Electrodynamics of Fulde-Ferrell-Larkin-Ovchinnikov superconducting state

M. Houzet and V. P. Mineev

Commissariat à l'Energie Atomique, DSM/DRFMC/SPSMS 38054 Grenoble, France

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The Ginzburg-Landau theory of the vortex lattice for clean isotropic three-dimensional superconductors is developed, including the space variation of currents and fields, in the limit when the critical field is mainly determined by the paramagnetic depairing effect and the orbital effect is of minor importance. Then, in addition to the Abrikosov vortex lattice, the formation of a Fulde-Ferrell-Larkin-Ovchinnikov state is favored. In this case, the diamagnetic superfluid currents mainly come from paramagnetic interaction of electron spins with the local magnetic field, and not from the kinetic energy response to the external field as usual. We find that the stable vortex lattice keeps its triangular structure as in the habitual Abrikosov mixed state, while the internal magnetic field acquires components perpendicular to the applied magnetic field. Experimental possibilities related to this prediction are discussed.

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I. INTRODUCTION

Orbital and paramagnetic effects are both important in the suppression of the superconducting state. While orbital effect leads to the formation of an Abrikosov vortex lattice below the orbital upper critical field $H_{c20} \approx \Phi_0/2\pi\xi_0^2$ in type-II superconductors,[1] the paramagnetic effect determines the paramagnetic limit of superconductivity $H_p = \Delta_0/\sqrt{2}\mu$,[2,3] and promotes the tendency to Cooper pairing with nonzero momentum—the so-called Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state.[4,5] Here Φ_0 is the flux quantum, $\xi_0 = \hbar v_F/\pi\Delta_0$ is the superconducting coherence length, Δ_0 is the superconducting gap at zero temperature, v_F is the Fermi velocity, and $\mu = g\mu_B/2$ is the electron magnetic moment.

The interplay of both effects takes place at a large enough ratio of $\sqrt{2}H_{c20}/H_p = \alpha_M$,[6] called the Maki parameter.[7] In this regime, the (H,T) phase diagram of an isotropic s-wave superconductor in the clean limit has been studied by means of a Ginzburg-Landau (GL) functional.[8] The critical assumption was made that screening supercurrents are not important, and the local field in the superconductor was taken equal to the external field. This is true when the GL parameter κ, defined as the ratio of London penetration depth λ_L and coherence length ξ_0, is large enough. For a clean superconductor, $\lambda_L = (m^* c^2/4\pi n_e e^2)^{1/2}$ and the GL parameter is related to the Maki parameter by

$$\kappa \approx \frac{\alpha_M}{\sqrt{k_F r_e (m^*/m)}}. \tag{1}$$

Here, n_e is the electronic density, k_F is the Fermi momentum, $r_e = e^2/mc^2$ is the classical radius of the electron, and m^*/m is the ratio of an effective to the bare electron mass. It is clear from this relation that, if the Maki parameter is large, the GL parameter has an even larger value; hence the assumption made in Ref. 8 seems reasonable.

If, however, one is interested in the magnetic response of superconductors in the FFLO state, it is necessary to relax this assumption. In the present paper, we develop the theory of the FFLO state in a clean isotropic three-dimensional superconductor, including the space variations of currents and fields.

The paper is organized as follows. In Sec. II, we introduce the free energy functional including the term describing the Zeeman interaction with a spatially nonuniform magnetic field. Although unimportant for the upper critical field determination (Sec. III) and for the vortex lattice structure found in Sec. IV, this term is crucial for the current and field distribution in the FFLO-modulated superconducting state (Sec. V). There we find that the internal magnetic field has components perpendicular to the applied magnetic field. The experimental possibilities related to this theoretical prediction are discussed in the Conclusion.

II. FREE ENERGY

We investigate the phase diagram of a superconductor near the upper critical field, but far from the critical temperature T_c. For this purpose, a GL expansion of the free energy in powers of the order parameter and its gradients is possible, provided the length scale of variation of the order parameter, determined by the magnetic length $\lambda = \sqrt{\Phi_0/2\pi B}$ at magnetic field B, remains large compared to the superconducting coherence length. This is indeed the case in the limit of large Maki parameter, when the critical field is mainly determined by the paramagnetic depairing effect: $\xi_0/\lambda \sim \sqrt{B/B_{c20}} \ll 1$.

