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Structure of the vortex lattice in the Fulde-Ferrell-Larkin-Ovchinnikov state

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In clean superconductors under a high magnetic field, when the upper critical field is determined by both orbital and paramagnetic effects, new solutions for the superconducting order parameter, with additional modulation of the vortex lattice along the field, must be realized. They correspond to the Fulde-Ferrell-Larkin-Ovchinnikov effect. In order to determine the structure of the vortex lattice in the new phases, we derive a modified Ginzburg-Landau functional in the high-κ limit, and we determine the corresponding phase diagram.

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I. INTRODUCTION

In 1964 Fulde and Ferrell[1] and Larkin and Ovchinnikov[2] demonstrated that a superconducting state with a spatially modulated order parameter may be stabilized when a large magnetic field or internal exchange field is acting on the electron spins (the paramagnetic effect). So far, to our knowledge, such a Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state has received no unambiguous experimental evidence. The main reasons are that such a state is very sensitive to impurities[3] and that the orbital effect of the magnetic field is usually more important than the paramagnetic one. However, type-II magnetic superconductors of the borocarbide family (R)Ni_2B_2C should be promising candidates for the FFLO-state observation in the paramagnetic phase. Indeed, the reentrant behavior of the upper critical field H_{c2} along the easy magnetic axis in $TmNi_2B_2C$ ($T_c = 10.5$ K, $T_N = 1.7$ K) indicates a strong internal exchange field.[4] In this paper we intend to clarify the structure of the FFLO state when the paramagnetic effect is important and the orbital one cannot be neglected, in order to describe the situation relevant to the experiment. For this, we develop a Ginzburg-Landau (GL) type functional for superconductors in the clean limit. Such an approach was developed in the pure paramagnetic limit by Buzdin and Kachkachi[5] and then extended to unconventional singlet superconductivity in the presence of impurities.[6] It should be considered as an aid for clarifying the structure of the FFLO state, and as a guide for a theory extended to all temperatures and magnetic fields. Hereafter, we assume that the paramagnetic effect is caused by Zeeman splitting and we note the internal field $I = \mu_B B$, where B is the local induction (assuming a Landé factor $g = 2$). Results should be easily generalized to the case of magnetic superconductors, where the main contribution to I comes from exchange interactions between electrons and polarized magnetic atoms.

Using Eilenberger equations,[7] we derive the GL free-energy functional for s-wave superconductivity in the presence of orbital and Zeeman effects in the weak-coupling limit, by supposing both a small value and slow variation of the order parameter Δ over the superconducting coherence length $\xi_0 = \hbar v_F / 2\pi k_B T_c$. In the pure paramagnetic limit, the free energy is valid near the second-order normal (N) to uniform (U) superconducting phase transition at the magnetic field $H_0(T)$. The nonuniform state appearance is related to the change in sign of the coefficient β at the gradient term $\beta|\nabla\Delta|^2$. In the standard GL functional, β is positive, but it occurs as a function of the field acting on the electron spins and goes to zero at $[T^* = 0.56T_c, H^* = H_0(T^*) = 1.07k_BT_c/\mu_B]$, being negative at $T < T^*$.[8] The negative coefficient β means that the modulated state has lower energy when compared to the nonuniform one. To find the modulation vector, one needs to incorporate into the GL functional the term with a second-order derivative $\delta|\nabla^2\Delta|^2$. In addition, in the BCS theory, simultaneously with the vanishing of the gradient term, the coefficient γ at the fourth-order term $\gamma|\Delta|^4$ vanishes, too.[8] Due to this particular property, one needs to add the higher-order terms $\sim|\Delta|^2|\nabla\Delta|^2$ and $|\Delta|^6$. Such a description is adequate in the vicinity of the tricritical point (TP) (T^*,H^*), where the modulation vector goes to zero. At the N/FFLO transition, the modulation of the order parameter with momentum $q = \sqrt{-\beta/2\delta}$ is favored, and both solutions e^{iqx} and e^{-iqx} are degenerate. It is the fourth-order term that breaks the degeneracy and selects either a cos(qx) modulation (LO phase according to widely used terminology) or a e^{iqx} modulation FF phase). By taking into account all the relevant terms in the functional, it is found that the N/FFLO transition is actually slightly first order into an LO state.[9] Moreover, the FFLO/U transition appears to be second order.[9]

