

## Current-current correlations in hybrid superconducting and normal-metal multiterminal structures

G. BIGNON<sup>1</sup>, M. HOUZET<sup>2</sup>, F. PISTOLESI<sup>1</sup> and F. W. J. HEKKING<sup>1</sup>

<sup>1</sup> *Laboratoire de Physique et Modélisation des Milieux Condensés, CNRS and Université Joseph Fourier - B.P. 166, 38042 Grenoble, France*

<sup>2</sup> *Commissariat à l'Énergie Atomique, DSM/DRFMC/SPSMS F-38054 Grenoble, France*

(received 15 October 2003; accepted in final form 30 April 2004)

PACS. 74.50.+r – Tunneling phenomena; point contacts, weak links, Josephson effects.

PACS. 74.45.+c – Proximity effects; Andreev effect; SN and SNS junctions.

PACS. 74.40.+k – Fluctuations (noise, chaos, nonequilibrium superconductivity, localization, etc.).

**Abstract.** – We consider a hybrid system consisting of two normal metal leads weakly connected to a superconductor. Current-current correlations of the normal leads are studied in the tunneling limit at subgap voltages and temperatures. We find that only two processes contribute to the cross-correlation: the crossed Andreev reflection (emission of electrons in different leads) and the elastic cotunneling. Both processes are possible due to the finite size of the Cooper pair. Noise measurements can thus be used to probe directly the superconducting wave function without the drawbacks appearing in average current measurements where the current is dominated by direct Andreev reflection processes. By tuning the voltages it is possible to change the sign of the cross-correlation. Quantitative predictions are presented both in the diffusive and ballistic regimes.

*Introduction.* – The Andreev reflection is the process of elastic transfer of two electrons forming a Cooper pair from the superconductor to the normal metal [1]. Electrons that are emitted in the normal metal cross the normal/superconductor interface within a distance of the order of the size of the Cooper pair. It is thus possible for the two electrons to be transmitted in two different normal leads, when the distance between the contacts is comparable with the pairs' size. This process, called crossed Andreev reflection (CAR), can be used to probe directly the spatial structure of the Cooper pair, but how can it be detected? Different authors have proposed to measure the conductance matrix in the multiterminal structure and to extract the distance-dependent contribution of the CAR [2–5]. However, this procedure has two main drawbacks. First, CAR comes along with elastic cotunneling (EC) [6], the transfer of an electron from one normal lead to the other one through the superconductor (conserving its spin and energy). Second, CAR and EC contributions to the average currents are dominated by direct Andreev reflection in each normal lead, *i.e.*, by the current associated with the transfer of two electrons in the same lead [5]. Recently, it was shown that strong electron-electron interactions in the normal leads could reduce this contribution [7]. Still considering the non-interacting case, we will show that one can pick up the CAR or the EC contribution directly by measuring the cross-correlation of currents in the two normal leads.

As a matter of fact, due to the discrete nature of charges, current fluctuates in time around its mean value. Current noise measurement revealed a powerful probe for electronic systems [8]. In particular, cross-correlation in multiterminal superconducting hybrid structures

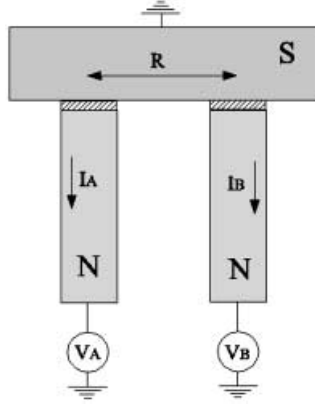


Fig. 1 – Schematic picture of the three-terminal device.

has deserved some attention since it was first anticipated [9, 10] and then predicted [11–18] to show a sign change with respect to normal metallic structures. A simplified explanation of this effect is the following. If electrons in the two leads are emitted from one Cooper pair, one expects a positive correlation, since both electrons appear at the same time in each lead. By contrast, electrons from a fermionic source due to Pauli principle arrive one by one in each channel and they are transmitted either in one or the other lead, leading to a negative correlation. Most calculations consider Y-shaped structures attached to a superconducting lead through a single junction [11–14, 16–18]. This geometry is the standard one for interference in optical experiments, but it leads to a variety of elementary processes for charge transfer in electronic samples. This makes the interpretation of the cross-correlation’s sign subtler [19].

