Cross correlation of incoherent multiple Andreev reflection

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We use a semiclassical theory to calculate the current correlations in a multiterminal structure composed of a quantum dot connected to all superconducting leads at arbitrary voltage and temperature. This theory holds when the proximity effect is suppressed in the dot. At low voltage $eV \ll \Delta$ (Δ is the superconducting gap in the leads), when charge transport is due to incoherent multiple Andreev reflections, the correlations are strongly enhanced compared to those in the normal state. Moreover, we predict that the cross correlation can be positive or negative depending on the properties of the quantum point contacts between the dot and the leads. We also discuss the effect of inelastic scattering in the dot.

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Nonequilibrium current noise measurements have been used to get information on the charge and statistics of current-carrying states in quantum coherent nanodevices [1]. For noninteracting electrons, the Pauli exclusion principle dictates that the zero-frequency cross correlation in a multiterminal structure is negative [2]. Early experiments realizing solid-state analogues of the Hanbury-Brown and Twiss experiment indeed revealed a negative correlation [3, 4]. In contrast, there is no such sign constraint in the presence of electronic interactions. A superconducting source which emits Andreev pairs of electrons that are collected in normal leads may indeed generate a positive cross correlation as predicted initially for single-channel conductors [5–7] and then for multichannel structures [8–11]. A positive correlation was also predicted in normal metallic systems in the presence of inelastic scattering [12], in Luttinger liquids [13], or in quantum dots attached to spin-polarized leads [14]. Recently, a positive cross correlation was measured in a quantum Hall system attached to a voltage probe [15], in single and coupled quantum dots in the Coulomb blockade regime [16, 17], and in a beam splitter connected to super-Poissonian electron sources [18]. The predicted sign change of the cross correlation in an hybrid superconducting structure remains to be observed.

In the present work, we report theoretically that positive cross correlations may also arise in the case of a normal (N) metallic dot contacted to all superconducting (S) leads. We will restrict to a semiclassical regime when the Josephson current is completely suppressed at sufficiently large voltage, temperature, or magnetic field, and electronic transport is achieved by means of quasiparticles only. The Andreev reflection process allows to transfer the Cooper pairs in the leads as pairs of electrons with subgap energy in the dot [19]. At low voltage and temperature, these electrons are trapped; they must perform several Andreev reflections in order to gain enough energy and escape as quasiparticles in the leads. Such a correlated process for charge transport in S/N/S junctions was described in Ref. [20] in case of a ballistic conductor, and in Ref. [21] in case of a diffusive one, and it is known as the incoherent multiple Andreev reflection (MAR). It produces a rich subharmonic gap structure in the current-voltage characteristics $I(V)$ when $eV = 2\Delta/n$. Here, n is the number of Andreev reflections and Δ is the superconducting gap. At low voltage $V \ll \Delta/e$, a large shot noise $S = (4/3)G\Delta$ was also predicted [22–24] (G is the conductance at low voltage) with respect to the full shot noise in normal state, $S = 2eI$. This result was interpreted as the shot noise of an effective carrier with charge ne; the additional $1/3$-factor comes from the diffusion of the subgap electrons in energy space and it is reminiscent of the Fano factor for the shot noise reduction in diffusive wires [25, 26]. The full counting statistics of current fluctuations substantiated this interpretation [27]. Noise measurements have shown a good agreement with the theory [28–30]. The incoherent MAR regime in two lead geometries seems well understood by now. Our work provides new insight by considering the cross correlation in multiterminal structures.

The semiclassical calculation of the current correlations is based on the Boltzmann-Langevin theory applied to electronic transport in a diffusive wire [25]. It was also used to analyse the sign change of the cross correlation in a device with hybrid terminals [10]. In the following, we present an extension of this theory in order to calculate the current noise in a geometry with all superconducting leads, at arbitrary temperature and voltage. We then derive analytical formulas at low voltage and temperature. This allows us to specify under which conditions a change in the sign of the cross correlation could be observed. Finally, we consider the effect of inelastic scattering in the dot.

