Magnetic-spin-echo response to a moving vortex lattice in a type-II superconductor

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We present a model for the reduction of the height of a NMR spin echo caused by driven motion of vortices in an isotropic type-II superconductor. Detailed calculations are carried out for a triangular vortex lattice and a small-amplitude quasistatic sinusoidal rocking of the direction of the applied magnetic field. The model exhibits a scaling behavior in the relevant variables that is independent of the geometry of the sample. It is applied to the case of a thin cylindrical sample with the external field applied perpendicular to the axis of the sample. By fitting the model to experimental measurements, absolute values of the penetration depth λ and the coherence length ξ can be obtained.

I. INTRODUCTION

It has recently been found that the NMR spin echo can be used as a sensitive local probe (averaged over the sample volume) of vortex motion in the mixed state of a type-II superconductor.[1,2] Such measurements probe the magnetic-field distribution of the vortex lattice and irreversible phenomena, such as vortex pinning,[3,4] rf-assisted flux motion,[5] etc. Through its sensitivity to the structure of the magnetic field, the spin-echo response can be used for volume measurements of the fundamental lengths in a superconductor, the penetration depth (λ) and probably the coherence length (ξ).

NMR (Refs. 6,7) and muon spin resonance[8] (μSR) measurements of λ based upon the local field distribution of the vortex structure have been reported in several cases. In the past, NMR measurements of λ have been possible only for samples with absorption lines narrow enough that the additional broadening caused by the vortex lattice can be seen in the absorption line shape. An advantage of the spin-echo measurement analyzed in this paper is that it can be used to measure λ even when, as is often the case, inhomogeneous broadening of the NMR absorption line is very large compared to the additional broadening caused by the vortex lattice.

A particularly valuable aspect of spin-echo measurements of ξ is that they would be related directly to the spatial variation of the order parameter in the superconductor and not inferred from measurements of the upper critical field (B_{c2}). Thus this measurement of ξ would not be affected by processes that modify B_{c2} but do not change ξ, such as Pauli limiting.[9]

In this paper we calculate the effect of a small, dynamic change in the orientation of the external magnetic field on the height of the spin-echo signal, show how it can be used to measure λ and ξ in an isotropic type-II superconductor, and apply it to the case of cylindrical sample geometry. Our purpose is to provide the theoretical framework for analyzing the corresponding experiments. The detailed assumptions of the model and approximations used for the calculations are discussed as they are introduced. Overall, the work presented here applies to small vortex lattice displacements in isotropic, strongly type-II superconductors for the magnetic-field range $2B_{c1} < B_0 < 0.25B_{c2}$ (B_{c1} is the lower critical field and B_0 is the magnitude of the externally applied field $\mathbf{B}_0$). First we review briefly the steps used to calculate the magnetic-field distribution of the vortex lattice. Particular emphasis is placed on the way in which it and its spatial derivatives scale with changes in B_0, λ, and ξ. Then we calculate the effect of a slow oscillation in the direction of $\mathbf{B}_0$ on the height of the spin echo. This model is then applied to the case of a thin, cylindrical sample with the average orientation of $\mathbf{B}_0$ perpendicular to its axis and the small oscillation parallel to the axis. Finally, we describe the steps that can be used to obtain values for λ and ξ from experimental measurements. SI units are used throughout this paper.

II. EFFECT OF VORTEX MOTION ON A MAGNETIC SPIN ECHO

The model we use for the NMR system to calculate the effect of vortex motion on the height of the spin echo assumes that the dynamic response of the NMR system is that of weakly interacting spins in a magnetic field and that the spins do not have any quadrupolar interaction, either because they have no quadrupole moment or because the symmetry of the lattice precludes an electric-field gradient at the site of the nuclei. In another paper[10] we analyze the case when quadrupolar interactions are significant and show that an additional effect is imposed on the echo height that can be evaluated theoretically and experimentally. It will be assumed that the frequency dependence of the vortex motion is quasistatic on the scale of the NMR frequency in the magnetic field.