In this paper, we consider the different situation (depicted in fig. 1) where *each* normal arm is *directly* connected to the superconductor through a tunnel interface. We find that CAR contributes to the cross-correlation with a positive sign, while EC contributes with a negative sign, and there are no spurious contributions from direct Andreev reflection. The CAR and EC contributions can be selected independently by tuning the voltages of the normal terminals. Cross-correlation in tunneling systems thus provides a direct way i) to probe both CAR and EC and ii) to measure the sign change of the correlations.

We present quantitative predictions for the dependence of the cross-correlation spectral noise on the distance between normal arms both in the ballistic and diffusive regimes.

*Effective Hamiltonian.* – Let us consider a conventional Bardeen-Cooper-Schrieffer superconductor weakly connected to two normal-metal leads called A and B (see fig. 1). To describe charge transport, we use the standard tunnel Hamiltonian:  $H = H_S + H_N^A + H_N^B + H_T^A + H_T^B$ , where  $H_N^\alpha$  and  $H_S$  describe the clean or impurity-disordered normal lead  $\alpha$  (with  $\alpha = A$  or  $B$ ) and the superconductor, respectively. Explicitly,  $H_N^\alpha = \sum_{\mathbf{k}\sigma} \xi_{\mathbf{k}} c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma}^\alpha$  and  $H_S = \sum_{\mathbf{q}\sigma} \zeta_{\mathbf{q}} d_{\mathbf{q}\sigma}^\dagger d_{\mathbf{q}\sigma} + \sum_{\mathbf{q}} [\Delta d_{\mathbf{q}\downarrow}^\dagger d_{-\mathbf{q}\uparrow}^\dagger + \Delta^* d_{-\mathbf{q}\uparrow} d_{\mathbf{q}\downarrow}]$  with  $\Delta$  the superconducting gap. The operators  $c_{\mathbf{k}\sigma}^\alpha$  and  $d_{\mathbf{q}\sigma}$  are the destruction operators for electrons of spin  $\sigma$  in the normal metal  $\alpha$  and in the superconductor, respectively. The indices  $\mathbf{k}$  and  $\mathbf{q}$  together with the spin indicate quantum numbers labelling the eigenstates in the disconnected leads. The tunneling part of the Hamiltonian is given by

$$H_T^\alpha = \sum_{\mathbf{k}\mathbf{q}\sigma} \left[ t_{\mathbf{k}\mathbf{q}}^\alpha c_{\mathbf{k}\sigma}^\dagger d_{\mathbf{q}\sigma} + t_{\mathbf{k}\mathbf{q}}^{\alpha*} d_{\mathbf{q}\sigma}^\dagger c_{\mathbf{k}\sigma}^\alpha \right], \quad (1)$$

where  $t_{\mathbf{k}\mathbf{q}}^\alpha = \int d\mathbf{r} d\mathbf{r}' t^\alpha(\mathbf{r}, \mathbf{r}') \psi_{\mathbf{k}}(\mathbf{r}) \psi_{\mathbf{q}}^*(\mathbf{r}')$  and  $t^\alpha(\mathbf{r}, \mathbf{r}')$  is the quantum amplitude for an electron to tunnel from position  $\mathbf{r}$  in the normal lead  $\alpha$ , to position  $\mathbf{r}'$  in the superconductor. We introduce the electrostatic potential by the standard gauge transformation  $\xi_{\mathbf{k}} \rightarrow \xi_{\mathbf{k}}^\alpha = \xi_{\mathbf{k}} + eV_\alpha$ .

We are interested in the subgap regime defined by  $|eV_A|, |eV_B|, k_B T \ll \Delta$ . Then, no excitation can be created in the superconductor over times larger than  $\hbar/\Delta$ . Under these conditions, in lowest order of perturbation theory, only two different processes contribute to charge transport: Andreev reflection and elastic cotunneling. In both cases, the corresponding quantum amplitude can be obtained using second-order perturbation theory [20]. Specifically, for the Andreev process, one can calculate the quantum amplitude for destroying a Cooper pair in the superconductor and create two electrons, one in the lead  $\alpha$  with spin  $\uparrow$  and the other in lead  $\beta$  with spin  $\downarrow$ . The result is