The system that we consider is a large quantum dot contacted to superconducting leads α $(= 1, 2, 3)$ through quantum point contacts characterized by their number of transmitting channels N_α and transparencies Γ_α, see fig. 1(a). The normal-state conductance at each contact is $G_\alpha = (2e^2/h)N_\alpha\Gamma_\alpha$. We consider the semiclassical case when G_α strongly exceeds the conductance quantum, so

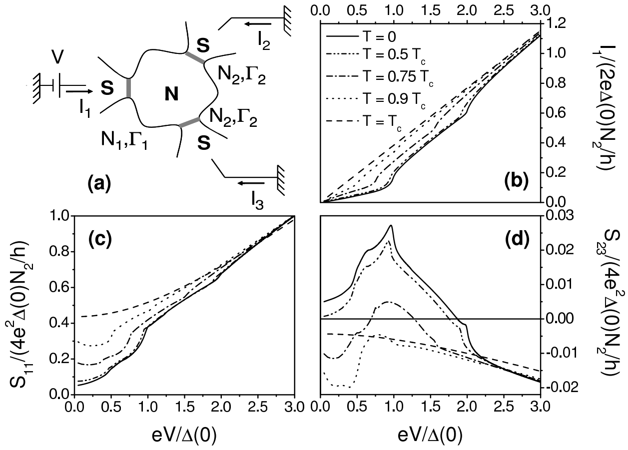

FIG. 1: (a) Schematic device of a quantum dot connected to superconducting leads through quantum point contacts with N_α channels of transparency Γ_α. (b) Voltage dependence of the current in a device with $N_1 = 10N_2$, $\Gamma_1 = 1$, and $\Gamma_2 = 0.2$, at various temperatures. (c) Noise and (d) cross correlation for the same device. Here, T_c is the critical temperature in the leads and $\Delta(0)$ is the superconducting gap at zero temperature.

that Coulomb blockade in the dot can be neglected. We also assume that the proximity effect is suppressed. This is realized when a small magnetic field destroying the coherence of Andreev pairs in the dot is applied, or when eV or kT exceeds the energy scale $\varepsilon^* = \delta \sum_\alpha N_\alpha \Gamma_\alpha$ associated to phase-coherent phenomena (δ is the mean-level spacing in the dot). In order to simplify the calculations, we take identical contacts with leads 2 and 3 ($N_2 = N_3$ and $\Gamma_2 = \Gamma_3$) and we assume that they are kept at the same potential, so that $V_1 = V$ and $V_2 = V_3 = 0$.

We start with the derivation of a Boltzmann-Langevin theory allowing to calculate the zero-frequency current correlators

$$S_{\alpha\beta} = 2\int dt \langle \Delta I_\alpha(t) \Delta I_\beta(0)\rangle \tag{1}$$

at arbitrary voltage and temperature. Here, $\Delta I_\alpha(t) = I_\alpha(t) - \bar{I}_\alpha$ is the fluctuating current in lead α ($\bar{I}_\alpha$ is its time average), $\langle \cdots \rangle$ denotes an ensemble average, and

$$I_\alpha(t) = \frac{e}{h}\int d\varepsilon [i^e_\alpha(\varepsilon,t) - i^h_\alpha(\varepsilon,t)] \tag{2}$$

is decomposed into an electron spectral particle current

$$\begin{aligned} i^e_\alpha(\varepsilon) &= N_\alpha T^N_\alpha(\varepsilon + eV_\alpha)[f^e(\varepsilon) - f_0(\varepsilon + eV_\alpha)] \\ &+ N_\alpha R^A_\alpha(\varepsilon + eV_\alpha)[f^e(\varepsilon) - f^h(\varepsilon + 2eV_\alpha)] \\ &+ \delta i^e_\alpha(\varepsilon), \end{aligned} \tag{3}$$