In the usual spin-echo measurement,[11] starting from an equilibrium nuclear magnetization in the local magnetic field $\mathbf{B}(\mathbf{r})$ a first rf pulse is applied that tips the spins into the plane perpendicular to $\mathbf{B}(\mathbf{r})$. After that, each spin (indexed by j) processes in its local field of magnitude $B_j(t)$ at the instantaneous angular frequency $\omega_j(t)$ given by

$$\omega_j(t)=\gamma B_j(t) , \tag{1}$$

where t is the time and γ is the nuclear gyromagnetic ratio in rad/s T. After a duration τ a second rf pulse is applied that reverses the sense of the precession, and at $t=2\tau$ a spin echo with a height $S_j(2\tau)$ is formed by the spin. The accumulated phase of the spin $\Phi_j(2\tau)$ due to a changing magnetic field is[12]

$$\Phi_j(2\tau)=\gamma\int_0^{\tau}\Delta B_j(t)dt-\gamma\int_{\tau}^{2\tau}\Delta B_j(t)dt \tag{2}$$

with

$$\Delta B_j(t)=B_j(t)-\frac{1}{2\tau}\int_0^{2\tau}B_j(t)dt\ , \tag{3}$$

where $\Delta B_j(t)$ is the instantaneous deviation of the local field from its time average over the interval 2τ. We define $\Delta B_j(t)$ in this way to focus explicitly on the change in magnetic field that has an effect on the height of the echo; the second term on the right of Eq. (3) is subtracted because it has none. The height per spin of the spin-echo signal of a given isochromat $[S_j(2\tau)]$ in terms of the corresponding height without vortex motion $[S_{0j}(2\tau)]$ is

$$S_j(2\tau)=S_{0j}(2\tau)\cos[\Phi_j(2\tau)]\ . \tag{4}$$

If $S_{0j}(2\tau)$ is assumed to be the same for all spins $[S_j(2\tau)=S_0]$, which is equivalent to the spin-phase memory time T_{2j} being the same for all spins, the effect on the entire spin system of a time dependent $B_j(t)$ is given by

$$\frac{S(2\tau)}{S_0}=\sum_j\cos[\Phi_j(2\tau)]\ . \tag{5}$$

Equation (5) will be used to calculate the effect of vortex motion on the echo height. The connection to vortex motion is through the time dependence of the local vortex magnetic field [Eq. (3)] that is caused by vortex motion.

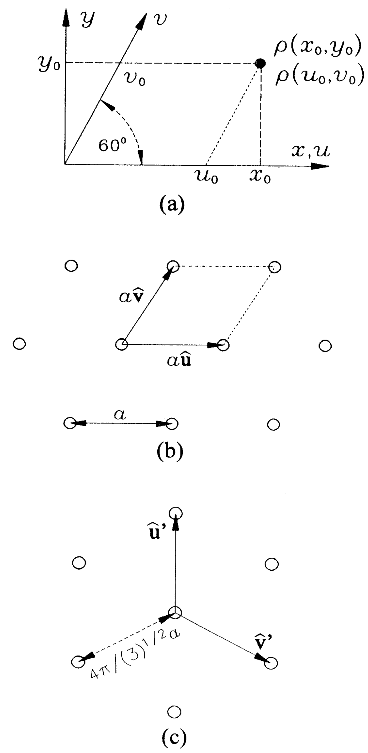

FIG. 1. Vortex lattice and coordinates. (a) Rectangular and hexagonal coordinates. The rectangular coordintes are x,y with unit vectors $\hat{\mathbf{x}},\hat{\mathbf{y}}$ and the hexagonal coordinates are u,v with unit vectors $\hat{\mathbf{u}},\hat{\mathbf{v}}$. (b) Direct triangular vortex lattice. The points of the lattice are indicated by the open circles, the primitive cell is shown by the dashed line, and the nearest-neighbor spacing is a. (c) Hexagonal reciprocal lattice with basis vectors $\hat{\mathbf{u}}',\hat{\mathbf{v}}'$ for the vortex structure. The lattice constants is $4\pi/\sqrt{3}a$.

III. TRIANGULAR VORTEX LATTICE MODEL

It is well established that the equilibrium arrangement of the vortices in the intermediate state of an isotropic type-II superconductor is a triangular lattice with a separation between adjacent points (a) whose dependence on the external field is set by the criterion that there be one quantum of flux (Φ_0) per primitive cell of the vortex lattice.[4] We label the Cartesian and triangular lattice axes by x,y and u,v, respectively. The relations between the unit basis vectors of the Cartesian $(\hat{\mathbf{x}},\hat{\mathbf{y}})$ and triangular $(\hat{\mathbf{u}},\hat{\mathbf{v}})$ lattices are given by [see Fig. 1(a)].