$$A_{\mathbf{k}\mathbf{p}}^{\alpha\beta} = \sum_{\mathbf{q}} t_{\mathbf{k}\mathbf{q}}^\alpha t_{\mathbf{p}-\mathbf{q}}^\beta u_{\mathbf{q}} v_{\mathbf{q}} \left[ \frac{1}{\xi_{\mathbf{k}}^\alpha + E_{\mathbf{q}}} + \frac{1}{\xi_{\mathbf{p}}^\beta + E_{\mathbf{q}}} \right], \quad (2)$$

where  $u_{\mathbf{q}}^2 = 1 - v_{\mathbf{q}}^2 = (1 + \zeta_{\mathbf{q}}/E_{\mathbf{q}})/2$  and  $E_{\mathbf{q}} = \sqrt{\Delta^2 + \zeta_{\mathbf{q}}^2}$  is the superconducting spectrum. The amplitudes  $A^{AA}$  and  $A^{BB}$  correspond to the injection of both electrons in the same lead, while  $A^{AB}$  and  $A^{BA}$  give the amplitude of the crossed Andreev reflection.

In a similar way, one can calculate the amplitude for elastic cotunneling ( $B, \sigma \rightarrow A, \sigma$ ):

$$T_{\mathbf{k}\mathbf{p}} = \sum_{\mathbf{q}} t_{\mathbf{k}\mathbf{q}}^A t_{\mathbf{p}\mathbf{q}}^{B*} \left[ \frac{v_{\mathbf{q}}^2}{E_{\mathbf{q}} + \xi_{\mathbf{k}}^\alpha} - \frac{u_{\mathbf{q}}^2}{E_{\mathbf{q}} - \xi_{\mathbf{p}}^\beta} \right]. \quad (3)$$

It is convenient to write an effective Hamiltonian that takes into account these two processes [21]:

$$H^{\text{eff}} = H_N^A + H_N^B + \sum_{\alpha, \beta} [J_{\alpha\beta} + \text{h.c.}] + T + T^\dagger, \quad (4)$$

where  $J_{\alpha\beta} = \sum_{\mathbf{k}\mathbf{p}} A_{\mathbf{k}\mathbf{p}}^{\alpha\beta} c_{\mathbf{k}\uparrow}^{\alpha\dagger} c_{\mathbf{p}\downarrow}^{\beta\dagger}$  and  $T = \sum_{\mathbf{k}\mathbf{p}\sigma} T_{\mathbf{k}\mathbf{p}\sigma} c_{\mathbf{k}\sigma}^{A\dagger} c_{\mathbf{p}\sigma}^B$ . This new Hamiltonian is equivalent to the initial one at low energies and allows to obtain current and noise straightforwardly.

*Currents and cross-correlation.* – Current operator in each normal lead is defined, as usual, by the time derivative of the particle number operator  $N_\alpha = \sum_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma}^{\alpha\dagger} c_{\mathbf{k}\sigma}^\alpha$ :  $I_\alpha = -e dN_\alpha/dt = -(ei/\hbar)[H^{\text{eff}}, N_\alpha]$  (sign conventions are defined in fig. 1). We thus obtain

$$I_A = \frac{ie}{\hbar} [2J_{AA} + J_{AB} + J_{BA} + T] + \text{h.c.}; \quad I_B = \frac{ie}{\hbar} [2J_{BB} + J_{AB} + J_{BA} - T] + \text{h.c.} \quad (5)$$

It is now possible to evaluate the current in both normal leads and their zero-frequency self- and cross-correlation defined as

$$S_{\alpha\beta} = \int_{-\infty}^{+\infty} dt \langle [\delta I_\alpha(t), \delta I_\beta(0)]_+ \rangle, \quad (6)$$

where  $\delta I_\alpha(t) = I_\alpha(t) - \langle I_\alpha \rangle$ ,  $\alpha$  and  $\beta$  take the values  $A$  or  $B$ , and the brackets denote quantum and statistical averaging. One can then apply standard time-dependent perturbation theory

to the lowest non-vanishing order ( $\sim t^4$ ). We obtain

$$\langle I_A \rangle = \frac{2e}{\hbar^2} [(N_{\rightarrow}^{AA} - N_{\leftarrow}^{AA}) + (N_{\rightarrow}^{AB} - N_{\leftarrow}^{AB}) + (N_{\rightarrow}^{\text{EC}} - N_{\leftarrow}^{\text{EC}})], \quad (7)$$

