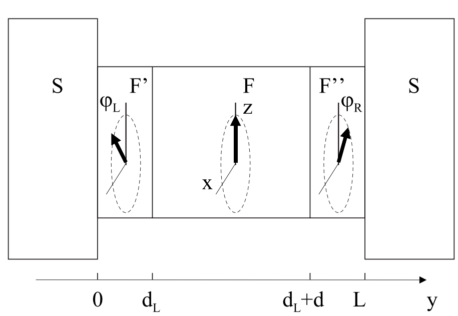

FIG. 1. Geometry of S/F′/F/F″/S junction. The arrows indicate noncollinear orientations of the magnetizations in each layer with thicknesses d_L, d, and d_R, respectively ($L=d_L+d+d_R$).

The considered system, if realized experimentally, could provide an excellent opportunity to study the crossover between the triplet and singlet Josephson effects with the rotation of the magnetic moment of any of the F layers.

Let us now calculate the supercurrent through a Josephson junction made of a ferromagnetic trilayer attached to superconducting leads, according to the geometry depicted in Fig. 1. We assume that the layers are in good electric contact and that the magnetizations in the layers have the same amplitude. The exchange field $\boldsymbol{h}$ acting on the spin of the conduction electrons is parallel to the magnetizations, with the following spatial dependence:

$$\boldsymbol{h}(y)=\begin{cases} h(\sin\phi_L\hat{x}+\cos\phi_L\hat{z}), & 0<y<d_L,\\ h\hat{z}, & d_L<y<d_L+d,\\ h(\sin\phi_R\hat{x}+\cos\phi_R\hat{z}), & d_L+d<y<L,\end{cases} \quad (1)$$

where d_L, d, and d_R are the thicknesses of each layer, and $L=d_L+d+d_R$ is the total length of the junction. Here we adopt the sames axes for space and spin quantization.

We first consider the diffusive limit, when the mean free path is shorter than the widths of the layers and coherence lengths. For simplicity, we also assume that the temperature is close to the critical temperature of the leads. Then, within the quasiclassical theory of superconductivity,[10] the current flowing through the junction is

$$I=\frac{GL}{e}\pi T\sum_{\omega>0}\text{Im Tr}[\hat{F}^*(y)\hat{\sigma}_y\hat{F}'(y)\hat{\sigma}_y], \quad (2)$$

where the anomalous Green's function $\hat{F}=F_0+\boldsymbol{F}\cdot\hat{\boldsymbol{\sigma}}$ is a matrix in spin space and solves the linearized Usadel equation in the ferromagnet:

$$-D\hat{F}''(y)+2\omega\hat{F}(y)+i\boldsymbol{h}(y)\cdot\{\hat{\boldsymbol{\sigma}},\hat{F}(y)\}=0 \quad (3)$$

(in units with $\hbar=k_B=1$). Here, G is the conductance of the junction in its normal state, D is the diffusion constant of the ferromagnet, $\omega=(2n+1)\pi T$ are the Matsubara frequencies at temperature T, $\hat{\sigma}_{i(i=x,y,z)}$ are the Pauli matrices, and the primes denote the derivative along the y direction. Depairing currents generated by the orbital effect have been neglected in Eq. (3), as is usually done for ferromagnetic layers with in-plane magnetization.[1]

The Usadel equation (3) is solved in the central F layer in terms of its values at the interfaces with the F′ and F″ layers:

$$F_0(y)\pm F_z(y)=[F_0(d_L)\pm F_z(d_L)]\frac{\text{sh}q_\pm(d_L+d-y)}{\text{sh}q_\pm d}+[F_0(d_L+d)\pm F_z(d_L+d)]\frac{\text{sh}q_\pm(y-d_L)}{\text{sh}q_\pm d},$$

$$F_x(y)=F_x(d_L)\frac{\text{sh}q_0(d_L+d-y)}{\text{sh}q_0 d}+F_x(d_L+d)\frac{\text{sh}q_0(y-d_L)}{\text{sh}q_0 d}, \quad (4)$$