at energy ε, and a hole spectral current $i^h_\alpha(\varepsilon) = -i^e_\alpha(-\varepsilon)$. Both depend on the nonequilibrium distribution functions for electrons, $f^e(\varepsilon)$, and holes, $f^h(\varepsilon) = 1 - f^e(-\varepsilon)$, in the dot and the Fermi distribution f_0 at temperature T in the leads. The first term in the r.h.s. of eq. (3) describes the quasiparticle tunnelling above the gap; the second term describes the Andreev reflection. Here, T^N_α and R^A_α are the normal-transmission and Andreev-reflection coefficients, respectively, for a uniform barrier with transparency Γ_α [19]. The last term $\delta i^e_\alpha(\varepsilon)$, as well as $\delta i^h_\alpha(\varepsilon) = -\delta i^e_\alpha(-\varepsilon)$, are Langevin sources; they describe the stochastic character of electron and hole transfer at the contacts. Their correlators will be specified below. The distribution functions in the dot are further determined by the kinetic equations:

$$\sum_\alpha i^e_\alpha = 0 \quad \text{and} \quad \sum_\alpha i^h_\alpha = 0, \tag{4}$$

which express the conservation of the spectral currents in the absence of inelastic scattering in the dot.

We briefly retrieve known results on the time-average current flowing through the device [20, 21]. It can be expressed as:

$$\bar{I}_\alpha = \frac{2eN_\alpha}{h}\sum_p \int_0^{eV} d\varepsilon D_\alpha(\varepsilon_p)[\bar{f}^e(\varepsilon_p - eV_\alpha) - \bar{f}^h(\varepsilon_p + eV_\alpha)], \tag{5}$$

where $\varepsilon_p = \varepsilon + 2peV$ (p is an integer), $D_\alpha = T^N_\alpha + 2R^A_\alpha$, and $\bar{f}^e$ (resp. $\bar{f}^h$) is the time-average electron (resp. hole) distribution function. In ensemble average, the Langevin source vanishes: $\langle \delta i^e \rangle = 0$. Then, eqs. (3)-(4) yield a set of equations coupling $\bar{f}^e$ and $\bar{f}^h$ at different energies, that can be solved numerically. By inserting the solution into eq. (5), we get the current-voltage characteristics $\bar{I}_1(V)$. The curve is strongly nonlinear and displays the subharmonic gap structure described above (see fig. 1(b)).

Now, we consider the current fluctuations. The Langevin source in eq. (3) generates the fluctuations of the distribution functions $\Delta f^{e/h}(t) = f^{e/h}(t) - \bar{f}^{e/h}$ and particle spectral currents $\Delta i^{e/h}_\alpha(t) = i^{e/h}_\alpha(t) - \bar{i}^{e/h}_\alpha$ around their time average. From eq. (3) we get:

$$\begin{aligned} \Delta i^e_\alpha(\varepsilon) = N_\alpha[T^N_\alpha(\varepsilon + eV_\alpha) + R^A_\alpha(\varepsilon + eV_\alpha)]\Delta f^e(\varepsilon) \\ -N_\alpha R^A_\alpha(\varepsilon + eV_\alpha)\Delta f^h(\varepsilon + 2eV_\alpha) + \delta i^e_\alpha(\varepsilon). \end{aligned} \tag{6}$$

at each contact. Moreover, eq. (4) yields:

$$\sum_\alpha \Delta i^e_\alpha = 0 \quad \text{and} \quad \sum_\alpha \Delta i^h_\alpha = 0. \tag{7}$$

Making use of eqs. (6)-(7), it is possible to relate the fluctuating distribution functions to the Langevin sources. Finally, we may express the cross correlators (1) in terms of the second moment of the Langevin sources.

We assume that each quantum point contact generates independent fluctuations of spectral particle current with a white noise. Then, the Langevin correlators are

$$\langle \delta i^e_\alpha(\varepsilon,t)\delta i^e_\beta(\varepsilon',0)\rangle = hN_\alpha\delta_{\alpha\beta}\delta(t)\delta(\varepsilon-\varepsilon')s^{ee}_\alpha(\varepsilon), \tag{8a}$$

$$\langle \delta i^e_\alpha(\varepsilon,t)\delta i^h_\beta(\varepsilon',0)\rangle = hN_\alpha\delta_{\alpha\beta}\delta(t)\delta(\varepsilon-\varepsilon'+2eV_\alpha)s^{eh}_\alpha(\varepsilon), \tag{8b}$$