$$S_{AA} = \frac{4e^2}{\hbar^2} [2(N_{\rightarrow}^{AA} + N_{\leftarrow}^{AA}) + (N_{\rightarrow}^{AB} + N_{\leftarrow}^{AB}) + (N_{\rightarrow}^{\text{EC}} + N_{\leftarrow}^{\text{EC}})], \quad (8)$$

$$S_{AB} = \frac{4e^2}{\hbar^2} [(N_{\rightarrow}^{AB} + N_{\leftarrow}^{AB}) - (N_{\rightarrow}^{\text{EC}} + N_{\leftarrow}^{\text{EC}})], \quad (9)$$

with  $N_{\rightleftharpoons}^{\alpha\beta} = 2\pi\hbar \sum_{\mathbf{k}\mathbf{p}} |A_{\mathbf{k}\mathbf{p}}^{\alpha\beta}|^2 H_{\rightleftharpoons}^{\text{And}}(\xi_{\mathbf{k}}, \xi_{\mathbf{p}}) \delta(\xi_{\mathbf{k}} + \xi_{\mathbf{p}} + eV_{\alpha} + eV_{\beta})$ ,  $N_{\rightleftharpoons}^{\text{EC}} = 2\pi\hbar \sum_{\mathbf{k}\mathbf{p}} |T_{\mathbf{k}\mathbf{p}}|^2 \times H_{\rightleftharpoons}^{\text{cot}}(\xi_{\mathbf{k}}, \xi_{\mathbf{p}}) \delta(\xi_{\mathbf{k}} - \xi_{\mathbf{p}} + eV_A - eV_B)$ , and  $H_{\rightleftharpoons}^{\text{And}}(\xi_{\mathbf{k}}, \xi_{\mathbf{p}}) = f(\xi_{\mathbf{k}})f(\xi_{\mathbf{p}})$ ,  $H_{\leftarrow}^{\text{And}}(\xi_{\mathbf{k}}, \xi_{\mathbf{p}}) = [1 - f(\xi_{\mathbf{k}})][1 - f(\xi_{\mathbf{p}})]$ ,  $H_{\rightarrow}^{\text{cot}}(\xi_{\mathbf{k}}, \xi_{\mathbf{p}}) = f(\xi_{\mathbf{k}})[1 - f(\xi_{\mathbf{p}})] = H_{\leftarrow}^{\text{cot}}(\xi_{\mathbf{p}}, \xi_{\mathbf{k}})$ , where  $f(\xi_{\mathbf{k}}) = \langle c_{\mathbf{k}\sigma}^{\dagger} c_{\mathbf{k}\sigma}^{\alpha} \rangle$  is the Fermi function.

Equation (7) for the current agrees with the result obtained by Falci *et al.* in ref. [5]. As mentioned above, the average current has three components: the direct Andreev ( $N^{AA}$  or  $N^{BB}$ ), the crossed Andreev ( $N^{AB}$ ), and the elastic cotunneling ( $N^{\text{EC}}$ ) currents. The direct Andreev term is instead absent in the cross-correlation given by eq. (9): cross-correlation is thus a direct measure of CAR and EC. It is convenient to introduce  $S^{\text{CAR}} = 4e^2/\hbar^2(N_{\rightarrow}^{AB} + N_{\leftarrow}^{AB})$  and  $S^{\text{EC}} = 4e^2/\hbar^2(N_{\rightarrow}^{\text{EC}} + N_{\leftarrow}^{\text{EC}})$  in terms of which we have  $S_{AB} = S^{\text{CAR}} - S^{\text{EC}}$ . Since  $N_{\rightleftharpoons}^{AB}$  and  $N_{\rightleftharpoons}^{\text{EC}}$  are always positive, this expression implies that CAR and EC processes contribute to  $S_{AB}$  with positive and negative sign, respectively. A simple interpretation of this fact is that CAR implies instantaneous currents of the *same* sign in both leads, while EC implies instantaneous currents of *opposite* signs.

To obtain more quantitative predictions, we follow the procedure of refs. [20, 21]. The main result is given in eq. (14) below, with the amplitudes given in eqs. (17) and (20) for the ballistic and diffusive regimes, respectively.