and $F_y=0$, as $\boldsymbol{h}$ has no component along the $\hat{y}$ direction. Here, $q_0=\sqrt{2\omega/D}$ and $q_\pm=\sqrt{2(\omega\pm ih)/D}$. As the amplitude of the exchange field is much larger than the critical temperature T_c, we may simplify $q_\pm\simeq(1\pm i)/\xi_f$, where $\xi_f=\sqrt{D/h}$ is the ferromagnetic coherence length and is much shorter than the superconducting coherence length $\xi_0=\sqrt{D/2\pi T_c}$. The solutions of Eq. (3) in the other layers, as well as their derivatives, should match Eq. (4) continuously at each interface. In the absence of interface barriers with the S leads, they should also take the values $\hat{F}(y=0,L)=\hat{F}^{L,R}$, where $\hat{F}^{L,R}=(\Delta/\omega)e^{\mp i\chi/2}$ are the bulk solutions in the leads. Here, Δ is the modulus of the superconducting gap and χ is the phase difference maintained between the leads. Close to T_c, the gap vanishes as $\Delta(T)=\{[8\pi^2/7\zeta(3)]k^2T_c(T_c-T)\}^{1/2}$. Here, we neglect self-consistency for the gap equation in the leads, as is usually done, assuming that the width of the S electrodes is much larger than that of the F layers, or that the Fermi velocity in the F layers is smaller.[2]

To proceed further with tractable formulas, we assume that the F′ and F″ layers are thin: $d_L,d_R\ll\xi_f$. Then, the solution in the F′ layer varies only slightly with y and can be put in the approximate form

$$\hat{F}(y)\simeq\hat{F}(d_L)+(y-d_L)\hat{F}'(d_L)-\frac{(y-d_L)^2}{d_L^2}[\hat{F}(d_L)-d_L\hat{F}'(d_L)-\hat{F}^L], \quad (5)$$

which satisfies the boundary conditions at $y=0$ and $y=d_L$. In addition, it should also solve the Usadel equation. Inserting Eq. (5) into (3), we get

$$\frac{D}{d_L^2}[\hat{F}(d_L)-d_L\hat{F}'(d_L)-\hat{F}^L]+\frac{i}{2}\boldsymbol{h}\cdot\{\hat{\boldsymbol{\sigma}},\hat{F}^L\}\simeq 0, \quad (6)$$

where the term $\omega\hat{F}^L$ was neglected (as $h\gg T$). Equation (6) yields the results

$$F_0(d_L)=F_0^L, \quad (7a)$$

$$F_x(d_L)=-i(d_L^2h/D)\sin\phi_L F_0^L, \quad (7b)$$

$$F_z(d_L)=-i(d_L^2h/D)\cos\phi_L F_0^L, \quad (7c)$$

provided that $d_L|\hat{F}'(d_L)|\ll|\hat{F}(d_L)|$, as can be checked consistently from Eq. (4) when $d_L\ll\xi_f$.

Similar results can be obtained for $\hat{F}(y=d_L+d)$. We can now evaluate Eq. (2), say at $y=d_L$, and we find $I=I_c\sin\chi$, where the critical current is:

$$I_c=\frac{2\pi TG}{e}\sum_{\omega>0}\frac{\Delta^2}{\omega^2}\left(\text{Re}\,\frac{q_+d}{\text{sh}q_+d}-\frac{q_0d}{\text{sh}q_0d}\frac{d_L^2d_R^2}{\xi_f^4}\sin\phi_L\sin\phi_R\right). \quad (8)$$

The first term in Eq. (8) comes from short range singlet (F_0) and triplet (F_z) components of the anomalous function $\hat{F}$. It equals the critical current of a S/F/S junction with length d.[11] Its sign oscillates with the variation of the ratio d/ξ_f. In particular, when $d\gg\xi_f$,

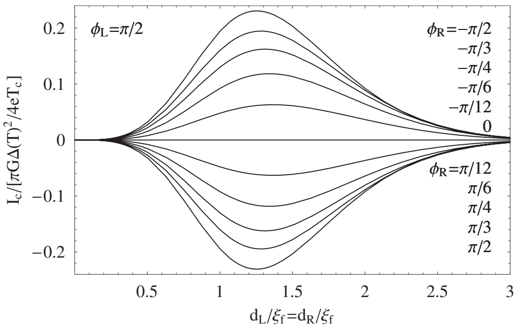

FIG. 2. Critical current induced by long range triplet proximity effect in S/F′/F/F″/S junction, in units of $[\pi G\Delta(T)^2/4eT_c]$, for varying length of F′ and F″ layers, at $d_L=d_R\sim\xi_f\ll d\ll\xi_0$, and for different orientations of the magnetization in the layers.

$$I_{cf}=\frac{\pi G}{2\sqrt{2}e}\frac{\Delta(T)^2}{T_c}\frac{d}{\xi_f}\sin\left(\frac{\pi}{4}+\frac{d}{\xi_f}\right)e^{-d/\xi_f}. \qquad (9)$$

Thus, its amplitude is also exponentially suppressed.

