Josephson current through a precessing spin

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Abstract. A study of the dc Josephson current between two superconducting leads in the presence of a precessing classical spin is presented. The precession gives rise to a time-dependent tunnel potential which not only creates different tunneling probabilities for spin-up and spin-down quasiparticles, but also introduces a time-dependent spin-flip term. In particular, we study the effects of the spin-flip term alone on the Josephson current between two spin-singlet superconductors as a function of precession frequency and junction transparency. The system displays a steady-state solution although the magnitude and nature of the current is indeed affected by the precession frequency of the classical spin.

Contacting a single molecule in a superconducting nanojunction is a challenging goal, especially if the molecule carries a magnetic moment that can precess in presence of a local magnetic field. The effect of a Josephson current on such a precession was considered by Zhu et al. [1]. Here we address the reverse problem, i. e. how the precession of a classical spin inside a junction affects the Josephson current [2]. As in Ref. [2] we assume a spin-dependent tunneling of quasiparticles but hereafter address the case of arbitrary transparency and precession frequency. We consider two s-wave superconductors coupled over a precessing classical spin, $\vec{S}$, positioned between them. The precession gives rise to a time-dependent tunneling term

$$\hat{H}_T = \hat{\psi}_L^\dagger \hat{v}_{LR}(t)\hat{\psi}_R + H.C. \qquad (1)$$

in the Hamiltonian where $\hat{\psi}_\alpha$ is the Nambu-spinor in lead $\alpha = \mathrm{R, L}$. The hopping matrix $\hat{v}_{LR}(t)\,(=\ \hat{v}_{RL}^\dagger(t))$ has a spin structure and may be parametrized into a spin-independent amplitude v_o and a spin-dependent part $v_s(\cos\vartheta\,\vec{S}_\parallel + \sin\vartheta\,\vec{S}_\perp(t))\cdot\vec{\sigma}$. The spin-quantization axis for the tunneling quasiparticles is given by the precession axis, $\vec{S}_\parallel$, and $\vec{S}_\perp(t)$ gives the instantaneous projection of the precessing spin in the plane. The spin-independent amplitude together with the parallel portion of the spin matrix, $v_o + v_s\cos\vartheta\,\vec{S}_\parallel\cdot\vec{\sigma}$, causes a difference in tunneling amplitude for spin-up and spin-down quasiparticles, while the perpendicular portion, $\sin\vartheta\,\vec{S}_\perp(t)$, induces spin flips. In this paper, we focus on how the Josephson current is affected by spin flips and let the spin direction rotate with an angular frequency ω in the plane thus setting $v_o = 0$ and $\vartheta = \pi/2$. For this case the hopping matrix in combined Nambu-spin space is

$$\hat{v}(t)_{LR} = v_s \begin{pmatrix} e^{-i\omega t\sigma_z}\sigma_x & 0 \\ 0 & e^{i\omega t\sigma_z}\sigma_x \end{pmatrix}. \qquad (2)$$

The tunneling amplitude v_s is further assumed to be energy-independent and to conserve momentum perpendicular to the interface.

To calculate the current through the spin, we solve the time-dependent boundary conditions imposed by $\hat{H}_T$ by using the quasiclassical formulation of the T-matrix method [3, 4]. This allows evaluation of the surface T-matrix, $\tilde{\check{t}}(t,t')$ by means of the extended hopping matrix, $\tilde{\check{v}}(t,t')$, and the surface Green's functions of the two uncoupled leads $\tilde{\check{g}}_\infty(t,t')$ as

$$\tilde{\check{t}}(t,t') = \tilde{\check{v}}(t,t') + \tilde{\check{v}}(t,t_1) \otimes \tilde{\check{g}}_\infty(t_1,t_2) \otimes \tilde{\check{t}}(t_2,t'). \quad (3)$$