$$\hat{\mathbf{u}}=\hat{\mathbf{x}},\quad \hat{\mathbf{v}}=\tfrac{1}{2}\hat{\mathbf{x}}+\frac{\sqrt{3}}{2}\hat{\mathbf{y}} \tag{6}$$

and the corresponding coordinate transformations are

$$x=u+\tfrac{1}{2}v\ ,\quad y=\frac{\sqrt{3}}{2}v \tag{7}$$

and

$$u=x-\frac{1}{\sqrt{3}}y\ ,\quad v=\frac{2}{\sqrt{3}}y\ . \tag{8}$$

The points of the triangular vortex lattice (lattice constant a) relative to the origin are

$$\mathbf{r}_P=a(l\hat{\mathbf{u}}+m\hat{\mathbf{v}})\ ,\quad l,m=0,\pm1,\pm2,\ldots \tag{9}$$

and the area per primitive cell [parallelogram on Fig. 1(b)] is $(\sqrt{3}/2)a^2$. Since, for the cases of interest here, the penetration of the external field is essentially complete and supported by the vortex lattice, the value of a is given by

$$\bar{B}=\frac{2\Phi_0}{\sqrt{3}a^2}\simeq B_0\ , \tag{10}$$

where $\bar{B}$ is the average field in the superconductor and we restrict B_0 to be somewhat greater than B_{c1}. The difference between $\bar{B}$ and B_0 is a result of the magnetization of the superconductor, which is small as long as B_0 is large enough. In what follows, we will assume that this approximation is sufficient in the context of the problem considered here; when a numerical value of $\bar{B}$ is needed, we will use B_0.

The corresponding reciprocal lattice is shown in Fig. 1(b). Its primitive translations are

$$\hat{\mathbf{u}}'=\frac{2\pi}{a}\left[\frac{2}{\sqrt{3}}\right]\hat{\mathbf{y}} \text{ and } \hat{\mathbf{v}}'=\frac{2\pi}{a}\left[\hat{\mathbf{x}}-\frac{1}{\sqrt{3}}\hat{\mathbf{y}}\right] . \qquad (11)$$

A. Magnetic-field and field-gradient distribution in the triangular vortex lattice

Our calculations will assume an external applied field of magnitude B_0 in the range $B_{c1} \ll B_0 \ll B_{c2}$ oriented along the z direction (B_{c1} and B_{c2} are the upper and lower critical fields, respectively). The currents of the vortex lattice will generate a magnetic field inside the superconductor (B_V) whose equilibrium direction is parallel to B_0. Its average value $\bar{B}$ is close to B_0 and it has a relatively small spatial variation over the vortex lattice. Our calculations will use the London model in reciprocal space to calculate B_V. They follow closely the review by Brandt.[13] The range of validity is $B_{c1} \ll B_0 < 0.25 B_{c2}$.

For the London model, three parameters determine the spatial variation of the magnetic field generated by the vortex lattice: B_0, the superconducting coherence length (ξ), and the penetration depth (λ). The main goal of the analysis presented here is to develop the model used to obtain experimental values for λ and ξ from spin-echo measurements.

The field in the superconductor (B_V) is given by[13]

$$B_v(\mathbf{r})=\bar{B}\sum_{\mathbf{k}}\frac{e^{-K^2\xi^2/2}e^{-i\mathbf{K}\cdot\mathbf{r}}}{1+K^2\lambda^2}=\bar{B}\sum_{\mathbf{k}}\frac{e^{-K^2\xi^2/2}\cos(\mathbf{K}\cdot\mathbf{r})}{1+K^2\lambda^2} , \qquad (12)$$

where $\mathbf{K}$ is a translation of the vortex reciprocal lattice and the exponential is replaced because B_V is real (the coefficients of the harmonic term are an even function of K). The points of the reciprocal lattice to be summed over are

$$\mathbf{K}_{lm}=l\hat{\mathbf{u}}'+m\hat{\mathbf{v}}'=\frac{2\pi}{a}\left[m\hat{\mathbf{x}}+\frac{2}{\sqrt{3}}\left[l-\frac{m}{2}\right]\hat{\mathbf{y}}\right] \qquad (13)$$

and the products involving $\mathbf{K}$ are

$$\mathbf{K}_{lm}\cdot\mathbf{r}=\frac{2\pi}{a}\left[mx+\frac{2}{\sqrt{3}}\left[l-\frac{m}{2}\right]y\right]=2\pi\left[\frac{u}{a}m+\frac{v}{a}l\right] \qquad (14)$$

and

$$K_{lm}^2=\frac{16\pi^2}{3a^2}(l^2+m^2-lm) . \qquad (15)$$

Since our calculations treat B_0,λ, and ξ as independent variables, it is useful to introduce the dimensionless variable β defined by

$$\beta=\frac{4\pi^2\bar{B}\xi^2}{\sqrt{3}\Phi_0} . \qquad (16)$$

Equation (12) can then be written as

$$B_V=\bar{B}\sum_{\text{all } l,m}\frac{\exp\{-\beta(l^2+m^2-lm)\}\cos\{2\pi[(u/a)m+(v/a)l]\}}{1+(8\pi^2\bar{B}\lambda^2/\sqrt{3}\Phi_0)(l^2+m^2-lm)} , \qquad (17)$$

where $\bar{B}$ is the contribution to the sum from the origin of the reciprocal lattice ($l,m=0$). It is also the spatial average of B_V.