As in Ref. 9, the free energy density was derived microscopically in the frame of the Gor'kov[10] and Eilenberger[11] and the Larkin-Ovchinnikov[12,13] formalisms (from here, we adopt units with $\hbar = k_B = c = 1$):[14]

$$\begin{aligned}\mathcal{F}_s = \mathcal{F}_{n0} + \frac{\boldsymbol{h}^2}{8\pi} + \alpha|\Delta|^2 + \beta|\Delta|^4 + \gamma|\boldsymbol{D}\Delta|^2 \\ + \delta\left(|\boldsymbol{D}^2\Delta|^2 + (2e\boldsymbol{h})^2|\Delta|^2 - \frac{2e}{3}(\Delta^*\boldsymbol{D}\Delta + \text{c.c.})\text{rot}\,\boldsymbol{h}\right) \\ + \zeta|\Delta|^2|\boldsymbol{D}\Delta|^2 + \eta[(\Delta^*)^2(\boldsymbol{D}\Delta)^2 + \text{c.c.}] + \varepsilon(h_z - B)|\Delta|^2.\end{aligned} \tag{2}$$

Here, $\mathcal{F}_{n0}$ is the free energy density in the normal state in the absence of magnetic field, $\boldsymbol{D} = -i\nabla + 2e\boldsymbol{A}$, $\boldsymbol{h} = \text{rot}\boldsymbol{A}$ is the local internal magnetic field, and the coefficients in the functional depend on both temperature T and induction B determined by the spatial average $\bar{\boldsymbol{h}} \equiv \boldsymbol{B} = B\hat{z}$:

$$\alpha = N_0\left[\ln\frac{T}{T_c} + 2\pi T\,\mathrm{Re}\sum_{\omega>0}\left(\frac{1}{\omega} - \frac{1}{\omega + i\mu B}\right)\right],$$

$$\gamma = \frac{\pi N_0 v_F^2}{12}K_3,\quad \beta = \frac{\pi N_0}{4}K_3,\quad \delta = -\frac{\pi N_0 v_F^4}{80}K_5,$$

$$\zeta = 8\eta = -\frac{\pi N_0 v_F^2}{6}K_5,\quad \varepsilon = -\pi N_0 \mu L_2, \tag{3}$$

where $N_0 = m^* k_F/2\pi^2$ is the normal density of states at the Fermi level, v_F is the Fermi velocity, $\omega = \pi T(2\nu+1)$ is the Matsubara frequency, and

$$K_n = 2T\,\mathrm{Re}\sum_{\omega>0}\frac{1}{(\omega + i\mu B)^n},\quad L_n = 2T\,\mathrm{Im}\sum_{\omega>0}\frac{1}{(\omega + i\mu B)^n}. \tag{4}$$

In the following paragraphs, let us explain the origin of additional terms in Eq. (2) compared to the usual GL theory. The standard form of the GL functional is given by the terms in the first line of Eq. (2) only. In the absence of the paramagnetic effect ($\mu B \to 0$), the coefficients α, β, and γ depend on temperature; α changes its sign at the critical temperature T_c of the superconducting transition, while β and γ keep positive values. The other terms in the functional are superfluous in order to describe the transition from the normal to the superconducting state.

In the paramagnetic limit, when the orbital effect is neglected ($\alpha_M \to \infty$), the functional (2) reduces to[15]

$$\mathcal{F}_s = \mathcal{F}_{n0} + \frac{B^2}{8\pi} + \alpha|\Delta|^2 + \beta|\Delta|^4 + \gamma|\nabla\Delta|^2 + \delta|\nabla^2\Delta|^2 + \zeta|\Delta|^2|\nabla\Delta|^2 - \eta[(\Delta^*)^2(\nabla\Delta)^2 + \mathrm{c.c.}], \tag{5}$$

with coefficients depending now on both the temperature and the magnetic field. The transition from the normal to a uniform superconducting state takes place at the critical field $B_c(T)$ defined by $\alpha=0$. Along this transition line, the coefficients β and γ are proportional and they become negative at $T < T^* \simeq 0.56T_c$. This defines the tricritical point (T^*, B^*) of the phase diagram, with $B^* = B_c(T^*) \simeq 1.07T_c/\mu$. At the tricritical point, the sign change of the coefficient γ signals a possible instability toward the FFLO state with spatial modulation of the order parameter Δ, while the sign change of the coefficient β signals a possible change of the order of the normal to superconducting phase transition. As shown in Ref. 15, higher-order terms must be retained in the functional density (2) to consider these effects. Below, we consider only the region of phase diagram where the transition from the normal to the superconducting state remains of second order. Thus we only retain up to fourth-order terms in Δ in Eq. (2), while higher-order terms in the gradient allow determination of the modulation wave vector in the FFLO state.