In a more realistic situation, the orbital effect cannot be neglected. In type-II superconductors, it induces a vortex lattice structure in the superconducting state (S) below H_{c2}. The coexistence of both FFLO and vortex modulations has been considered in the exact calculation of $H_{c2}(T)$ assuming that the transition is second order.[10,11] At the transition, the orbital effect forces the solutions for the order parameter to be eigenfunctions of the operator $[-i\nabla - (2\pi/\phi_0)\mathbf{A}]^2$, where $\mathbf{A}$ is the potential vector, and $\phi_0 = \pi\hbar c/|e|$ is the flux quantum. This problem is equivalent to that of a charged particle in a constant magnetic field. The eigenvalues are $[2\pi B(2n+1)/\phi_0 + q^2]$, where the Landau level (LL) n quantizes perpendicular motion, and q is the continuous wave vector in the direction parallel to the field. In the absence of a paramagnetic effect, H_{c2} corresponds to the lowest eigenvalue, thus $n = 0$ (lowest Landau level, or LLL) and $q = 0$ (no parallel motion). When the paramagnetic effect is strong and temperature is decreased, higher H_{c2} may be obtained for $q \neq 0$ when $\alpha_M > 1.8$ (that is, FFLO modulation)[10] and/or $n\neq 0$ when $\alpha_M>9$,[11] giving rise to new solutions for Δ. Here, the Maki parameter $\alpha_M=\sqrt{2}H_{c2}^{orb}(0)/H_P(0)$ weighs the paramagnetic and orbital effects;[12] $H_{c2}^{orb}(0)=0.165\phi_0/\xi_0^2$ is the orbital limit at $T=0$, and $H_P(0)=1.25k_BT_c/\mu_B$ is the Chandrasekhar-Clogston paramagnetic limit.

The main goal of our paper is to calculate the structure of Δ near H_{c2} in the presence of both orbital and paramagnetic effects and in the frame of the modified GL functional for high-κ superconductors (κ is the Ginzburg-Landau parameter). Indeed, we expect that the need to consider a new basis of Landau functions for Δ should affect strongly the form of the resulting vortex lattice. In three-dimensional superconductors, it is speculated that the FFLO phase consists of the vortex lattice with additional modulation of the order parameter along the direction of the applied magnetic field. That is, the structure of the order parameter corresponds to the modulation along all three directions. Such a method was already employed by us in quite a different context of two-dimensional superconductivity (see Ref. 13, for more details). In Sec. II, we derive the free-energy functional in the vicinity of TP. In Sec. III, we determine the phase diagram. We find that two new superconducting FFLO phases appear at high magnetic fields between N and S states and below some triple point at temperature $T_0<T^*$. Very near T_0, the phase is of the FF type, if we characterize it by the nature of the modulation along the field. At lower temperatures, it is of the LO type. N/FF and N/LO transitions near T_0 are of the second order; N/LO transitions at lower temperature becomes first order. Transition to the S state is expected to be second order. Let us stress that other attempts have been made to calculate the structure of Δ from direct numerical minimization of the complete Eilenberger equations.[14] However, effects such as the appearance of an FF state and a change of the transition order into an LO state have not been revealed. We believe that our method, though limited to some restrictive conditions, gives better understanding of the situation and can give some hints for further numerical calculations.