It is instructive to write Eq. (17) as

$$B_V=\bar{B}+\frac{\sqrt{3}\Phi_0}{8\pi^2\lambda^2}\sum_{l,m\neq 0}\frac{\exp\{-\beta(l^2+m^2-lm)\}\cos\{2\pi[(u/a)m+(v/a)l]\}}{\sqrt{3}\Phi_0/8\pi^2\bar{B}\lambda^2+(l^2+m^2-lm)} . \qquad (18)$$

The first term in the denominator of the summand is normally small in comparison to the second (it is 1.2×10^{-3} for $\lambda=300$ nm and $B_0=1$ T). Under these circumstances, the variation of the magnetic field relative to its average value, $\Delta B_V=B_V-\bar{B}$, is

$$\Delta B_V\simeq\frac{\sqrt{3}\Phi_0}{8\pi^2\lambda^2}\sum_{l,m\neq 0}\frac{\exp\{-(4\pi^2\bar{B}\xi^2/\sqrt{3}\Phi_0)(l^2+m^2-lm)\}\cos\{2\pi[(u/a)m+(v/a)l]\}}{(l^2+m^2-lm)} . \qquad (19)$$

Equation (19) is a very good approximation that we use to calculate ΔB_V. It shows several useful properties that can be exploited when calculating ΔB_V: (1) the contribution from $\bar{B}$ and ξ scales as the product $\bar{B}\xi^2$, (2) the contribution from λ is proportional to $1/\lambda^2$, and (3) the contributions from λ and $\bar{B}\xi^2$ appear in separate factors.

For comparison with experiments, ΔB_V is calculated on a grid of P points on the u axis and Q points on the v axis of the primitive cell of the vortex lattice. Their coordinates are

$$\frac{u_p}{a}=\frac{p}{P},\quad p=0,\ldots,P-1,\quad \frac{v_q}{a}=\frac{q}{Q},\quad q=0,\ldots,Q-1 . \qquad (20)$$

In such calculations, we normally use $P=Q=50$ for a total of 2500 points in the primitive cell. The number of points needed for the reciprocal lattice sum is determined mainly by the argument of the exponential in Eq. (19). It has converged rather well when l and/or m is large enough (l_{max}) that

$$\beta l_{\max}^2 = \frac{4\pi^2 \bar{B} \xi^2 l_{\max}^2}{\sqrt{3}\Phi_0} \geq 5 \ . \tag{21}$$

Substitution of the magnetic field $\bar{B} \simeq B_0 = 1$ T and the relatively low value $\xi = 2$ nm gives $l_{\max} \geq 10$. In our calculations to fit experiments[2] in NbTi alloy at $B_0 = 1$ T ($\xi \simeq 4$ nm), we have used the conservative value $l_{\max} = 15$.

Below, it is shown that the spatial derivatives of ΔB_V are often important for analyzing the response of a spin echo to vortex motion. These derivatives are calculated at p,q in the primitive cell of the vortex lattice using

$$\frac{\partial \Delta B_V(p,q)}{\partial u} \simeq \frac{P-1}{a}[B_V(p+1,q) - B_V(p,q)] \tag{22}$$

and a similar expression for the other gradient. To make the parametric dependence of the field derivatives more evident, we then use $b(\bar{B}\xi^2,p,q)$ to represent the summation on the right-hand side of Eq. (19), so that

$$\frac{\partial \Delta B_V(p,q)}{\partial u} = \frac{1}{8\pi\lambda^2}\left[\frac{3\sqrt{3}\Phi_0\bar{B}}{2}\right]^{1/2} \delta b_p(\bar{B}\xi^2,p,q) \ , \tag{23}$$

where

$$\delta b_p(\bar{B}\xi^2,p,q) = (P-1)[b(\bar{B}\xi^2,p+1,q) - b(\bar{B}\xi^2,p,q)] \tag{24}$$

and

$$\frac{\partial \Delta B_V(p,q)}{\partial v} = \frac{1}{8\pi\lambda^2}\left[\frac{3\sqrt{3}\Phi_0\bar{B}}{2}\right]^{1/2} \delta b_q(\bar{B}\xi^2,p,q) \tag{25}$$

with

$$\delta b_q(\bar{B}\xi^2,p,q) = (Q-1)[b(\bar{B}\xi^2,p,q+1) - b(\bar{B}\xi^2,p,q)] \ . \tag{26}$$

These expressions are useful because they show how the field derivatives depend on λ, ξ, and $\bar{B}$. Also, once the values of ΔB_V (or b) are calculated on the grid, it is a simple step to obtain the two derivatives from them.