In the vicinity of the tricritical point, the coefficients in Eq. (3) are readily evaluated. From the equation $\alpha=0$, we get

$$B_c(T) - B^* \simeq -0.31\frac{T - T^*}{\mu}, \tag{6}$$

while, along this line,

$$\gamma \simeq 0.090\frac{N_0 v_F^2 (T - T^*)}{T_c^3},$$

$$\delta \simeq 0.0011\frac{N_0 v_F^4}{T_c^4},$$

$$\varepsilon \simeq 0.80\frac{N_0 \mu}{T_c}. \tag{7}$$

Note that $\delta > 0$. The other coefficients in Eq. (3) are readily obtained from the relations (7).

In the presence of both paramagnetic and orbital effects, the functional (2) differs from Eq. (5) by the substitution $\nabla \to i\boldsymbol{D}$ in the gradient terms and the inclusion of other terms with the same order. A functional similar to Eq. (2) was derived in the limit of $\kappa \to \infty$.[8,9] At finite values of κ, the coordinate-dependent deviation of the magnetic field from the external field manifests itself not only in gradient terms, but also in a Zeeman interaction with electron spins. This results in the last term in the functional, which is absent in Refs. 8, 9, and 13, and which simply corresponds to local decrease (enhancement) of the critical temperature $T_c(B)$ when $h_z(\boldsymbol{x}) > B$ ($< B$). Note that the coefficient ε is proportional to B. Hence, the corresponding term in the functional (2) is negligibly small in the ordinary GL region near $T_c(B \to 0)$

Before concluding this section, we emphasize that the functional (2) is appropriate to describe the transition from the normal to the superconducting state in the vicinity of the tricritical point and at large Maki parameter, when the characteristic space variations of the superconducting order parameter due to FFLO modulation and/or vortex state formation much exceed the superconducting coherence length. Let us also mention that the theory presented in this work addresses only the case of clean, isotropic, three-dimensional superconductors. It could be easily generalized to arbitrary impurity disorder, shape of the Fermi surface, and pairing symmetry of the superconducting state.[9]

III. UPPER CRITICAL FIELD

At the second-order phase transition between the normal and superconducting states, the magnetic field is uniform, $\boldsymbol{h}_{c2} = B\hat{z}$. The linearized gap equation obtained from Eq. (2),

$$\alpha\Delta + \gamma\boldsymbol{D}^2\Delta + \delta[(\boldsymbol{D}^2)^2 + \lambda^{-4}]\Delta = 0, \tag{8}$$

where $\lambda^{-1} = \sqrt{2eB}$ is the inverse magnetic length, is solved using

$$\Delta = \varphi_0(x,y)f(z),\quad \boldsymbol{D}^2\Delta = \left(\frac{1}{\lambda^2} + q^2\right)\Delta, \tag{9}$$

where $\varphi_0(x,y)$ is the linear combination of Landau wave functions with level $n=0$ for the particle with charge $2e$ under magnetic field B, multiplied by an exponentially modulated function along the $\hat{z}$ direction, $f(z)=e^{\pm iqz}$. (Note that, in principle, higher Landau levels could also be considered. But they may be realized only at low temperatures when the transition from the normal to the superconducting state with lowest Landau level has turned first order;[9] see Sec. IV.)