The $\otimes$ product implies time convolution of common time-arguments and matrix multiplication in Keldysh-Nambu-spin space while the tilde refers to the extended "reservoir space" as described in Ref. [3]. Carrying out the convolutions, we find two things: first, the T-matrix equation depends only on differences of time-arguments, leading to a steady-state solution where the precession causes a spin-dependent side-band coupling $\varepsilon \pm \omega$. Secondly, the T-matrices can be decomposed into a spin-up and spin-down component along $\vec{S}_\parallel$ with spin-dependent energy shifts $\varepsilon \pm \omega$. This kind of decomposition is described e.g. in Ref. [3]. For the spin-up component T-matrix in the left lead, we write (right lead is straightforward $L \leftrightarrow R$)

$$\check{t}_{L\uparrow}(\varepsilon) = \left[\frac{1}{|v_s|^2}\check{g}^{-1}_{R\downarrow,\infty}(\varepsilon-\omega) - \check{g}_{L\uparrow,\infty}(\varepsilon)\right]^{-1}. \quad (4)$$

The spin-down component T-matrix is given by interchanging the spin and the sign of the energy shift. Now, the Green's functions on each side (L,R) of the interface can be evaluated as [4, 5]

$$\begin{aligned} \check{g}^{in}_{L/R} &= \check{g}_{L/R,\infty} + (\check{g}_{L/R,\infty} + i\pi\check{1})\check{t}_{L/R}(\check{g}_{L/R,\infty} - i\pi\check{1}) \\ \check{g}^{out}_{L/R} &= \check{g}_{L/R,\infty} + (\check{g}_{L/R,\infty} - i\pi\check{1})\check{t}_{L/R}(\check{g}_{L/R,\infty} + i\pi\check{1}). \end{aligned} \quad (5)$$

The Green's functions are divided into incoming and outgoing propagators on either side of the interface depending on the direction of the group velocity $\boldsymbol{v_f}$ of the involved "electron-like" quasiparticles relative to the surface normals $\hat{\boldsymbol{n}}_{L,R}$, e.g. $\boldsymbol{v_f} \cdot \hat{\boldsymbol{n}}_{L,R} < 0$ for an incoming propagator. The charge current for a given superconducting phase difference, φ, across the junction and spin-precession frequency is calculated from the Keldysh component $\hat{g}^K$ as

$$j_{L/R}(\varphi) = \frac{e}{2\hbar}\int\frac{d\varepsilon}{4\pi i}\mathrm{Tr}\left[\hat{\tau}_3\left\{\hat{g}^{K,in}_{L/R}(\varepsilon) - \hat{g}^{K,out}_{L/R}(\varepsilon)\right\}\right] = \frac{e}{4\hbar}\int d\varepsilon\mathrm{Tr}\left\{\hat{\tau}_3\left[\check{t}_{L/R}(\varepsilon), \check{g}_{L/R,\infty}(\varepsilon)\right]^K\right\} \quad (6)$$

We will analyze the current-carrying processes across the precessing spin first focusing on the equilibrium case, i.e. $\omega = 0$. In this limit the kernel in eq. (6) has a rather simple form and the current-phase relation at arbitrary transparency, $\mathcal{D} = 4v_s^2/(1+v_s^2)^2$, and temperature, T, is

$$j(\varphi) = -\frac{e}{2\hbar}\Delta\frac{\mathcal{D}\sin\varphi}{\sqrt{1-\mathcal{D}\cos^2\frac{\varphi}{2}}}\tanh\frac{\varepsilon_J(\varphi)}{2T} \quad (7)$$

(we drop indices L/R and compute all currents on the left side from now on). This expression is identical to the one without spin flips [6] up to an extra phase shift of π. The current is carried by Andreev-bound states with phase dispersion $\varepsilon_J(\varphi) = \pm\Delta\sqrt{1-\mathcal{D}\cos^2\frac{\varphi}{2}}$. The spin flip couples the "spin-up" spinor, $\hat{\psi}_\uparrow = (\psi_{\uparrow\boldsymbol{p}_f}, \psi^\dagger_{\downarrow-\boldsymbol{p}_f})^{\mathrm{T}}$, to the "spin-down" spinor, $\hat{\psi}_\downarrow = (\psi_{\downarrow\boldsymbol{p}_f}, \psi^\dagger_{\uparrow-\boldsymbol{p}_f})^{\mathrm{T}}$, causing the junction to always be in a π-state.