Introducing  $U = (V_A + V_B)/2$  and  $G_Q = 2e^2/h$ ,  $S^{\text{CAR}}$  can be rewritten as

$$S^{\text{CAR}} = 2G_Q \int d\varepsilon A(\varepsilon) [f(\varepsilon + eU)(1 - f(\varepsilon - eU)) + f(\varepsilon - eU)(1 - f(\varepsilon + eU))], \quad (10)$$

with  $F(\zeta; \xi_A, \xi_B) = 2\pi u(\zeta)v(\zeta)\{(E(\zeta) + eV_A - \xi_A)^{-1} + (E(\zeta) + eV_B + \xi_B)^{-1}\}$ , and

$$A(\varepsilon) = \int d\zeta d\zeta' F(\zeta; \varepsilon - eU, \varepsilon + eU) F(\zeta'; \varepsilon - eU, \varepsilon + eU) \Xi(\varepsilon - eU, \varepsilon + eU; \zeta, \zeta'). \quad (11)$$

The function  $\Xi$  reads

$$\begin{aligned} \Xi(\xi_A, \xi_B; \zeta, \zeta') &= \int_{\mathcal{V}_A} d\mathbf{r}_1 d\mathbf{r}_2 \int_{\mathcal{V}_B} d\mathbf{r}_3 d\mathbf{r}_4 \int_{\mathcal{V}_S} d\mathbf{r}'_1 \dots d\mathbf{r}'_4 t^{A*}(\mathbf{r}_1, \mathbf{r}'_1) t^A(\mathbf{r}_2, \mathbf{r}'_2) \cdot \\ &\quad \cdot t^{B*}(\mathbf{r}_3, \mathbf{r}'_3) t^B(\mathbf{r}_4, \mathbf{r}'_4) K_{\xi_A}(\mathbf{r}_1, \mathbf{r}_2) K_{\xi_B}(\mathbf{r}_3, \mathbf{r}_4) K_{\zeta}(\mathbf{r}'_2, \mathbf{r}'_3) K_{\zeta'}(\mathbf{r}'_4, \mathbf{r}'_1), \end{aligned} \quad (12)$$

where  $K_{\xi}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{\mathbf{k}} \delta(\xi - \xi_{\mathbf{k}}) \psi_{\mathbf{k}}(\mathbf{r}_1) \psi_{\mathbf{k}}^*(\mathbf{r}_2)$  is the single-particle spectral function,  $\mathcal{V}_A$ ,  $\mathcal{V}_B$ ,  $\mathcal{V}_S$  indicate the volumes of leads A and B, and of the superconductor.

We assume that tunneling only occurs with constant amplitude and between neighboring points lying on the junctions surface  $\mathcal{S}_{\alpha}$ . Following refs. [6, 20], one can then trade the  $t$ -,  $K_{\xi_A}$ -, and  $K_{\xi_B}$ -dependence for the normal-state tunnel conductances per unit surface,  $g^A$  and  $g^B$  (plus the density of states of the superconductor,  $\nu_S$ ):

$$\begin{aligned} \Xi(\xi_A, \xi_B; \zeta, \zeta') &\simeq \Xi(\zeta, \zeta') = \\ &= \frac{\hbar^2}{16\pi^2 e^4 \nu_S^2} \int_{\mathcal{S}_A} \int_{\mathcal{S}_B} d^2\mathbf{r}_A d^2\mathbf{r}_B \times \times g^A(\mathbf{r}_A) g^B(\mathbf{r}_B) K_{\zeta}(\mathbf{r}_A, \mathbf{r}_B) K_{\zeta'}(\mathbf{r}_B, \mathbf{r}_A). \end{aligned} \quad (13)$$

Fermi functions in eq. (10) restrict the range of variation of  $|\varepsilon|$  to be at most of the order of  $\max(k_B T, |eV_A|, |eV_B|) \ll \Delta$ . One can then safely discard voltage and  $\varepsilon$ -dependence of  $F$  in eq. (11). A similar procedure can be applied to  $S^{\text{EC}}$  as well. We finally obtain

$$S_{AB} = 2eG_Q \left[ (V_A + V_B) \coth \left( \frac{eV_A + eV_B}{2k_B T} \right) A^{\text{CAR}} - (V_A - V_B) \coth \left( \frac{eV_A - eV_B}{2k_B T} \right) A^{\text{EC}} \right] \quad (14)$$