The second term in Eq. (8) comes from the long range triplet component (F_x) and yields

$$I_{ct}=-I_{cn}(d_L^2d_R^2/\xi_f^4)\sin\phi_L\sin\phi_R, \qquad (10)$$

where I_{cn} is the critical current in the S/N/S junction:[2]

$$I_{cn}=\frac{G}{e}2\pi T\sum_{\omega>0}\frac{q_0d}{\mathrm{sh}q_0d}\frac{\Delta^2}{\omega^2}. \qquad (11)$$

In particular, in junctions with length $d\ll\xi_0$, $I_{cn}=(\pi G\Delta^2/4eT_c)$. The small prefactor $(d_L^2d_R^2/\xi_f^4)$ in Eq. (10) comes from the simplifying assumption $d_L,d_R\ll\xi_f$ that we used in the calculation. As explained in the introductory part of the papers, I_{ct} would be reduced by the exponential factor $e^{-(d_L+d_R)/\xi_f}$ at $d_L,d_R\gg\xi_f$. Thus, at optimal size $d_L,d_R\sim\xi_f$, the second term $I_{ct}\sim -I_{cn}\sin\phi_L\sin\phi_R$ is much larger than the first one, I_{cf}, provided that the magnetic layers have noncollinear orientations. For arbitrary lengths $d_L,d_R\sim\xi_f$, the critical current originating from the long range triplet correlation only, at $\xi_f\ll d\ll\xi_0$, was also obtained from Eqs. (2) and (3) (see Fig. 2). We see that the triplet contribution to the critical current may be observed on the experiment only in a rather small interval of the F′ and F″ layers thickness: $d_L,d_R\sim(0.5-2.5)\xi_f$.

A dependence of the critical current on the orientations of the magnetizations in successive F layers similar to Eq. (8) was obtained in Refs. 8 and 9. We note that the sign of the long range component of the critical current can be tuned with these orientations. This component is absent in the case of only two layers with opposite[12] or even noncollinear magnetizations.[13]

The Usadel equations would easily allow generalizing the result (8) obtained here. (i) Qualitatively, the above result should not rely on the assumption that the temperature is close to T_c and it would be preserved even at smaller temperature. (ii) Barrier interfaces between the layers and the leads would decrease both short range and long range contributions to the critical current.[9] (iii) Equation (1) may also describe the case of a ferromagnet with magnetic domains and thin domain walls (few atomic lengths). If the domain walls are large, the long range triplet contribution will be decreased by the factor $\xi_f/\delta_w\ll 1$, where δ_w is the domain wall width, in analogy with the theory of enhanced critical temperature in S/F bilayers due to domain-wall superconductivity.[14]

Note an interesting possibility to separate the triplet and singlet Josephson effects even for a relatively thin central F layer $d\sim\xi_f$. Indeed, if its thickness is around the first critical value $(3\pi/4)\xi_f$ [see Eq. (9)], the temperature variation may serve as a fine tuning and provoke the 0-π transition.[15,16] For the S/F′/F/F″/S system, the singlet component would vanish at such temperature and only the triplet critical current would be observed.

We consider now the clean limit. The supercurrent flowing through the junction is now given by

$$I=-\frac{2\pi TG}{e}\sum_{\omega>0}\int\frac{d\Omega_{\boldsymbol{n}}}{4\pi}n_x\,\mathrm{Im}\,\mathrm{Tr}[\hat{f}^*_{-\boldsymbol{n}}(y)\hat{\sigma}_y\hat{f}_{\boldsymbol{n}}(y)\hat{\sigma}_y], \qquad (12)$$

where $\hat{f}_{\boldsymbol{n}}(y)$ solves the Eilenberger equation in the F layer

$$\boldsymbol{v}\cdot\boldsymbol{\nabla}\hat{f}_{\boldsymbol{n}}(y)+2\omega\hat{f}_{\boldsymbol{n}}(y)+i\boldsymbol{h}\cdot\{\hat{\boldsymbol{\sigma}},\hat{f}_{\boldsymbol{n}}(y)\}=0. \qquad (13)$$

Here, $\boldsymbol{v}=v\boldsymbol{n}$ is the Fermi velocity, $\boldsymbol{n}$ is a unit vector, and G is the Sharvin conductance of the ballistic junction in its normal state. In addition, the solution of Eq. (13) should be continuous, and match with the bulk solution in the S lead that the electrons come from. That is, $\hat{f}_{\boldsymbol{n}}(y=0)=\hat{F}^L$ if $n_y>0$, $\hat{f}_{\boldsymbol{n}}(y=L)=\hat{F}^R$ if $n_y<0$. Again, we neglect self-consistency for the gap equation in the leads.