where the fluctuation powers $s^{ee}_\alpha(\varepsilon)$ and $s^{eh}_\alpha(\varepsilon)$ are determined by the transmission coefficients for all possible charge-transfer processes and the time-average distribution functions on each side of the contact,

$$\begin{aligned} s^{ee}_\alpha = {} & T^N_\alpha[\bar f^e(1-f_\alpha)+f_\alpha(1-\bar f^e)-T^N_\alpha(\bar f^e-f_\alpha)^2] \\ & +R^A_\alpha[\bar f^e(1-\bar f^h_\alpha)+\bar f^h_\alpha(1-\bar f^e)-R^A_\alpha(\bar f^e-\bar f^h_\alpha)^2] \\ & -2T^N_\alpha R^A_\alpha(\bar f^e-\bar f^h_\alpha)(\bar f^e-f_\alpha), \end{aligned} \tag{9a}$$

$$\begin{aligned} s^{eh}_\alpha = {} & -R^A_\alpha[\bar f^e(1-\bar f^h_\alpha)+\bar f^h_\alpha(1-\bar f^e)-R^A_\alpha(\bar f^e-\bar f^h_\alpha)^2] \\ & +T^N_\alpha R^A_\alpha(\bar f^e-\bar f^h_\alpha)^2-4T^N_\alpha T^A_\alpha(\bar f^e-f_\alpha)(\bar f^h-f_\alpha). \end{aligned} \tag{9b}$$

Here, we used short notations $\bar f^h_\alpha=\bar f^h(\varepsilon+2eV_\alpha)$, $f_\alpha = f_0(\varepsilon+eV_\alpha)$, $T^N_\alpha=T^N_\alpha(\varepsilon+eV_\alpha)$, $R^A_\alpha=R^A_\alpha(\varepsilon+eV_\alpha)$, and $T^A_\alpha=T^A_\alpha(\varepsilon+eV_\alpha)$ is the coefficient for Andreev transmission [19]. Eqs. (9) may be derived microscopically [31] from the quantum mechanical generating functional for the current fluctuations in a contact between normal and superconducting reservoirs with non-Fermi distribution functions [32]. The terms proportional to T^N_α and R^A_α express the partition noise due to normal transmission and Andreev reflection, respectively; other terms originate from the interference between two-particle scattering processes. Altogether, they allow to reproduce the noise at a single N/S contact [33].

Combining eqs. (1)-(9), we can evaluate numerically the noise and cross correlation at arbitrary voltage and temperature. The results are illustrated for a specific device in fig. 1(c-d). A subharmonic gap structure is clearly visible. The cross correlation for this device is positive at low voltage and temperature $kT\ll\Delta$. At $eV\gg\Delta$, the normal state result $S_{23}<0$ for non-interacting fermions is recovered. For not so low temperatures, the cross correlation's sign results from the competition [22] between the positive contribution due to MAR and the negative contribution due to quasiparticle transfer above the gap. Remarkably, the positive sign persists in a finite voltage range.

In the following, we specify the optimal conditions to observe the sign change of the correlation. For this, we address the low voltage and temperature regime in more details. For subgap energies, only normal and Andreev-reflection processes may take place ($T^N=T^A=0$). Then, the time average of eq. (2) yields:

$$\bar i^e_\alpha(\varepsilon) = N_\alpha R^A_\alpha(\varepsilon+eV_\alpha)[\bar f^e(\varepsilon)-\bar f^h(\varepsilon+2eV_\alpha)], \tag{10}$$

and $\bar i^e_\alpha(\varepsilon-eV_\alpha)=-\bar i^h_\alpha(\varepsilon+eV_\alpha)$. Combining eq. (10) with the time average of eq. (4), we may express $\bar f^h(\varepsilon)$ as a function of $\bar f^e(\varepsilon)$ and $\bar f^e(\varepsilon+2eV)$. Substituting it back into eq. (4) and taking the limit of small voltage, we get a diffusion equation in energy space when $|\varepsilon|<\Delta$:

$$\frac{\partial \bar i^e_1(\varepsilon)}{\partial\varepsilon}=0 \quad \text{where} \quad \bar i^e_1(\varepsilon) = -2eV\mathcal{D}(\varepsilon)\frac{\partial \bar f^e}{\partial\varepsilon}, \tag{11}$$