with  $A^i = \int d\zeta d\zeta' \Phi^i(\zeta) \Phi^i(\zeta') \Xi(\zeta, \zeta')$ , where  $i = \{\text{CAR}, \text{EC}\}$ ,  $\Phi^{\text{CAR}}(\zeta) = 2\pi \Delta / (\Delta^2 + \zeta^2)$  and  $\Phi^{\text{EC}}(\zeta) = 2\pi \zeta / (\Delta^2 + \zeta^2)$ . Equation (14) singles out the voltage and temperature dependence of the cross-correlation. The spatial dependence due to the coherent propagation of electrons in the superconductor between the two junctions is isolated in the amplitudes  $A^{\text{CAR}}$  and  $A^{\text{EC}}$ . The same amplitudes appear in the expressions for the current obtained from (7):

$$\langle I_A \rangle = I_A^o(V_A) + G_Q (V_A + V_B) A^{\text{CAR}} + G_Q (V_A - V_B) A^{\text{EC}}. \quad (15)$$

Here  $I_A^o(V_A)$  is the direct Andreev contribution discussed by Hekking and Nazarov [20] giving rise to the non-linear  $I$ - $V$  characteristic. The second and third terms on the right-hand side of eq. (15) give instead the CAR and EC contribution to the current and they have been discussed in refs. [2–5]. For completeness we give the current auto-correlation (noise):

$$S_{AA} = 4eI_A^o(V_A) \coth(eV_A/k_B T) + S^{\text{CAR}} + S^{\text{EC}}. \quad (16)$$

The first term of the right-hand side was obtained in ref. [21] and dominates the noise.

Comparing eqs. (14) and (15), the advantage of measuring the low-temperature cross-correlation over the mean current is apparent. Indeed, in  $S_{AB}$  for  $T = 0$ , there is no direct term. This allows to measure directly  $A^{\text{CAR}}$  or  $A^{\text{EC}}$  by setting  $V_A = V_B$  or  $V_A = -V_B$ , respectively. In contrast, the same procedure would not work by measuring the current. As we will see below,  $A^{\text{CAR}} \approx A^{\text{EC}} = A$ , thus the current becomes  $\langle I_A \rangle = I_A^o(V_A) + 2G_Q A V_A$ , with no residual dependence on  $V_B$ . The interesting  $A$ -dependence is thus hidden by the direct term  $I^o$ . In order to raise the amplitude degeneracy, some authors [3, 5, 15, 22–24] proposed to perform the experiment with spin-polarized normal leads, and measure the drag current  $I_A(V_A = 0, V_B)$ . Detection of Aharonov-Bohm oscillations has also been proposed to distinguish the crossed and direct terms [4] in a ring geometry where EC cannot be detected. This is not necessary if  $S^{AB}$  is measured. A last remark on the temperature dependence is in order. For temperatures  $k_B T \gg |eV_A|, |eV_B|$ , cross-correlation is completely suppressed.

We discuss now briefly the amplitude in the two interesting regimes: ballistic and diffusive. In the ballistic regime one finds [4, 5]

$$\begin{bmatrix} A^{\text{CAR}} \\ A^{\text{EC}} \end{bmatrix} = \int_{S_A} \int_{S_B} d^2 \mathbf{r}_A d^2 \mathbf{r}_B \frac{g^A(\mathbf{r}_A) g^B(\mathbf{r}_B)}{G_Q^2} \begin{bmatrix} \sin^2(k_F |\mathbf{r}_A - \mathbf{r}_B|) \\ \cos^2(k_F |\mathbf{r}_A - \mathbf{r}_B|) \end{bmatrix} \frac{e^{-2|\mathbf{r}_A - \mathbf{r}_B|/\xi_b}}{(k_F |\mathbf{r}_A - \mathbf{r}_B|)^2}, \quad (17)$$

where  $\xi_b = \hbar v_F / \Delta$  is the superconducting coherence length and  $v_F, k_F$  the Fermi velocity and momentum. In multichannel junctions, the trigonometric functions in (17) are averaged out, giving the same numerical factor. Thus  $A^{\text{CAR}} \approx A^{\text{EC}} \equiv A_b$ . For junctions with a typical size much smaller than the distance,  $R$ , between the two contacts, the spatial dependence is given by  $A_b \sim e^{-2R/\xi_b} / (k_F R)^2$ . The exponential decay is related to the characteristic size of the Cooper pairs in the superconducting lead. The algebraic prefactor further reduces the intensity of the effect on the scale of the Fermi wavelength as discussed in refs. [4, 5, 25]. Cross-contributions are expected to be small in ballistic systems.