II. FREE ENERGY NEAR THE TRICRITICAL POINT

We derive the free-energy functional for a clean, type-II superconductor ($\kappa\gg 1$) in magnetic field by making use of the quasiclassical Eilenberger formalism.[7] Note that we consider the orbital effect in semiclassical approximation. Indeed, the effects we discuss here are quite different from those related to electron level quantization.[15] Such quantization may start to play a role at extremely low temperatures $T<T_c^2/E_F$, where E_F is the Fermi energy, whereas we consider the situation near the tricritical temperature $T\sim 0.5T_c$. Near the normal-to-superconducting phase transition, the superconducting screening current is negligible. So the local magnetic field $\mathbf{B}=B\hat{z}=\mathrm{rot}\,\mathbf{A}$ may be considered as homogeneous, as well as the exchange field I acting on the electron spins. Then, the Eilenberger equations on the averaged Green functions $g_\sigma(\mathbf{r},\mathbf{v},\omega)$, $f_\sigma(\mathbf{r},\mathbf{v},\omega)$, and $f_\sigma^\dagger(\mathbf{r},\mathbf{v},\omega)$, with spin index $\sigma=\pm$ at temperature T are

$$(\hbar\omega+i\sigma I)f_\sigma+\frac{\hbar}{2}\mathbf{v}\cdot\mathbf{\Pi}f_\sigma=\sigma\Delta g_\sigma,$$

$$(\hbar\omega+i\sigma I)f_\sigma^\dagger-\frac{\hbar}{2}\mathbf{v}\cdot\mathbf{\Pi}^*f_\sigma^\dagger=\sigma\Delta^* g_\sigma,$$

$$g_\sigma^2+f_\sigma f_\sigma^\dagger=1. \tag{1}$$

Here, $\Delta(\mathbf{r})$ is the superconducting order parameter, $\omega=(2n+1)\pi k_BT/\hbar$ are the Matsubara frequencies, $\mathbf{\Pi}=\mathbf{\nabla}-(2i\pi/\phi_0)\mathbf{A}$, and $\mathbf{v}=v_F\hat{v}$ is the Fermi velocity. These equations should be solved self-consistently with the condition

$$\Delta(\mathbf{r})\ln\frac{T_c}{T}=\pi k_BT\sum_\omega\left[\frac{\Delta(\mathbf{r})}{\hbar\omega}-\int\frac{d^2\Omega_{\mathbf{v}}}{4\pi}f_+(\mathbf{r},\mathbf{v},\omega)\right]. \tag{2}$$

The characteristic paramagnetic length scale for the order parameter is $2\pi/q$, where q is the FFLO modulation vector. The orbital length scale is of the order of $\sqrt{\phi_0/B}$. We assume that $q\xi_0\ll 1$ and $\epsilon_0=2\pi B\xi_0^2/\phi_0\ll 1$, or equivalently $B\ll H_{c2}(0)$. These conditions are fulfilled in the vicinity of TP and for large values of α_M. Then, near the N/S second-order transition, we can expand Eqs. (1) and (2) up to sixth order in Δ and in the operator $\mathbf{v}\cdot\mathbf{\Pi}$. By considering all the relevant terms we find that these equations derive from the difference of free-energy density between the superconducting and normal state

$$\mathcal{F}=N(0)V^{-1}\int d^3r\Bigg\{\alpha|\Delta|^2+\beta|\mathbf{\Pi}\Delta|^2+\delta\Bigg[|\mathbf{\Pi}^2\Delta|^2+\frac{4e^2\mathbf{B}^2}{\hbar^2c^2}|\Delta|^2\Bigg]+\gamma|\Delta|^4+\mu|\Delta|^2|\mathbf{\Pi}\Delta|^2+\frac{\mu}{8}[\Delta^{*2}(\mathbf{\Pi}\Delta)^2+\Delta^2(\mathbf{\Pi}^*\Delta^*)^2]+\eta|\Delta|^6\Bigg\}. \tag{3}$$