At large Maki parameter, the upper critical field is close to $B_c(T)$. We note that $\alpha \simeq \varepsilon(B-B_c(T))$ and Eq. (8) then defines a field

$$B(q) = B_c(T) - \frac{\gamma}{\varepsilon}(\lambda^{-2}+q^2) - \frac{\delta}{\varepsilon}[(\lambda^{-2}+q^2)^2+\lambda^{-4}]. \quad (10)$$

The upper critical field B_{c2} is found by taking the maximum of $B(q)$ with respect to q. Analysis of Eq. (10) shows that, in the presence of the orbital effect, the critical temperature $\tilde{T}^*$ below which FFLO modulation appears is defined by the equation

$$\gamma + 2\delta/\lambda^2 = 0. \quad (11)$$

From Eqs. (11) and (7), we find that $\tilde{T}^*$ is decreased compared to its value T^* in the absence of the orbital effect.[8] Using $\Delta_0 \simeq 1.76T_c$, $H_{c20} \simeq 0.212\Phi_0/\xi_0^2$, and $\alpha_M \simeq 7.39(v_F^2/\Phi_0\mu T_c)$ for a clean, isotropic, three-dimensional superconductor, we find

$$\tilde{T}^* \simeq T^* - 1.19\frac{T_c}{\alpha_M}. \quad (12)$$

That is, at $T>\tilde{T}^*$, the usual superconducting state with $q=0$ appears with critical field

$$B_{c2}(T) = B_c(T) - \frac{2eB_c(T)\gamma}{\varepsilon} - \frac{8e^2B_c(T)^2\delta}{\varepsilon}, \quad (13)$$

while, at $T<\tilde{T}^*$, the FFLO state appears with finite q,

$$\gamma + 2\delta(q^2+\lambda^{-2}) = 0, \quad (14)$$

and critical field

$$B_{c2}(T) = B_c(T) + \frac{\gamma^2}{4\delta\varepsilon} - \frac{4e^2B_c(T)^2\delta}{\varepsilon}. \quad (15)$$

IV. PHASE DIAGRAM AND VORTEX LATTICE STRUCTURE

In order to determine the structure of the vortex lattice state, following a variational procedure similar to that of Refs. 1 and 16, consideration of higher-order terms in the free energy density (2) is required. Just below the upper critical line defined by $B_{c2}(T)$, the magnetic field is partially screened by supercurrents and we decompose $\boldsymbol{h}=\boldsymbol{B}+\boldsymbol{h}_1$, with $B \lesssim B_{c2}$ and $\overline{\boldsymbol{h}_1}=\boldsymbol{0}$, and, correspondingly, $\boldsymbol{A}=\boldsymbol{A}_0+\boldsymbol{A}_1$. At $T>\tilde{T}^*$, the conventional Abrikosov state (A state) is realized, and $f(z)=1$. At $T<\tilde{T}^*$, the FFLO state is realized and two possible modulations could appear at $B<B_{c2}(T)$: the so-called FF state with exponential modulation, $f(z)=\exp(iqz)$, and the LO state with sinusoidal modulation, $f(z)=\sqrt{2}\sin qz$.

By minimizing the free energy, we may determine the lattice geometry[1] and, below $\tilde{T}^*$, the temperature range when the FF or LO state is favored. For this, we write the spatial average of the free energy density (2) in the form

$$\overline{\mathcal{F}_s} = \frac{B^2+\overline{\boldsymbol{h}_1^2}}{8\pi} + \overline{\mathcal{F}_2(\Delta,\boldsymbol{A})} + \overline{\mathcal{F}_4(\Delta,\boldsymbol{A})}, \quad (16)$$

where $\mathcal{F}_2$ and $\mathcal{F}_4$ collect together quadratic and quartic terms with respect to Δ, respectively. By making the substitution $\Delta \to (1+\epsilon)\Delta$ in $\overline{\mathcal{F}_s}$ and requiring that linear-in-ϵ terms vanish,[16] we get

$$0 = \overline{\mathcal{F}_2(\Delta,\boldsymbol{A})} + 2\overline{\mathcal{F}_4(\Delta,\boldsymbol{A})}$$
$$\simeq \overline{\mathcal{F}_2(\Delta,\boldsymbol{A}_0)} + \overline{\boldsymbol{A}_1 \cdot \frac{\delta\mathcal{F}_2}{\delta\boldsymbol{A}}(\Delta,\boldsymbol{A}_0)} + 2\overline{\mathcal{F}_4(\Delta,\boldsymbol{A}_0)}. \quad (17)$$