If the precession frequency is finite, the Josephson current changes into a steady-state non-equilibrium current. The spin-flip process is now accompanied with a loss (gain) of an energy

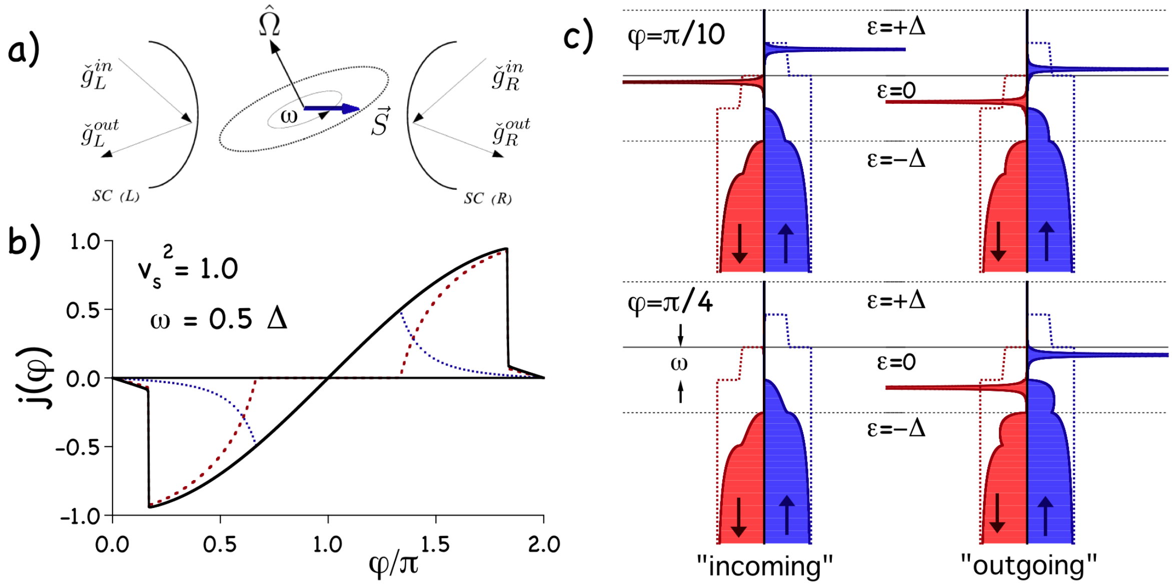

Figure 1. a) The considered system b) The current-phase relation for unit transmission $|v_s|^2 = 1$ and precession frequency $\omega = 0.5\Delta$. The red dashed line is the current contribution from Andreev scattering states within the gap and the dotted blue line is the current carried by spin-scattered continuum states. The unit for the current is $e\Delta/\hbar$. c) The *electron-like* function $g^{<,in/out}_{\downarrow/\uparrow}(\varepsilon)$ giving the combined information of the junction states and their occupation is plotted for two different phase differences. The dashed lines are the spin- and direction- dependent non-equilibrium occupation factors (ranging from unity to zero as a function of energy). The incoming scattering states have a phase dispersion opposite to that of the outgoing ones and hence the incoming and the outgoing states carry current in opposite directions across the spin.

ω when a spin-up (spin-down) electron or a spin-down (spin-up) hole traverses the junction. In the effective t-matrix, eq. (4), this is seen as a Green's function at a given energy, ε, on one side of the junction connected to Green's functions at energies $\varepsilon \pm \omega$ on the other side. We neglect the back-action on the precessing spin for exchanging energy ω. For a nonzero frequency ω the current given in eq. (6) does not deliver a simple expression as in eq. (7), but one can still investigate the current-carrying contributions of the spectra analytically. The current is now carried both by Andreev scattering states $\varepsilon_{J,\uparrow/\downarrow}$ and by an energy region $\pm\omega$ around the gap edges at $\pm\Delta$ as shown for unit transparency in panel b) in figure 1. The effective gap in energy is reduced from 2Δ to $2\Delta - \omega$ as states in the continuum within ω of the gap are scattered into the gapped region. For a nonzero precession frequency, the Andreev scattering states are given as