Examples of ΔB_V and $\partial B_V/\partial u$ using the relations discussed in this section are shown in Fig. 2.

B. Change of the local magnetic field by vortex motion

Now we describe the calculation steps for the effect of quasistatic vortex motion on the height of the nuclear spin echo in two limiting cases. If the displacement of the vortex lattice is comparable to or larger than the vortex lattice constant a, one simply calculates B_V for the appropriate displacements and uses them with Eq. (3). In this case, points outside the primitive cell are translated back into it with a lattice translation. When doing this calculation, Eqs. (7) and (8) are used to convert from the triangular vortex lattice to the Cartesian laboratory coordinates.

If, however, the displacement $d\mathbf{r}$ is small enough compared to a, the change can be calculated using the small-displacement approximation:

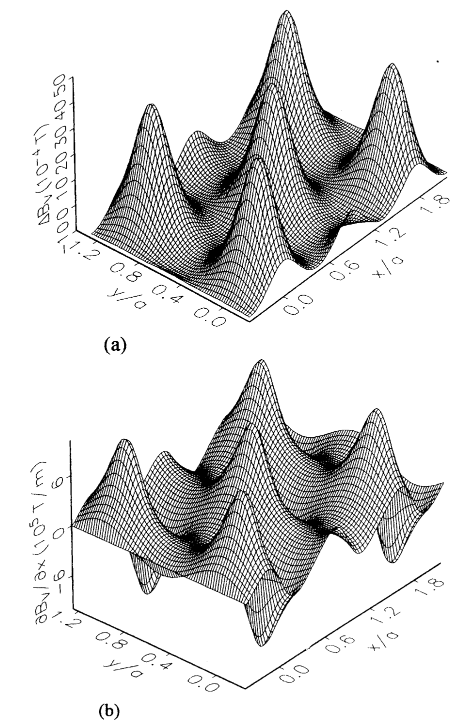

FIG. 2. (a) Local magnetic field of the vortex lattice calculated from Eq. (19). (b) $\partial B_V/\partial x$ obtained from Eq. (22) and (a). The parameters used are $B_0 = 1$ T, $\lambda = 300$ nm, and $\xi = 4$ nm.

$$dB_V = \nabla B_V \cdot d\mathbf{r} = \frac{\partial B_V}{\partial x}dx + \frac{\partial B_V}{\partial y}dy \ . \tag{27}$$

In this case, a matrix of values for ∇B_V can be calculated once and further evaluation done through the inner product with $d\mathbf{r}$. It is helpful to use the triangular vortex lattice as the basis for the corresponding numerical calculation and to transform the result to rectangular coordinates using

$$B_V(x,y) = B_V[u(x,y), v(x,y)] \ , \tag{28}$$

$$\frac{\partial B_V}{\partial x} = \frac{\partial B_V}{\partial u} \ , \quad \frac{\partial B_V}{\partial y} = -\frac{1}{\sqrt{3}}\frac{\partial B_V}{\partial u} + \frac{2}{\sqrt{3}}\frac{\partial B_V}{\partial v} \ , \tag{29}$$

and

$$dB_V = \frac{\partial B_V}{\partial u}dx + \left[\frac{2}{\sqrt{3}}\frac{\partial B_V}{\partial v} - \frac{1}{\sqrt{3}}\frac{\partial B_V}{\partial u}\right]dy \ . \tag{30}$$

Equation (30) is used to calculate dB_V using the vortex displacement in Cartesian components. The numerically computed values of the derivatives in the triangular vortex lattice are obtained with Eqs. (23) and (25).