By variation of the free energy with respect to $\boldsymbol{A}$, we get the Maxwell equation relating the internal magnetic field to screening currents:

$$\frac{1}{4\pi}\text{rot}\,\boldsymbol{h}_1 = \boldsymbol{j}_s = -\frac{\delta\mathcal{F}_2}{\delta\boldsymbol{A}}(\Delta,\boldsymbol{A}_0), \quad (18)$$

up to second-order terms Δ. We insert Eq. (18) into Eq. (17) and integrate the second term by parts. Then Eq. (17) yields

$$\overline{|\Delta|^2} = -\frac{1}{2}\frac{\overline{\mathcal{F}_2(\Delta,\boldsymbol{A}_0)}/\overline{|\Delta|^2}}{[\overline{\mathcal{F}_4(\Delta,\boldsymbol{A}_0)} - \overline{\boldsymbol{h}_1^2}/8\pi]/(\overline{|\Delta|^2})^2}, \quad (19)$$

where the right-hand side depends on the structure of the order parameter only. Near the transition, we note that $\overline{\mathcal{F}_2(\Delta,\boldsymbol{A}_0)} \simeq \varepsilon[B-B_{c2}(T)]\overline{|\Delta|^2}$. Inserting Eq. (19) into Eq. (16), we obtain:

$$\overline{\mathcal{F}_s} = \frac{B^2}{8\pi} - \frac{\varepsilon^2[B-B_{c2}(T)]^2}{4[\overline{\mathcal{F}_4(\Delta,\boldsymbol{A}_0)} - \overline{\boldsymbol{h}_1^2}/8\pi]/(\overline{|\Delta|^2})^2}. \quad (20)$$

The equilibrium vortex lattice structure is thus the one that minimizes the denominator of the second term in the RHS of Eq. (20).

At large enough GL parameter, the denominator in the second term of Eq. (20) is dominated by its first term. Noting that the gap function (9) obeys the properties:

$$(\boldsymbol{D}_\perp\Delta)^2 = 0, \quad \overline{|\Delta|^2|\boldsymbol{D}_\perp\Delta|^2} = \frac{1}{2\lambda^2}\overline{|\Delta|^4}, \quad (21)$$

and using Eq. (14) in the FF or LO state, we find

$$\frac{\overline{\mathcal{F}_4(\Delta,\boldsymbol{A}_0)}}{(\overline{|\Delta|^2})^2} = \pi N_0\beta_A \times \begin{cases} \dfrac{K_3}{4} - \dfrac{v_F^2K_5}{12\lambda^2} & \text{in the A state,} \\ -\dfrac{K_3}{6} + \dfrac{v_F^2K_5}{24\lambda^2} & \text{in the FF state,} \\ \dfrac{K_3}{36} - \dfrac{v_F^2K_5}{48\lambda^2} & \text{in the LO state,} \end{cases} \quad (22)$$

where

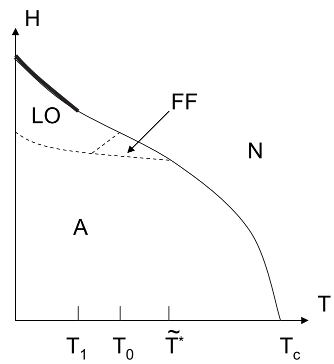

FIG. 1. Schematic phase diagram of a clean three-dimensional superconductor with large Maki parameter. Thin (thick) line is for second- (first-)order transition. Transitions shown with dashed lines are not discussed in the present work.

$$\beta_A = \frac{\overline{|\varphi_0|^4}}{(\overline{|\varphi_0|^2})^2} \tag{23}$$

is the Abrikosov parameter.[1]

We can now discuss the nature of the superconducting state below the upper critical line $B_{c2}(T)$ at $\kappa \to \infty$.[8,9] We already found in Sec. III that the standard A state is realized at $T > \tilde{T}^*$. According to Eq. (22), the coefficient $\overline{\mathcal{F}_4}$ remains positive for these temperatures. Thus the transition from the normal (N) to the A state is of the second order. At $T < \tilde{T}^*$, the coefficient $\overline{\mathcal{F}_4}$ corresponding to the FF state is smaller than the one corresponding to the LO state, and remains positive in the temperature range defined by $9/28 > \lambda^2 K_3 / v_F^2 K_5 > 3/10$, that is, $T_0 < T < \tilde{T}^*$, where