Solving Eq. (13) at $0<y<d_L$ and $n_y>0$, we find for $\hat{f}_{\boldsymbol{n}}(y)\equiv f_0+\boldsymbol{f}.\hat{\boldsymbol{\sigma}}$ that

$$f_0\pm(\sin\phi_Rf_x+\cos\phi_Rf_z)=(\Delta/\omega)e^{-i\chi/2}e^{-2(\omega\pm ih)y/v_y},$$

$$\sin\phi_Rf_z-\cos\phi_Rf_x=0. \qquad (14)$$

Then, using continuity of $\hat{f}$ at $y=d_L$ and solving Eq. (13) at $d_L<y<d_L+d$, we find

$$f_0\pm f_z=\alpha e^{-2(\omega\pm ih)(y-d_L)/v_y}(c_{d_L}\mp is_{d_L}\cos\phi_L),$$

$$f_x=-i\alpha\sin\phi_Ls_{d_L}e^{-2\omega(y-d_L)/v_y}, \qquad (15)$$

where $\alpha=(\Delta/\omega)e^{-i\chi/2}e^{-2\omega d_L/v_y}$, and we use the shortened notations $s_{d_L}=\sin(2hd_L/v_y)$, $c_{d_L}=\cos(2hd_L/v_y)$. A similar solution can be found for $\hat{f}$ at $n_y<0$. The supercurrent (12) is then conveniently evaluated at $y=d_L+d/2$, and we find $I=I_c\sin\chi$, where

$$I_c=\frac{4\pi TG}{e}\sum_{\omega>0}\int_0^1dn_yn_y\frac{\Delta^2}{\omega^2}e^{-2\omega L/v_y}$$
$$\times(c_dc_{d_L}c_{d_R}-c_ds_{d_L}s_{d_R}\cos\phi_L\cos\phi_R-s_dc_{d_L}s_{d_R}\cos\phi_R$$
$$-s_ds_{d_L}c_{d_R}\cos\phi_L-s_{d_L}s_{d_R}\sin\phi_L\sin\phi_R). \qquad (16)$$

To proceed further, we assume that $d_L,d_R\ll\xi_f\ll d$, where the ferromagnetic coherence length $\xi_f = v/h$ in the clean limit is much shorter than the superconducting coherence length $\xi_0 = v/2\pi T_c$. Then,

$$I_c \simeq \frac{4\pi TG}{e}\sum_{\omega>0}\int_0^1 dn_y n_y \frac{\Delta^2}{\omega^2} e^{-2\omega d/v_y} \times\left[\cos\left(\frac{2hd}{v_y}\right) - \sin\left(\frac{2hd_L}{v_y}\right)\sin\left(\frac{2hd_R}{v_y}\right)\sin\phi_L \sin\phi_R\right]. \tag{17}$$

Here, the first term comes from the short range proximity effect. It coincides with the critical current of a clean S/F/S junction with length d. In particular, at $\xi_f \ll d \ll \xi_0$, it yields[17]

$$I_{cf} = -\frac{\pi\Delta^2 G}{2eT_c}\frac{\xi_f}{2d}\sin\left(\frac{2d}{\xi_f}\right). \tag{18}$$

The second term comes from the long range triplet proximity effect and yields (for $d_L \sim d_R \ll \xi_f \ll d \ll \xi_0$)

$$I_{ct} = -\frac{\pi\Delta^2 G}{2eT_c}\left(\frac{4d_L d_R}{\xi_f^2}\ln\frac{\xi_f}{2(d_L+d_R)}\right)\sin\phi_L \sin\phi_R. \tag{19}$$

It is small under the assumption $d_L, d_R \ll \xi_f$. On the other hand, at $d_L, d_R \gg \xi_f$, the critical current (19) would be suppressed by the factor $\xi_f^2/d_L d_R \ll 1$, due to the short range proximity effect in the F′ and F″ layers. Again, we expect a maximum of the critical current at $d_L \sim d_R \sim \xi_f$, with amplitude $I_{ct} \propto -I_{cn}\sin\phi_L \sin\phi_R$, where $I_{cn} = (\pi\Delta^2 G/4eT_c)$ is the critical current of a clean S/N/S junction with $d \ll \xi_0$. The dependence of the critical current on the orientations of the magnetizations in the F layers is similar to the diffusive case.