$N(0)$ is the density of states at the Fermi level, V the volume of the sample, and we introduced

$$\alpha=\ln\frac{T}{T_c}+2\pi k_BT\,\mathrm{Re}\sum_{\omega>0}\left(\frac{1}{\hbar\omega}-\frac{1}{\hbar\omega+iI}\right),$$

$$\beta=\frac{(\hbar v_F)^2K_3}{12},\qquad \gamma=\frac{K_3}{4},$$

$$\delta=-\frac{(\hbar v_F)^4K_5}{80},\qquad \mu=-\frac{(\hbar v_F)^2K_5}{6},\qquad \eta=-\frac{K_5}{8},$$

where

$$K_n=2\pi k_BT\,\mathrm{Re}\sum_{\omega>0}\frac{1}{(\hbar\omega+iI)^n}.$$

In the vicinity of TP, δ is positive. Thus, we introduce the dimensionless order parameter $\psi(\tilde{\mathbf{r}})=\Delta(\mathbf{r})/2\pi k_BT_c\sqrt{\epsilon_0}$, spatial coordinate $\tilde{\mathbf{r}}=\sqrt{\epsilon_0}\mathbf{r}/\xi_0$, free-energy density $\tilde{\mathcal{F}}=\mathcal{F}/(2\pi k_B T_c)^2 N(0)(\delta\epsilon_0^3/\xi_0^4)$, and coefficients $\tilde{\alpha}=(\alpha\xi_0^4/\delta\epsilon_0^2+1)$, $\tilde{\beta}=\beta\xi_0^2/\delta\epsilon_0$. On the whole, we obtain the free-energy density

$$\tilde{\mathcal{F}}=\tilde{V}^{-1}\int d^3\tilde{r}\left\{\tilde{\alpha}|\psi|^2+\tilde{\beta}|\tilde{\mathbf{\Pi}}\psi|^2+|\tilde{\mathbf{\Pi}}^2\psi|^2+3\tilde{\beta}|\psi|^4+\frac{40}{3}|\psi|^2|\tilde{\mathbf{\Pi}}\psi|^2+\frac{5}{3}[\psi^{*2}(\tilde{\mathbf{\Pi}}\psi)^2+\psi^2(\tilde{\mathbf{\Pi}}^*\psi^*)^2]+10|\psi|^6\right\}. \quad (4)$$

In this formulation, $\tilde{\alpha}$ and $\tilde{\beta}$ play, respectively, the roles of renormalized magnetic field and temperature.

Analyzing the conditions at the second-order N/S transition, we find that ψ must be an eigenfunction of the operator $\tilde{\mathbf{\Pi}}^2$. Thus, the corresponding upper critical field depends on the Landau level n and dimensionless wave vector $\tilde{q}$, and the actual one is given by

$$\tilde{\alpha}_c=\max_{n,\tilde{q}}[-\tilde{\beta}(2n+1+\tilde{q}^2)-(2n+1+\tilde{q}^2)^2].$$

Accordingly, we get

$$\tilde{\alpha}_c=-(\tilde{\beta}+1) \quad \text{if} \quad \tilde{\beta}>-2,$$

$$\tilde{\alpha}_c=\frac{\tilde{\beta}^2}{4} \quad \text{if} \quad \tilde{\beta}<-2. \quad (5)$$

When $\tilde{\beta}>-2$, the critical field corresponds to LLL ($n_c=0,\tilde{q}_c=0$), whereas when $\tilde{\beta}<-2$, it corresponds to ($n_c=0,\tilde{q}_c^2=-(\tilde{\beta}+2)/2$), but for $\tilde{\beta}<-4$, it is still possible to have the same critical field with $n_c\neq 0$. At this stage, we obtain the coordinates of the N/S/FFLO triple point ($\tilde{\alpha}=1,\tilde{\beta}=-2$).