$$T_0 \simeq \tilde{T}^* - 0.085 \frac{T_c}{\alpha_M}. \tag{24}$$

Thus a second-order transition takes place from the N to the FF state for these temperatures. The coefficient $\overline{\mathcal{F}_4}$ corresponding to the LO state is smaller than the one corresponding to the FF state at $T < T_0$ and remains positive at $\lambda^2 K_3 / v_F^2 K_5 > 3/4$, that is, $T_1 < T < T_0$, where

$$T_1 \simeq \tilde{T}^* - 1.785 \frac{T_c}{\alpha_M}. \tag{25}$$

Thus, a second-order transition takes place from the N to the LO state for these temperatures, while the transition into the LO state becomes of the first order at $T < T_1$, when the coefficient $\overline{\mathcal{F}_4}$ corresponding to the LO state becomes negative. These results are summarized schematically in the phase diagram shown in Fig. 1. In the figure, transition lines between the different superconducting phases have also been included for completeness, but their study falls beyond the scope of the present work. Note also that, at all temperatures, the free energy (20) is minimized when β_A is minimal, that is, for a triangular vortex lattice (with $\beta_A = 1.1596$).

V. ELECTRODYNAMICS OF VORTEX STATES

To consider the situation at finite GL parameter we need to evaluate the term $\overline{\boldsymbol{h}_1^2}$ in Eq. (20) where $\boldsymbol{h}_1$ solves the Maxwell equation (18) with

$$\boldsymbol{j}_s = \boldsymbol{j}_{\text{kin}} + \boldsymbol{j}_Z, \tag{26a}$$

$$\boldsymbol{j}_{\text{kin}} = -2e\left((\gamma\Delta + 2\delta \boldsymbol{D}^2\Delta)(\boldsymbol{D}\Delta)^* - \frac{\delta}{3}\text{rot rot}[\Delta(\boldsymbol{D}\Delta)^*] + \text{c.c.} \right) + 8e^2\delta \boldsymbol{B} \times \boldsymbol{\nabla}|\Delta|^2, \tag{26b}$$

$$\boldsymbol{j}_Z = -\varepsilon\, \text{rot}(|\Delta|^2 \hat{z}). \tag{26c}$$

Here, $\boldsymbol{j}_{\text{kin}}$ originates from the usual superconducting kinetic energy response to the spatially varying magnetic field. The Zeeman current $\boldsymbol{j}_Z$ arises from the interaction energy of the superconducting diamagnetic correction to the normal-metal paramagnetic moment, with the spatially varying magnetic field $\boldsymbol{h}_1$.

Making use of the form (9) for the gap and its property (at Landau level $n=0$)

$$\Delta^*(\boldsymbol{D}_\perp \Delta) + \text{c.c.} = \text{rot}(|\Delta|^2 \hat{z}), \tag{27}$$

we find

$$\boldsymbol{j}_s = \boldsymbol{j}_{s,\perp} + \boldsymbol{j}_{s,\parallel}, \tag{28a}$$

$$\boldsymbol{j}_{s,\perp} = -\text{rot}\left[\left(\tilde{\epsilon} + \frac{2e\delta}{3}\nabla^2 \right) |\Delta|^2 \hat{z} \right], \tag{28b}$$

where $\tilde{\epsilon} = \epsilon + 2e(\gamma + 4\delta/\lambda^2)$ in the A state, and, with use of Eq. (14), $\tilde{\epsilon} = \epsilon + 4e\delta/\lambda^2$ in the FF and LO states. And for the longitudinal component, again with use of Eq. (14), we have in the lowest order in $|\Delta|^2$

$$\boldsymbol{j}_{s,\parallel} = \begin{cases} 0 & \text{in the A and LO states,} \\ -4e\frac{q\delta}{3}\nabla^2 |\Delta|^2 \hat{z} & \text{in the FF state.} \end{cases} \tag{28c}$$

Let us note that in the A or LO state, the supercurrents flow in planes perpendicular to the external field only, while in the FF state, they also flow in the parallel direction. Close to the critical temperature $\tilde{T}^*$, a simple estimation shows that

$$\left| \frac{\boldsymbol{j}_{\text{kin}}}{\boldsymbol{j}_Z} \right| \approx \frac{1}{\alpha_M^2}. \tag{29}$$