and $\mathcal{D}(\varepsilon)=[1/N_1R^A_1(\varepsilon)+1/2N_2R^A_2(\varepsilon)]^{-1}$. Above the gap, the electrons get thermalized by the leads: $\bar f^e(\varepsilon)=f_0(\varepsilon)$. Eq. (11) for the distribution function is easily solved. Then, the conductance $G=\bar I_1/V|_{V\to0}$ is [21, 27]:

$$G=\frac{4e^2}{h}\left[\frac{1}{2\Delta}\int_{-\Delta}^{\Delta}\frac{d\varepsilon}{\mathcal{D}(\varepsilon)}\right]^{-1}. \tag{12}$$

Evaluation of eq. (12) yields $G=[1/G^*_1+1/2G^*_2]^{-1}$ where $G^*_\alpha=(12e^2/h)N_\alpha\Gamma^2_\alpha/(8-8\Gamma_\alpha+3\Gamma^2_\alpha)$.

Below the gap, the Langevin sources in electron and hole channels are fully correlated: $\delta i^e_\alpha(\varepsilon-eV_\alpha) = -\delta i^h_\alpha(\varepsilon+eV_\alpha)$. Using eq. (7), we deduce that the fluctuating spectral current through lead 1 is conserved: $\Delta i^e_1(\varepsilon+2peV,t)=\Delta i_1(t)$ for $|\varepsilon+2peV|<\Delta$. At low voltage, it obeys:

$$\frac{\Delta i_1}{\mathcal{D}(\varepsilon)}=\left[-2eV\frac{\partial\delta f^e(\varepsilon)}{\partial\varepsilon}+\frac{\delta i^e_1(\varepsilon)}{N_1R^A_1(\varepsilon)}-\frac{\delta i^e_2(\varepsilon)+\delta i^e_3(\varepsilon)}{2N_2R^A_2(\varepsilon)}\right]. \tag{13}$$

Above the gap, the distribution function and spectral current do not fluctuate. When eq. (13) is integrated over energies $-\Delta<\varepsilon<\Delta$, the term $\propto\delta f^e$ then cancels and we get a relation between Δi_1 and the Langevin sources. We can now express the fluctuating currents in the leads: $\Delta I_1(t)=(4e\Delta/h)\Delta i_1(t)$ and $\Delta I_{2,3}(t)=-\Delta I_1(t)/2\pm(e/h)\int_{-\Delta}^{\Delta}d\varepsilon[\delta i^e_2(\varepsilon,t)-\delta i^e_3(\varepsilon,t)]$ in terms of the Langevin sources. As $\bar f^e\approx\bar f^h$, the Langevin correlators (9) reduce to $s^{ee}_\alpha=-s^{eh}_\alpha=2R^A_\alpha\bar f^e(1-\bar f^e)$. Then, we get the noise

$$S_{11}=\frac{4hG^2}{e^2}\int_{-\Delta}^{\Delta}d\varepsilon\frac{\bar f^e(\varepsilon)[1-\bar f^e(\varepsilon)]}{\mathcal{D}(\varepsilon)} \tag{14}$$

and the cross correlation

$$S_{23}=\frac{S_{11}}{4}-\frac{2e^2}{h}\int_{-\Delta}^{\Delta}d\varepsilon N_2R^A_2(\varepsilon)\bar f^e(\varepsilon)[1-\bar f^e(\varepsilon)] \tag{15}$$

From eq. (14), we derive the expected result $S_{11}=(4/3)G\Delta$. The cross correlation is also readily evaluated, but the general expression is quite cumbersome and is not reported here. We give some limiting cases:

$$S_{23}=\begin{cases} -\frac{16e^2\Delta}{3h}\frac{N_2^2}{N_1+2N_2} & \text{for} \quad \Gamma_1=\Gamma_2=1, \\ -\frac{72e^2\Delta}{35h}N_2 & \text{for} \quad \Gamma_1\ll1,\, \Gamma_2=1, \\ \frac{11e^2\Delta}{30h}N_2\Gamma_2^2 & \text{for} \quad \Gamma_1=1,\, \Gamma_2\ll1, \\ \frac{e^2\Delta}{30h}\frac{N_2\Gamma_2^2(11N_1\Gamma_1^2-38N_2\Gamma_2^2)}{N_1\Gamma_1^2+2N_2\Gamma_2^2} & \text{for} \quad \Gamma_1,\Gamma_2\ll1. \end{cases} \tag{16}$$