The Josephson current through a half metal (HM) with one spin band only is expected to vanish.[5,18] However, spin-flip processes taking place at S/F interfaces have been suggested to promote triplet correlation and induce a finite supercurrent through the device.[18–20] The quasiclassical theory presented in this work assumes that the ferromagnetic exchange field is much smaller than the Fermi energy. Therefore, it is not well suited to address quantitatively the case of HMs, when they are comparable. Qualitatively, the noncollinear layers F′ and F″ with thicknesses of the atomic scale would play the role of spin-flip scatterers with inverse scattering time τ_{sf}^{-1} proportional to the spin band splitting h. Then, the order of magnitude for the triplet-induced supercurrent can be obtained from Eq. (19) by noting that the reduction factor $d_L d_R/\xi_f^2$ (up to the logarithmic term) is proportional to $1/(\tau_{sf}E_F)^2$, where E_F is the Fermi energy. It is thus proportional to the probability for an electron from the minority spin band to be transferred through a HM by spin-flip processes at the interfaces with the leads.

In conclusion, we determined the Josephson current through a ferromagnetic trilayer. For collinear (parallel or antiparallel) magnetizations in the layers, the Josephson current is small due to the short range proximity effect in superconductor/ferromagnet structures. For noncollinear magnetizations, we determined the conditions for the Josephson current to be dominated by another contribution originating from the long range triplet proximity effect. In practice, the triplet Josephson current may be observed in systems with the lateral layer thickness of the order of ξ_f only.

The structures studied offer an interesting possibility to study the interplay between the Josephson current and dynamic precessing of the magnetic moment. Indeed, we may expect strong coupling between ferromagnetic resonance (or/and spin waves) and the Josephson current—in particular, the additional harmonics generation in the ac Josephson effect.

We acknowledge Norman Birge and Louis Jansen for a critical reading of the manuscript and useful comments.

[1] A. I. Buzdin, Rev. Mod. Phys. **77**, 935 (2005).
[2] A. A. Golubov, M. Yu. Kupriyanov, and E. Il'ichev, Rev. Mod. Phys. **76**, 411 (2004).
[3] F. S. Bergeret, A. F. Volkov, and K. B. Efetov, Rev. Mod. Phys. **77**, 1321 (2005).
[4] I. Sosnin, H. Cho, V. T. Petrashov, and A. F. Volkov, Phys. Rev. Lett. **96**, 157002 (2006).
[5] R. S. Keizer, S. T. B. Goennenwein, T. M. Klapwijk, G. Miao, G. Xiao, and A. Gupta, Nature (London) **439**, 825 (2006).
[6] Ya. V. Fominov, A. F. Volkov, and K. B. Efetov, Phys. Rev. B **75**, 104509 (2007).
[7] M. L. Kulic and I. M. Kulic, Phys. Rev. B **63**, 104503 (2001).
[8] A. F. Volkov, F. S. Bergeret, and K. B. Efetov, Phys. Rev. Lett. **90**, 117006 (2003).
[9] V. Braude and Yu. V. Nazarov, Phys. Rev. Lett. **98**, 077003 (2007).
[10] K. D. Usadel, Phys. Rev. Lett. **25**, 507 (1970).
[11] A. I. Buzdin and M. Yu. Kuprianov, Pis'ma Zh. Eksp. Teor. Fiz. **53**, 308 (1991) [JETP Lett. **53**, 321 (1991)].
[12] Ya. M. Blanter and F. W. J. Hekking, Phys. Rev. B **69**, 024525 (2004).
[13] B. Crouzy, S. Tollis, and D. A. Ivanov, Phys. Rev. B **75**, 054503 (2007).
[14] M. Houzet and A. I. Buzdin, Phys. Rev. B **74**, 214507 (2006).
[15] V. V. Ryazanov, V. A. Oboznov, A. Yu. Rusanov, A. V. Veretennikov, A. A. Golubov, and J. Aarts, Phys. Rev. Lett. **86**, 2427 (2001).
[16] V. A. Oboznov, V. V. Bol'ginov, A. K. Feofanov, V. V. Ryazanov, and A. I. Buzdin, Phys. Rev. Lett. **96**, 197003 (2006).
[17] A. I. Buzdin, L. N. Bulaevskii, and S. V. Panyukov, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 147 (1982) [JETP Lett. **35**, 178 (1982)].
[18] M. Eschrig, J. Kopu, J. C. Cuevas, and Gerd Schön, Phys. Rev. Lett. **90**, 137003 (2003).
[19] Y. Asano, Y. Tanaka, and A. A. Golubov, Phys. Rev. Lett. **98**, 107002 (2007).
[20] M. Eschrig and T. Löfwander, arXiv:cond-mat/0612533 (unpublished).