$$\varepsilon_{J,\uparrow} = \frac{\omega}{2} \pm \Delta\sqrt{1 + f(\omega,\varphi,v_s)}, \quad \varepsilon_{J,\downarrow} = -\frac{\omega}{2} \pm \Delta\sqrt{1 + f(\omega,\varphi,v_s)} \tag{8}$$

where $f(\omega,\varphi,v_s) = \frac{8v_s^4}{(1-v_s^4)^2}\cos^2\frac{\varphi}{2} + \left(\frac{\omega}{2\Delta}\right)^2 - \frac{(1+v_s^4)^2}{(1-v_s^4)^2}\sqrt{\frac{4v_s^4}{(1+v_s^4)^2}(1+\cos\varphi)^2 + 4\left(\frac{1-v_s^4}{1+v_s^4}\right)^2\left(\frac{\omega}{2\Delta}\right)^2}$. Starting at zero phase difference across the junction both $\varepsilon_{J,\uparrow}$ and $\varepsilon_{J,\downarrow}$ come in two phase branches lying symmetrically around $\pm\omega/2$. These *scattering states* exist and disperse with phase difference φ until they touch the gap edge at $\varphi_c(\omega)$ after which they merge with the continuum. The distinct states reappear again for phase differences larger than $2\pi - \varphi_c(\omega)$. In panel b) of figure 1 one can extract $\varphi_c \approx 2\pi/3$ for unit transparency.

The scattering states come with a non-equilibrium distribution function. In the top row of panel c) in figure 1 we see that the spin-down scattering states lie just below the Fermi level ($\varepsilon = 0$) and should be occupied in an equilibrium situation. At the same φ the spin-up scattering states are just above the Fermi level and should be unoccupied. The dynamic spin flip scatters between the two states and creates an effective occupation of both. This is indicated by the dashed lines which are the calculated occupation factors of the scattering states. The degree of occupation depends on precession frequency and the amount of back-scattering ($\mathcal{D}$). In the lower row of panel c) the phase difference is increased so that the spin-down state has dispersed above $\varepsilon = 0$ and is now unoccupied causing both scattering states to vanish in $g_{\downarrow}^{<,in}(\varepsilon)$ and $g_{\uparrow}^{<,in}(\varepsilon)$, resulting in a large and abrupt increase in absolute value in the current-phase relation as seen in panel b) in figure 1.

There are two spectral contributions to the current; the scattering and continuum states dominate for low and high precession frequencies respectively with a sharp break point $\omega = \Delta$. At and above this frequency both phase branches for scattering states are occupied giving a severely reduced contribution to the current. This competition gives a non-monotonous critical current as shown in figure 2. This is most pronounced for unit transparency but present also for lower values. In the tunnel limit, $v_s << 1$, the critical current has a peak at $\omega = 2\Delta$. This peak is a logarithmic divergency coming from resonance between the gap edges and the precession frequency. The divergency is cut off either by a finite temperature or by properly accounting for higher order tunneling as done by summing up the full t-matrix in equation (4), see also Ref. [7].

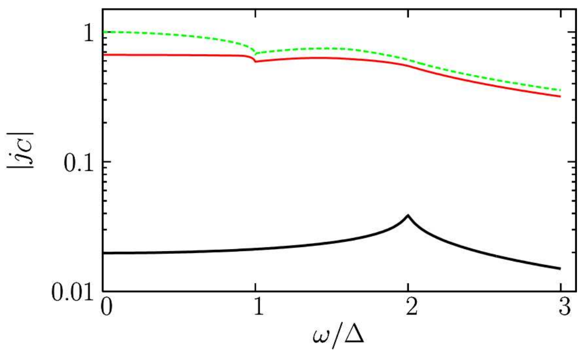

Figure 2. The critical current, $|j_c|$, as a function of precession frequency ω for $v_s^2 = 1.0, 0.5$, and 0.01 from top to bottom computed at $T = 0.0001\Delta$. The unit for current is $e\Delta/\hbar$, and note the different scales of $|j_c|$. Note the logarithmic scale for the critical current axis.

Acknowledgments

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