IV. SINUSOIDAL FIELD ROCKING

In this section we calculate the reduction of the spin echo caused by rocking the magnetic field sinusoidally through a small angle. It is done with a small field $\delta B_\perp(t) = \delta B_\perp \cos(\omega t)$ that is applied perpendicular to the large, static applied field B_0. The resultant rocking angle (ψ) is

$$\psi(t)=\frac{\delta B_\perp}{B_0}\cos(\omega t)=\psi_0\cos(\omega t)\ . \tag{31}$$

Corresponding to this angle there will be a local maximum displacement $\Delta\mathbf{r}_j(t)$ of the vortex lattice that depends on the geometry of the sample and the extent to which the vortex lattice alignment follows the rocking of the magnetic field. For our calculation it will be assumed that the alignment of the vortex lattice follows perfectly and instantaneously the magnetic-field rocking (quasistatic limit). Also, it will be assumed that the midpoint of each vortex in the sample is the fixed point about which the vortex lattice rocking occurs.[14] Finally, it will be assumed here that the sample dimensions and ψ_0 are both small enough that the local motion of the vortex lattice is small compared to their separation; i.e., $\Delta r_j \ll a$. With these assumptions, the small-displacement approximation is used to calculate the local change in the magnetic field:

$$B_j(t)-B_j(0)=\nabla B_j\cdot\Delta\mathbf{r}_j\cos(\omega t)\ , \tag{32}$$

which gives

$$\Phi_j(2\tau)=\gamma\nabla B_j\cdot\Delta\mathbf{r}_j\left[\int_{t-\tau}^{t}\cos(\omega t)dt-\int_{t}^{t+\tau}\cos(\omega t)dt\right] \tag{33}$$

when the origin of time is placed at the second pulse. By performing the integration, making the substitution $\omega t=\phi$ for the phase of the rocking motion, and writing

$$\nabla B_j\cdot\Delta\mathbf{r}_j=\Delta r_j\left[\frac{\partial B_j}{\partial x}\cos\theta+\frac{\partial B_j}{\partial y}\sin\theta\right] \tag{34}$$

(θ is the angle between $\Delta\mathbf{r}_j$ and the x direction of the vortex lattice), we obtain

$$\Phi_j(2\tau)=\frac{4\psi_0}{\omega}\sin\phi\sin^2\left[\frac{\omega\tau}{2}\right]\left[\frac{\Delta r_j}{\psi_0}\right]\times\gamma\left[\frac{\partial B_j}{\partial x}\cos\theta+\frac{\partial B_j}{\partial y}\sin\theta\right]\ . \tag{35}$$

Thus the reduction of the spin-echo height for the jth spin is

$$\frac{S_j(2\tau)}{S_{j0}}=\cos\left[\frac{4\psi_0}{\omega}\sin\phi\sin^2\left[\frac{\omega\tau}{2}\right]\left[\frac{\Delta r_j}{\psi_0}\right]\times\gamma\left[\frac{\partial B_j}{\partial x}\cos\theta+\frac{\partial B_j}{\partial y}\sin\theta\right]\right]\ . \tag{36}$$

To calculate the effect of all the spins requires the explicit specification of Δr_j, numerical values for the spatial derivatives of the vortex field, averaging over θ to account for a "polycrystalline" vortex lattice, and summing over all the nuclei. For the latter, we index the nuclei by their position $\mathbf{r}_S$ in the sample with the volume V (the subscript S distinguishes the sample coordinates from the direct space coordinates of the vortex lattice). Because there are no expected correlations between the field derivatives, θ, and $\mathbf{r}_S$, the summations for each of them can be carried out independently. The part from the field derivatives of the vortex lattice is done by averaging over N discrete points p,q of the vortex lattice primitive cell and that associated with θ is computed by averaging over it. These steps then lead to

$$\frac{S(2\tau)}{S_0}=\frac{1}{\pi NV}\int_V d^3\mathbf{r}_S\times\int_0^{\pi}d\theta\sum_{p,q}\cos\left[\zeta\left[\frac{\Delta r(\mathbf{r}_S)}{\psi_0}\right]G(p,q,\theta)\right]\ , \tag{37}$$

where we have introduced the scaling variable ζ and modeled the vortex field term with $G(p,q,\theta)$ using

$$\zeta=\frac{4\psi_0}{\omega}\sin^2\left[\frac{\omega\tau}{2}\right]\sin\phi\ , \tag{38}$$

$$\zeta\simeq\omega\psi_0\tau^2\sin\phi,\quad \omega\tau\ll 1\ , \tag{39}$$

$$G(p,q,\theta)=\gamma\left[\frac{\partial B(p,q)}{\partial x}\cos\theta+\frac{\partial B(p,q)}{\partial y}\sin\theta\right]\ . \tag{40}$$

One significant result of this model is that the parameters determined by the vortex lattice (through G), the sample dimensions, and ζ appear as a product in the argument of the cosine. This means that the model can be tested experimentally to determine if this form of scaling is obeyed. Some preliminary measurements[2] indicate that it is.