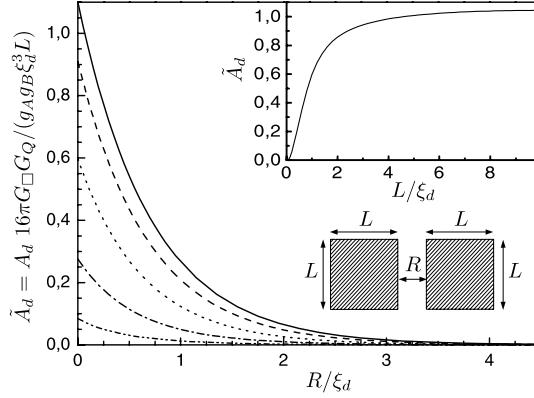


Fig. 2 – Amplitude  $A_d$  for two square contacts on a diffusive superconductive film. From top to bottom:  $L/\xi = \infty, 5, 2, 1, 0.5$ . Dependence on the distance  $R$  for different side lengths  $L$ . Inset: dependence on the side length for  $R = 0$ .

In diffusive superconductors, the coherence length  $\xi_d = \sqrt{\hbar D/\Delta}$  ( $D$  is the diffusion constant) and the distance between the two junctions largely exceed the elastic mean free path  $l_e$ . Impurity averaging is performed on (13) [23,24] within the usual approximation  $k_F l_e \gg 1$ :

$$\langle K_\zeta(\mathbf{r}_A, \mathbf{r}_B) K_{\zeta'}(\mathbf{r}_B, \mathbf{r}_A) \rangle_{\text{imp}} = \frac{\nu_S}{2\pi} [P_{\zeta-\zeta'}(\mathbf{r}_A, \mathbf{r}_B) + P_{\zeta'-\zeta}(\mathbf{r}_B, \mathbf{r}_A)]. \quad (18)$$

The Cooperon  $P_e$  satisfies the diffusion equation in the superconducting lead:

$$-\hbar D \Delta_{\mathbf{r}_1} P_e - i\epsilon P_e = \delta^3(\mathbf{r}_1 - \mathbf{r}_2). \quad (19)$$

Then, we find

$$A_d \equiv A^{\text{CAR}} = A^{\text{EC}} = \frac{\hbar^2}{32\pi e^4 \nu_S} \int d^2 \mathbf{r}_A d^2 \mathbf{r}_B g^A(\mathbf{r}_A) g^B(\mathbf{r}_B) P_{2i\Delta}(|\mathbf{r}_A - \mathbf{r}_B|). \quad (20)$$

If the superconducting electrode is a thin film with a thickness  $d \ll \xi_d$  (characterized by its sheet conductance  $G_\square = 2e^2 \nu_S D d$  in the normal phase), we can use the picture of two-dimensional electron diffusion. Then we find  $P_{2i\Delta}(R) = (1/\pi \hbar D d) K_0(\sqrt{2}R/\xi_d)$ , where  $K_0$  is the modified Bessel function of the second order. In a thicker film,  $P_{2i\Delta}(R) = (1/4\pi \hbar D R) \times e^{-\sqrt{2}R/\xi_d}$ . Comparing with the ballistic 3D case, one finds that the ratio of the two is  $(3/64\pi)(R/l_e)$  when  $R \ll \xi_d < \xi$ . Thus there is a more favorable algebraic dependence on  $R$ , but with a small prefactor. We calculate explicitly the value of  $A_d$  when the normal leads are square films of side  $L$  deposited at distance  $R$  over a superconducting film and the tunnel conductances at the barriers are uniform. The results are presented in fig. 2. As expected, the distance dependence is essentially determined by  $P_{2i\Delta}(R)$ . The size dependence is more interesting. For large values of  $L/\xi_d$ ,  $A_d(R=0)$  grows linearly with  $L$ ; this is due to the fact that the main contribution comes from a small stripe of width  $\xi_d$  and length  $L$  in each contact. This can be used experimentally to increase the signal, there are no drawbacks in increasing the lateral side of the contacts. For small contact sizes  $L \ll \xi_d$ , the whole contact contributes with the same value giving  $A_d \sim L^4 P_{2i\Delta}(R)$ .