Hence, we may neglect $\boldsymbol{j}_{\text{kin}}$ in the perpendicular component of the current. In the A and LO states, the Maxwell equation (18) acquires the following form:

$$-\frac{\text{rot}\,\boldsymbol{h}_1}{4\pi} \simeq \varepsilon\, \text{rot}|\Delta|^2 \hat{z}. \tag{30}$$

In the FF state, we may keep the small term corresponding to the parallel-current component and we get

$$-\frac{\text{rot}\,\boldsymbol{h}_1}{4\pi} \simeq \varepsilon\, \text{rot}|\Delta|^2 \hat{z} + \frac{4e\delta q}{3}\nabla^2 |\Delta|^2 \hat{z}. \tag{31}$$

Let us recall that in the conventional Abrikosov state in the vicinity of the critical temperature, where the coefficient γ is positive, the supercurrents are diamagnetic: that is, they create a magnetic moment directed opposite to the external field. The value of ε is always positive; hence as in the Abrikosov case the orbital currents in FF- and LO-modulated states are also diamagnetic despite their Zeeman origin.

Let us now determine the distribution of fields in the vortex state. The component $\boldsymbol{h}_1$ of the magnetic field is periodic and is found from the Maxwell equations (30) or (31), and

$$\mathrm{div}\,\boldsymbol{h}_1 = 0, \tag{32}$$

together with the condition $\overline{\boldsymbol{h}_1}=\boldsymbol{0}$. In the A state, these equations are solved using

$$-\frac{\boldsymbol{h}_1}{4\pi} = \varepsilon(|\Delta|^2 - \overline{|\Delta|^2})\hat{z}. \tag{33}$$

In the FF state, they are solved with

$$-\frac{\boldsymbol{h}_1}{4\pi} = \varepsilon(|\Delta|^2 - \overline{|\Delta|^2})\hat{z} + \frac{4e\delta q}{3}(\hat{z}\times\boldsymbol{\nabla}|\Delta|^2). \tag{34}$$

In the LO state, we search for the solution in the following form:

$$-\frac{\boldsymbol{h}_1}{4\pi} = \varepsilon(|\Delta|^2 - \overline{|\Delta|^2})\hat{z} + 2q\,\boldsymbol{\nabla}[\chi(x,y)\sin 2qz], \tag{35}$$

and we find that $\chi(x,y)$ is an auxiliary function which solves

$$-\nabla_\perp^2\chi + 4q^2\chi = \varepsilon|\varphi_0|^2. \tag{36}$$

In the FF and LO states, in contrast with the situation in the Abrikosov vortex state, the internal field has components perpendicular to the applied magnetic field.

Evaluating $\overline{\boldsymbol{h}_1^2}$ in the A and FF states, we get (neglecting the transverse component of the field in the FF state)

$$\frac{\overline{\boldsymbol{h}_1^2}}{8\pi(\overline{|\Delta|^2})^2} = 2\pi\varepsilon^2(\beta_A - 1). \tag{37}$$

In the LO state, we get

$$\frac{\overline{\boldsymbol{h}_1^2}}{8\pi(\overline{|\Delta|^2})^2} = 2\pi\varepsilon^2\left(\frac{3}{2}\beta_A - 1 + \frac{2q^2\overline{\chi|\varphi_0|^2}}{\varepsilon}\right). \tag{38}$$

A rough evaluation of the last term in Eq. (38) can be done by estimation of $-\nabla_\perp^2\chi \approx \lambda^{-2}\chi$ in Eq. (36). Consequently, as in Eq. (37), we obtain[17]

$$\frac{\overline{\boldsymbol{h}_1^2}}{8\pi(\overline{|\Delta|^2})^2} = 2\pi\varepsilon^2(C\beta_A - 1), \tag{39}$$

where in the A and FF states $C=1$ and for the LO state

$$C_{\mathrm{LO}} = \frac{3}{2} + \frac{2q^2\lambda^2}{1+4q^2\lambda^2}. \tag{40}$$