III. STRUCTURE OF THE SUPERCONDUCTING STATE

In this section, we investigate the most favorable structure for the order parameter in the S state, just below the transition. At the transition $\alpha=\tilde{\alpha}_c$, the order parameter may be any linear combination $\Psi(\tilde{\mathbf{r}})$ of the corresponding eigenfunctions at Landau level n and wave-vector amplitude $\tilde{q}$. Now, we keep $\tilde{\beta}$ constant and we consider $\tilde{\alpha}\lesssim\tilde{\alpha}_c$. The order parameter should be obtained by the minimization of Eq. (4), where the fourth-order terms are now retained. Therefore, we consider the next approximation $\psi=\Psi+\psi^{(1)}$. According to the perturbation theory, to be a solution, it should obey $\int\Psi^*\psi^{(1)}=0$. After straightforward calculation, we obtain the free energy up to the fourth order:

$$\tilde{\mathcal{F}}=(\tilde{\alpha}-\tilde{\alpha}_c)\overline{|\Psi|^2}+\left(3\tilde{\beta}\overline{|\Psi|^4}+\frac{40}{3}\overline{|\Psi|^2|\tilde{\mathbf{\Pi}}\Psi|^2}+\frac{5}{3}[\overline{\Psi^{*2}(\tilde{\mathbf{\Pi}}\Psi)^2}+\overline{\Psi^2(\tilde{\mathbf{\Pi}}^*\Psi^*)^2}]\right),$$

where the overbar stands for the spatial average. Every spatial average depends only on the geometry of the structure of the vortex lattice. Therefore, the free energy

$$\tilde{\mathcal{F}}=-\frac{(\tilde{\alpha}-\tilde{\alpha}_c)^2}{4B_4}$$

is minimum when the coefficient

$$B_4=\frac{3\tilde{\beta}\overline{|\Psi|^4}+\frac{40}{3}\overline{|\Psi|^2|\tilde{\mathbf{\Pi}}\Psi|^2}+\frac{5}{3}[\overline{\Psi^{*2}(\tilde{\mathbf{\Pi}}\Psi)^2}+\overline{\Psi^2(\tilde{\mathbf{\Pi}}^*\Psi^*)^2}]}{(\overline{|\Psi|^2})^2}$$

is minimum also. Note that such a coefficient is analogous, though much more cumbersome, to the one derived by Abrikosov $\overline{|\Psi|^4}/(\overline{|\Psi|^2})^2$.[16]

We are now considering the case when $\tilde{\alpha}_c$ is given by $n=0$. When $\tilde{\beta}>-2$, there is no modulation along the z axis, the order parameter realizes the usual two-dimensional vortex lattice $\varphi_0(\tilde{x},\tilde{y})$, and $B_4=(3\tilde{\beta}+\frac{20}{3})\beta_4$, where $\beta_4=\overline{|\varphi_0|^4}/(\overline{|\varphi_0|^2})^2$. To derive this we used the properties $(\tilde{\mathbf{\Pi}}_\perp\Psi)^2=0$ and $\overline{|\Psi|^2|\tilde{\mathbf{\Pi}}_\perp\Psi|^2}=\frac{1}{2}\overline{|\Psi|^4}$, where $\tilde{\mathbf{\Pi}}_\perp=(\tilde{\Pi}_x,\tilde{\Pi}_y)$. The minimum value for B_4 is positive, and it is given by the triangular vortex lattice for which $\beta_4^{\text{tri}}=1.1596$.

Now, we consider the situation when $\tilde{\beta}<-2$. In such case, we expect the order parameter to realize either the exponentially modulated (FF) state

$$\Psi(\tilde{x},\tilde{y},\tilde{z})=\varphi_0(\tilde{x},\tilde{y})e^{i\tilde{q}\tilde{z}}, \quad (6)$$

and $B_4^{\text{FF}}=(-2\tilde{\beta}-\frac{10}{3})\beta_4$; or the sinusoidally modulated (LO) state

$$\Psi(\tilde{x},\tilde{y},\tilde{z})=\varphi_0(\tilde{x},\tilde{y})\sin\tilde{q}\tilde{z}, \quad (7)$$

and $B_4^{\text{LO}}=\frac{1}{3}(\tilde{\beta}+5)\beta_4$. From the analysis of both values for B_4, we conclude that the most favorable vortex lattice re-