The dependence of the cross correlation with the ratio of transmitting channels is plotted for a given set of contact transparencies in fig. 2. This, together with eq. (16) illustrates that a positive, MAR induced cross correlation is predicted when some backscattering takes place at the leads 2 and 3 ($\Gamma_2<1$) and $N_1\Gamma_1^2\gtrsim N_2\Gamma_2^2$ (or $G^*_1\gtrsim G^*_2$), approximately.

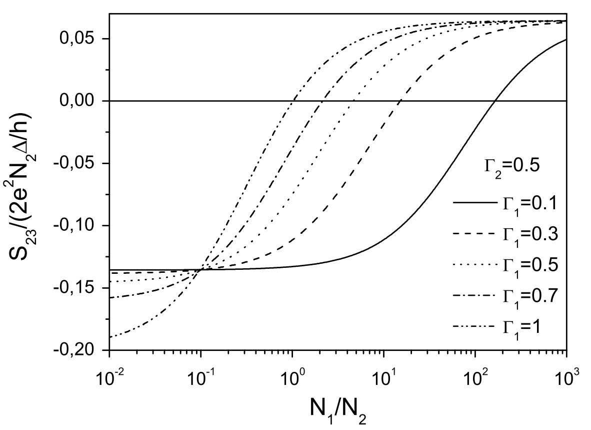

FIG. 2: Zero-temperature cross correlation *vs.* the ratio of channel numbers at the contacts for different values of the contact transparencies. (Note that the curves do not intersect exactly at the same point.)

Up to now, inelastic scattering in the dot has been neglected. In fact, in the MAR regime, the diffusion time for subgap electrons scales as V^{-2}. At low enough voltage, it exceeds the electron-electron collision time $\tau_{\rm ee}$ and the above results do not apply. In the strong inelastic regime, the subgap electrons get thermalized with an effective temperature $T_{\rm e}$ which is determined by the compensation of the energy inflow due to the Andreev reflections and the energy outflow due to inelastic collisions [23]. As a result, we obtain $kT_{\rm e} \simeq \Delta/\ln\lambda$, where $\lambda \sim \Delta^2/G^A V^2 \tau_{\rm ee}\delta$. Then, the noise is given by a fluctuation dissipation relation (FDR): $S_{11} = 4G^A kT_{\rm e}$. Here, $G^A_\alpha = (4e^2/h)N_\alpha\Gamma^2_\alpha/(2-\Gamma_\alpha)^2$ is the Andreev conductance at each contact for $eV, kT \ll \Delta$ and $G^A = 2G^A_1 G^A_2/(G^A_1 + 2G^A_2)$. Moreover, the cross correlation $S_{23} = -2(G^A G^A_2/G^A_1)kT_{\rm e}$ stays negative. When $V \to 0$, $T_{\rm e}$ vanishes and electron-phonon collisions cannot be neglected. Taking them into account would yield the FDR at the phonon temperature [22].

A strong motivation for measuring the cross correlation in multiterminal hybrid structures is that the devices with a superconducting emitter attached to metallic leads have been proposed as a source of Einstein-Podolsky-Rosen pairs of electrons and noise measurement could serve as a diagnosis of their entanglement [34]. In our proposal, the origin of the cross correlation is semiclassical and no signature of entanglement should be expected. Nevertheless, its detection would be an important stage toward more elaborated experiments.

In conclusion, we have presented a semiclassical theory of current noise in a superconducting multiterminal structure. We have shown that charge transfer in the regime of incoherent multiple Andreev reflection generates a large cross correlation compared to the normal-state result. While the cross correlation is negative when the leads are in normal state, we have found that it may become positive in the superconducting state.

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