*Conclusion.* – We calculated the current-current cross-correlation in multiterminal hybrid structures consisting of two normal-metallic leads attached to a superconductor through

tunnel barriers. We found that varying the voltage biases between the normal leads and the superconductor allows to probe separately two non-local mechanisms for the charge transfer in such systems. We evaluated this effect for realistic devices and we found that it may be observable in diffusive structures, provided that the two normal arms are deposited at a distance not larger than few tens of nanometers (of the order of the superconducting coherence length in the diffusive regime). In view of the recent observation of the doubled shot noise in a superconducting/normal metal tunnel junction [26], we think that measuring such effects is within the reach of present technology.

\* \* \*

We thank D. FEINBERG for pointing out the importance of the shape of the contact in order to optimize the signal. GB, FP, and FWJH acknowledge financial support from CNRS/ATIP-JC 2002 and Institut Universitaire de France. MH acknowledges financial support from the ACI-JC no 2036 from the French Ministry of Research.

#### REFERENCES

- [1] ANDREEV A. F., *Sov. Phys. JETP*, **19** (1964) 1228 (ANDREEV A. F., *Zh. Eksp. Teor. Fiz.*, **46** (1964) 1823).
- [2] BYERS J. M. and FLATTÉ M. E., *Phys. Rev. Lett.*, **74** (1995) 306.
- [3] DEUTSCHER G. and FEINBERG D., *Appl. Phys. Lett.*, **76** (2000) 487.
- [4] RECHER P., SUKHOROUKOV E. V. and LOSS D., *Phys. Rev. B*, **63** (2001) 165314.
- [5] FALCI G., FEINBERG D. and HEKKING F. W. J., *Europhys. Lett.*, **54** (2001) 255.
- [6] AVERIN D. V. and NAZAROV YU. V., *Phys. Rev. Lett.*, **65** (1990) 2446.
- [7] BENA C. *et al.*, *Phys. Rev. Lett.*, **89** (2002) 037901; RECHER P. and LOSS D., *Phys. Rev. B*, **65** (2002) 165327; *Phys. Rev. Lett.*, **91** (2003) 267003.
- [8] For a recent review see BLANTER Y. M. and BÜTTIKER M., *Phys. Rep.*, **336** (2000) 1.
- [9] ANANTRAM M. P. and DATTA S., *Phys. Rev. B*, **53** (1996) 16390.
- [10] MARTIN T., *Phys. Lett. A*, **220** (1996) 137.
- [11] TORRÈS J. and MARTIN T., *Eur. Phys. J. B*, **12** (1999) 319.
- [12] TORRÈS J., MARTIN T. and LESOVIK G. B., *Phys. Rev. B*, **63** (2001) 134517.
- [13] LESOVIK G. B., MARTIN T. and BLATTER G., *Eur. Phys. J. B*, **24** (2001) 287.
- [14] SCHECHTER M., IMRY Y. and LEVINSON Y., *Phys. Rev. B*, **64** (2001) 224513.
- [15] TADDEI F. and FAZIO R., *Phys. Rev. B*, **65** (2002) 134522.
- [16] BÖRLIN J., BELZIG W. and BRUDER C., *Phys. Rev. Lett.*, **88** (2002) 197001.
- [17] SAMUELSON P. and BÜTTIKER M., *Phys. Rev. Lett.*, **89** (2002) 046601.
- [18] SAMUELSON P. and BÜTTIKER M., *Phys. Rev. B*, **66** (2002) 201306.
- [19] For general reviews see also *Quantum Noise in Mesoscopic Systems*, edited by NAZAROV YU. V. (Kluwer, Dordrecht) 2002.
- [20] HEKKING F. W. J. and NAZAROV Y. V., *Phys. Rev. Lett.*, **71** (1994) 1625; *Phys. Rev. B*, **49** (1994) 6847.
- [21] PISTOLESI F., BIGNON G. and HEKKING F. W. J., ArXiv:cond-mat/0303165.
- [22] MELIN R. and FEINBERG D., *Eur. Phys. J. B*, **26** (2002) 101.
- [23] CHTCHELKATCHEV N. M. and BURMISTROV I. S., *Phys. Rev. B*, **68** (2003) 140501R; CHTCHELKATCHEV N. M., *JETP Lett.*, **78** (2003) 230.
- [24] FEINBERG D., *Eur. Phys. J. B*, **36** (2003) 419.
- [25] PRADA E. and SOLS F., ArXiv:cond-mat/0307500.
- [26] LEFLOCH F., HOFFMANN C., SANQUER M. and QUIRION D., *Phys. Rev. Lett.*, **88** (2002) 197001.