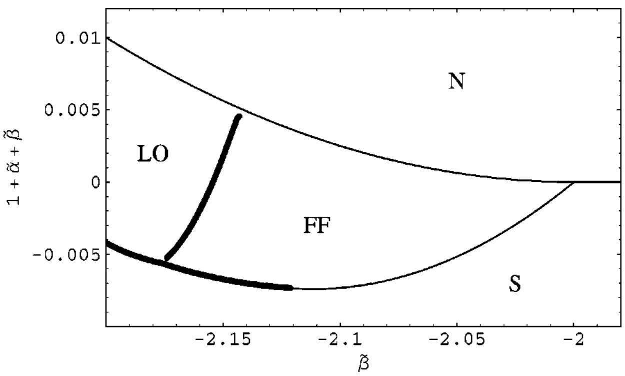

FIG. 1. Phase diagram in the $(\tilde{\beta},\tilde{\alpha}+\tilde{\beta}+1)$ plane. Thin lines stand for second-order transition, thick lines for first order. The peculiar choice of coordinates permits to dilate the region where the FF state is favored. It would be much more reduced in a standard (T,H)-phase diagram.

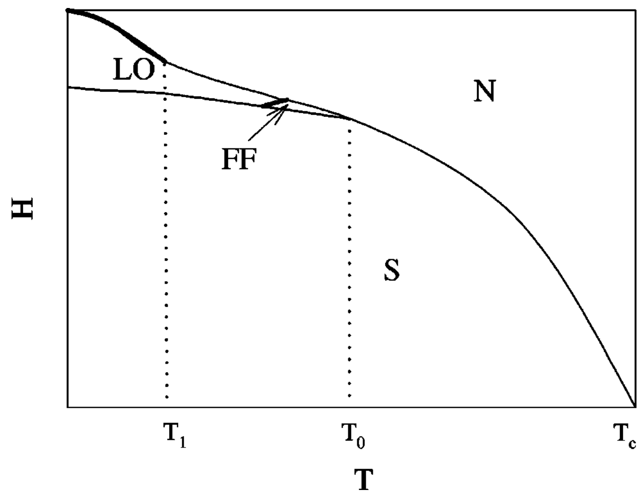

FIG. 2. Qualitative phase diagram. Thin lines stand for second order transition, thick lines for first order.

mains the triangular one, and the transition is of the second order into the FF state for $-\frac{15}{7}<\tilde{\beta}<-2$, second order into the LO state for $-5<\tilde{\beta}<-\frac{15}{7}$, and first order into the LO state for $\tilde{\beta}<-5$. Indeed, at $\tilde{\beta}=-5$, B_4 reverses its sign, and the transition can no longer be treated by neglecting sixth-order terms in Eq. (4).

Determination of the transition between the different S/FF/LO superconducting states requires numerical minimization of Eq. (4). However, we may use the first-harmonic approximation when considering the periodic FFLO modulation along the field. This approximation was employed in a similar discussion of the commensurate/incommensurate transition in ferroelectrics,[17] and it consists of assuming that the order parameter still has one of the forms (6) and (7) with the triangular vortex lattice, except that the modulation wave vector is now deduced from the minimization of Eq. (4), i.e.,

$$\tilde{\mathcal{F}}_{\mathrm{FF}}=[(\tilde{\alpha}-\tilde{\alpha}_c)+(\tilde{q}^2-\tilde{q}_c^2)^2]\overline{|\varphi_0|^2}+[B_4^{\mathrm{FF}}+10(\tilde{q}^2-\tilde{q}_c^2)]\overline{|\varphi_0|^4} \tag{8}$$

in the case of the FF state or

$$\tilde{\mathcal{F}}_{\mathrm{LO}}=[(\tilde{\alpha}-\tilde{\alpha}_c)+(\tilde{q}^2-\tilde{q}_c^2)^2]\overline{|\varphi_0|^2}+\left[B_4^{\mathrm{LO}}+\frac{25}{3}(\tilde{q}^2-\tilde{q}_c^2)\right]\overline{|\varphi_0|^4} \tag{9}$$

in the case of the LO state. Note that the S state corresponds to Eq. (8) when $\tilde{q}=0$. From the analysis of Eqs. (8) and (9), we conclude that the S/FF transition is of the second order for $\tilde{\beta}>-2.12$, and first order below. The S/LO and FF/LO transitions are also of the first order. Finally there is another S/FF/LO triple point at $\tilde{\beta}=-2.18$. All the results are summarized in the phase diagram represented in Fig. 1.