An implicit assumption of the spatial part of the integration done in Eq. (37) is that all parts of the sample contribute equally to the spin echo. This will be true if its dimensions are small enough that the penetration of the rf magnetic field is nearly complete. If this is not the case, a spatial weighting factor to account for incomplete penetration can be added to the integrand. Such a weighting factor does not, however, change the scaling behavior implied by Eq. (37).

Cylindrical sample geometry

Now consider the integral over V for a sample that is a thin wire. We assume it is thin enough that the rf field penetration is complete. The geometry of the field rocking, which is that used in some of our experiments,[2] is indicated on Fig. 3. A cylindrical sample of radius d and length l has its axis parallel to the z_S axis (vertical) of the

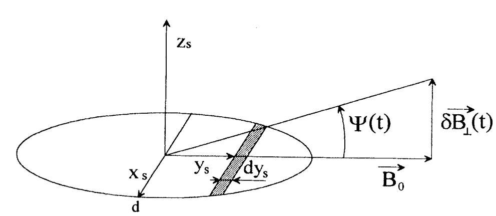

FIG. 3. Cylindrical sample field rocking geometry.

laboratory reference frame and its Cartesian coordinates in the horizontal plane are x_S, y_S centered on the axis of the cylinder. The large, static magnetic field B_0 is applied in the horizontal plane along the y_S direction and the small rocking field $\delta B_\perp(t) = \delta B_\perp \cos(\omega t) \ll B_0$ is applied along the z_S axis. Its effect is to oscillate the field up and down through the angle $\psi(t)$ given by Eq. (31).

The amplitude of the motion at y_S in a thin circular slab perpendicular to the z_S axis is $\Delta r(y_S) = y_S \psi_0$ and the area of the slab with the same amplitude of motion is $d\sqrt{1-(y_S/d)^2}dy_S dz_S$. It then follows that $S(2\tau)/S_0$ for this cylindrical geometry is

$$\frac{S(2\tau)}{S_0} = \frac{4}{\pi^2 N d} \int_0^\pi d\theta \sum_{p,q} \int_0^d dy_S \left[1 - \left[\frac{y_S}{d}\right]^2\right]^{1/2} \times \cos[y_S \zeta G(p,q,\theta)] \ . \quad (41)$$

Integration over y_S gives

$$\frac{S(2\tau)}{S_0} = \frac{2}{N\pi} \int_0^\pi d\theta \sum_{p,q} \frac{1}{\zeta d G(p,q,\theta)} J_1[\zeta d G(p,q,\theta)] \ , \quad (42)$$

where J_n is the nth-order Bessel function of the first kind. One of the important properties of Eq. (42) shows explicitly the scaling behavior with regard to ξ, G, and the sample dimensions (d) mentioned above.

The approach used to evaluate Eq. (42) depends on whether the argument of J_1 is small or large. When it is kept less than 1.7 by limiting ζ, ψ_0, and d ("small-argument approximation"), the approximation $2J_1(z)/z \simeq 1-(0.92/8)z^2$ contributes an error of less than in 3.4% of the maximum value of the change in the echo height, $1-2J_1(z)/z$. The factor 0.92 is a correction to the series expansion of $J_1(z)$ to improve the fit over a broader range of z. For this approximation, $1-2J_1(z)/z$ is 0.3 at $z=1.7$. Using this approximation and integrating over θ gives for the fractional change in the echo height

$$1 - \frac{S(2\tau)}{S_0} = \frac{(0.92)\sqrt{3}\zeta^2 d^2 \gamma^2 \Phi_0 \overline{B}}{512\pi^2\lambda^4} \times \langle (\delta b_p)^2 + (\delta b_q)^2 - \delta b_p \delta b_q \rangle \ , \quad (43)$$

where the angular brackets denote an average of the corresponding dimensionless field-gradient terms over the primitive cell of the vortex lattice. Equation (43) is especially informative because it shows, within its range of validity, the way in which the echo height reduction scales with ζ, d, and λ. Also, only the second moments of the field gradients in the vortex lattice affect the result. Higher-order terms in a polynomial fit include higher moments of the field gradient and higher powers of λ. They will, of course, preserve the scaling behavior associated with ζ.