Inserting the result for $\overline{\boldsymbol{h}_1^2}$ in Eq. (20) and using Eq. (22), we can present the free energy in the usual form:[1,16]

$$\overline{\mathcal{F}_s} = \frac{B^2}{8\pi} - \frac{[B - B_{c2}(T)]^2}{8\pi[1+\beta_A(2\kappa_{\mathrm{eff}}^2 - C)]}, \tag{41}$$

with the effective, temperature-dependent GL parameter κ_{eff} defined by

$$\kappa_{\mathrm{eff}}^2 = \frac{N_0}{4\varepsilon^2}\times\begin{cases} \dfrac{K_3}{4} - \dfrac{v_F^2K_5}{12\lambda^2} & \text{in the A state,}\\[2ex] -\dfrac{K_3}{6} + \dfrac{v_F^2K_5}{24\lambda^2} & \text{in the FF state,}\\[2ex] \dfrac{K_3}{36} - \dfrac{v_F^2K_5}{48\lambda^2} & \text{in the LO state.}\end{cases} \tag{42}$$

We note that the form (41) of the free energy requires $\kappa_{\mathrm{eff}}^2>0$, that is, $T>T_1$. Below T_1, the normal to FFLO state transition becomes of the first order and requires higher-order terms in the gap to be retained in Eq. (2). Moreover, at $T>T_1$, in analogy with type-II superconductors, the free energy (41) is indeed minimized with the vortex lattice state only if $\kappa_{\mathrm{eff}}>\sqrt{C/2}$. The situation in the vicinity of the point $\kappa_{\mathrm{eff}}=\sqrt{C/2}$ requires a special investigation similar to Ref. 18 at $T\to T_c$. Evaluating κ_{eff} at the temperature $\widetilde{T}^*$ corresponding to the tricritical point for N, A, and FF states, we find

$$\kappa_{\mathrm{eff}}^2 \simeq \frac{\kappa^2}{\alpha_M^3}. \tag{43}$$

The theory of the vortex lattice in the FFLO state thus applies at

$$\frac{\kappa^2}{\alpha_M^3} \sim \frac{1}{k_Fr_e}\frac{m}{m^*}\frac{1}{\alpha_M} \gtrsim 1. \tag{44}$$

We now determine the diamagnetic response. The applied magnetic field is found from the thermodynamic relation

$$H = 4\pi\frac{\partial\mathcal{F}_s}{\partial B}. \tag{45}$$

From the relation $B=H+4\pi M$, we obtain the magnetization induced in the superconducting state at a given applied magnetic field:

$$M(H) = -\frac{1}{4\pi}\frac{B_{c2}-H}{(2\kappa_{\mathrm{eff}}^2 - C)\beta_A}. \tag{46}$$

We note that the derivative of the induced magnetization in the superconducting state with respect to the applied magnetic field varies with the temperature along the upper critical line $[T, H_{c2}(T)]$. Its most peculiar features are (1) it increases abruptly close to the temperature T_0, as $C=1$ at $T>T_0$ when the transition is from the N to the FF state, and $C>3/2$ at $T<T_0$ when the transition is from the N to the LO state; (2) it diverges at $T<T_1$ when the transition into the LO state becomes of the first order. Feature 1 occurs close to the triple point for coexistence of the N, FF, and LO states, and it is related to the nature of the FF-LO transition, which is of the first order.[8]

The determination of the magnetic field distribution is also important for nuclear magnetic resonance (NMR). Because of the different field distributions (33), (34), and (35) in the A, FF, and LO states, respectively, we may expect distinct NMR line shapes in each of these states.

VI. CONCLUSION

The experimental search for the FFLO state is not very easy due to the absence of a particular feature distinguishing it from the ordinary Abrikosov mixed state. We already pointed out peculiarities of the magnetization and NMR in the FFLO state. More importantly, the solutions (34) and (35) for the field distribution demonstrate that the internal field has components perpendicular to the applied magnetic field in both FF and LO states, in contrast with the situation in the Abrikosov vortex state. The transverse component of the oscillating field in the FF state is negligibly small in comparison with the longitudinal oscillating component, while both components of the oscillating field have comparable value in the LO state.

The effect could be revealed experimentally by means of small-angle scattering of neutrons *polarized* parallel to the external field; namely, the transition to the LO (and possibly to the FF) state should manifest itself by a strong increase of scattering with neutron spin flip. Another possibility to reveal the LO and FF states is related to application of the muon spin resonance technique by making measurements of the relaxation rate of the precessional motion of muon spins polarized along the external field direction.

Finally, it should be noted that the appearance of a space-oscillating transverse field component in the LO and FF states found here for the isotropic *s*-wave superconductor has a model-independent character, and will also be present in anisotropic materials with a different type of superconducting pairing.

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