At $\tilde{\beta}<-4$, we noted previously that new solutions with higher LL give the same critical field. In particular, at $\tilde{\beta}=-4$, the LO solution described in the previous paragraph is in competition with the vortex lattice constructed with $(n=1)$ Landau functions in the absence of perpendicular modulation. We verified that the last solution is not favored. This result strongly suggests that the cascade of transitions of the second order into higher LL (Ref. 11) is suppressed by the first-order transition into LLL, though with modulation.

Though we neglected the current in the order-parameter structure determination, we may calculate it from the free-energy functional (3). In particular, the macrosopic current along the z direction is $\int dx dy j_z$, where

$$j_z=2i|e|N(0)\left(\beta\Delta\partial_z\Delta^*+\frac{2\delta}{3}\mathbf{\Pi}^{*2}\Delta^*\partial_z\Delta+\frac{4\delta}{3}\mathbf{\Pi}\partial_z\Delta\cdot\mathbf{\Pi}^*\Delta^*-\mathrm{c.c.}\right). \tag{10}$$

Then, we find that the condition (5) for determining the wave vector in the FFLO state gives exactly the vanishing of the calculated current. Such a point may look counterintuive, in particular regarding the absence of current in the FF state. Still we stress that expression (10) contains additional higher-order derivative terms compared to standard GL theory, for the same reasons as were necessary to modify the free-energy functional. In the pure paramagnetic limit, this point was already noticed in Ref. 2.

To conclude this section, we consider how the pure paramagnetic limit is recovered. When α_M grows to infinity, all the particular points on Fig. 1 collapse onto TP. Then, our result at the N/FFLO transition is in agreement with that of the first-order transition into a one-dimensional sinusoidally modulated state.[9] The disagreement of predicted transition orders at the FFLO/U transition can be explained. Indeed, the result of the transition of the second order could only be made by considering a large number of harmonics in describing the periodically modulated order parameter.[9] So, it is natural that the first-harmonic approximation misses this point, and the S/FFLO transition is mostly of the second order.

IV. CONCLUSION

From our results, we predict that in clean, paramagnetically limited superconductors, the FFLO effect consists of enhancing the upper critical field by inducing a new superconducting phase where the vortex lattice persists, and FFLO modulation takes place along the direction of the field (see Fig. 2). The temperature below which this happens is lower than that of the tricritical point in the pure paramagnetic limit, and its displacement is about $T^* - T_0 \sim 1.2 T_c/\alpha_M$. Moreover, the transition becomes first order below some temperature T_1, such as $T_0 - T_1 \sim 4 T_c/\alpha_M$. The superconducting state may be of two kinds. The first kind is characterized by exponential modulation near the transition to the N state. It takes place on a narrow region near T_0 ($\Delta T \sim 0.08 T_c/\alpha_M$). Despite the e^{iqz} factor in the order parameter, no bulk current flows in the sample at equilibrium. The second kind has sinusoidal modulation near the transition also. In this state, there are parallel planes of nodes of Δ. They should induce strong modifications in the out-of-equilibrium properties of the superconductor, such as pinning. In the particular case of $TmNi_2B_2C$, we expect $T_0 = 2.4$ K,[18] corresponding to not too large $\alpha_M \sim 3-4$. Extrapolating our estimates to this compound, we thus expect the FF phase to be favored in a region not much larger than $(0.1-0.2)$ K, and the temperature where there is the change of transition order should be too reduced to be observable. Thus, major hope of observing the FFLO state would be to explore the properties inside the LO state.

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