Even though the dependence of $1-S(2\tau)/S_0$ on $\overline{B}$ and ξ is less evident, there are some important comments to be made in regard to it. The coherence length appears only as part of the factor $\overline{B}\xi^2$ in the argument of the exponential in Eq. (19); i.e., in the terms δb_p and δb_q. When $\overline{B}\xi^2$ is small, the corresponding cutoff has little or no effect on the sum, which happily converges even without it. The consequence is that for a low enough magnetic field, $S(2\tau)/S_0$ is independent of ξ and the only remaining dependence on $B_0(\simeq\overline{B})$ is that shown explicitly in the prefactor in Eq. (43). At high magnetic fields, $S(2\tau)/S_0$ depends on both λ and ξ. Thus, in principle, from a measurement at low field one can obtain a value for λ which may then be combined with a high-field measurement to obtain ξ. In this way, it should be possible with spin echoes to measure the two fundamental lengths of the superconducting state through their effect on the magnetic-field distribution in the sample. Numerical simulations that we do not discuss in detail here[15] support this statement. This possibility can be understood qualitatively on the following basis. For small displacements of the vortex lattice, the spin echo is sensitive mainly to the gradient of the magnetic field, which is largest close to a vortex core (see Fig. 2). Away from the core, the gradient is determined mainly by λ. When $\overline{B}$ is small, only a small fraction of the vortex lattice is close to the cores and the volume-averaged square of the gradient is determined by λ. For large values of B_0, a substantial fraction of the sample is close to the vortex cores, with the result that a considerable part of the effect on $S(2\tau)/S_0$ is governed by ξ.

When the small-argument approximation is not valid, Eq. (31) is calculated numerically for each point in the primitive cell of the reciprocal lattice using as parameters ζ, $\overline{B}$, d, ξ, and λ. For most experiments, the first three are known and the last two obtained by adjusting them to fit the experimental result. As discussed above, when $\overline{B}\xi^2$ is small enough, $S(2\tau)/S_0$ is independent of ξ, so that the only adjustable parameter is then λ.

It should be pointed out that in field rocking measurements there is another contribution to the attenuation of the spin echo which may occur that has nothing to do with superconductivity. When present, it must be taken into account in the analysis of the experiments. It occurs when the nucleus under study has an electric quadrupole moment and is located in an electric-field gradient.[11] Under these circumstances, rocking the field also changes the quadrupolar interaction in a way that may further reduce the height of the spin echo. A detailed discussion of this effect is beyond the scope of this paper. A preliminary analysis of this effect[10] indicates, however, that its effect on the spin echo has a scaling behavior similar to that for vortex motion presented in this paper and that it is a factor which multiplies the echo reduction by vortex motion. It can be evaluated experimentally by measuring it in the normal state where vortices do not occur.[10]

V. DISCUSSION AND CONCLUSIONS

The calculations presented here have left out many features that should be part of a more complete treatment of the problem covered in this paper. Here, we discus some of them that should be treated in subsequent work.

One of the most obvious is to analyze other geometries, such as a slab or a film with various alignments of the magnetic field. Another point that could be important is our approximation that T_2 is the same for all spins being measured. When the NMR line has a significant structure, different parts of the spectrum may have different values of T_2.

Some of the most interesting further developments are related to the superconducting state itself. The model should be expanded to include anisotropic superconductors. We believe that, by exploiting polycrystalline or single-crystal samples and different alignments of the magnetic field, substantial progress can be made in microscopic measurements of the anisotropies of the fundamental lengths in such materials, which are of enormous current interest, both for technological reasons and for a fundamental understanding of the pairing interactions responsible for superconductivity. One should also go beyond the perfect triangular lattice to include distortions induced by pinning centers, etc.

Another topic for further development is a more complete treatment of the vortex motion. A key approximation of this paper has been that the vortex motion follows the change in alignment of the external field exactly and symmetrically around the symmetry center of the sample. Even for symmetrical samples, it is reasonable to expect that randomly placed pinning centers and surface effects should challenge this approximation to some extent, with the consequence that the actual motion has important effects that are not included in the model at its present level of development. One effect is that in the elastic regime of vortex lattice distortion there is a restoring force[16] that should reduce the displacement of the vortices to less than what it would be if the external field were followed freely. Another consideration is nonreversed vortex motion associated with pinning centers. Another further development would be to go beyond the quasistatic approximation for both the vortex dynamics and the nuclear spin dynamics.

The focus of this paper has been on small changes in the *direction* of the applied magnetic field. A change in its *magnitude* should introduce the additional features of vortex nucleation and annihilation. Such measurements are easily accessible experimentally, but they may be rather difficult to relate to a microscopic model.

In conclusion, we have presented the details of a model that calculates the reduction of the height of a NMR spin echo caused by driven motion of vortices in a type-II superconductor. Application for the model using a triangular vortex lattice to small-amplitude sinusoidal field rocking exhibits a scaling behavior in the relevant variables that relates experimental measurements to λ and ξ. The details of the model are worked out for right circular cylindrical sample geometry with the applied field perpendicular to the sample axis.

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